Errata for Amacher, Ollikainen, and Koskela, Economics of Forest Resources, The MIT Press, 2009

Chapters 1-6, 8, 10-12

This will be a continuously evolving document. Please send queries to either Greg Amacher (gamacher@vt.edu) or Markku Ollikainen (markku.ollikainen@helsinki.fi). An update of this errata adding the remaining chapters 7 and 9 will be posted during Spring of 2010.

A note about first order conditions in Chapters 2-3, and 5, 10:

In many cases, the nonzero term $(1-e^{-rT})^{-1}e^{-rT}$ that is left premultiplying the entire Faustmann first order condition after its derivation has been intentionally deleted in order to ease interpretation. In these cases, we used the language: the first order condition can be 'expressed' or 'reexpressed' as…..

Similar instances occur in other chapters such as Chapter 10 when Faustmann like first order conditions are derived for interpretation.

Chapter 2[1]

--Page 26, equation in text below equation 2.13: A *p* is missing in the second term.

--Page 27, equation 2.15: the correct equation is:

$$0 = [f'(T) - rf(T) - rf(T)e^{-rT}(1-e^{-rT})^{-1}]dp + r(1-e^{-rT})^{-1}dc - \left(pf(T) + V + r\frac{dV}{dr}\right)dr$$
$$+[pf''(T) - rpf'(T)]dT$$

--Page 27, equations 2.16a – 216c: Denominators are incorrect and should read: $pf''(T) - rpf'(T)$.

--Page 27, equation 2.16b should equal

$$\frac{dT^F}{dr} = \frac{pf(T) + V + r\frac{dV}{dr}}{pf''(T) - rpf(T)}$$

[1] These errors were pointed out by Richard (Bin) Mei.

--Page 27, text discussing price comparative static result, the first order condition should read

$$f'(T) - rf(T) - rf(T)e^{-rT}(1-e^{-rT})^{-1} = -p^{-1}[rc(1-e^{-rT})^{-1}] < 0$$

--Page 27, Real Interest Rate Effect. There is no error, but we present the derivation for clarity:

$$V_{Tr} = -pf(T) - V + \frac{r}{(1-e^{-rT})^2}\left[Tpf(T)e^{-rT}(1-e^{-rT}) + Te^{-rT}(pf(T)e^{-rT} - c)\right]$$

Rearrange next to obtain: $-(pf(T)+V) + \frac{rTpf(T)e^{-rT}}{(1-e^{-rT})} + \frac{rTe^{-rT}}{(1-e^{-rT})}V$

And to find that $-(pf(T)+V) + \frac{rTe^{-rT}}{(1-e^{-rT})}(pf(T)+V) = -(pf(T)+V)\left[1 - \frac{rTe^{-rT}}{(1-e^{-rT})}\right]$.

Divide the last bracket term by e^{rT} to obtain

$$-(pf(T)+V)\left[1 - \frac{rT}{(e^{rT}-1)}\right] = -(pf(T)+V)\left[\frac{e^{rT}-1-rT}{(e^{rT}-1)}\right].$$

--Page 31, equation (2.19b). There is no error but this result could use some clarification. The Faustmann formula in the presence of the yield tax is defined by equation (2.19a), and we derive the comparative static effect in terms of the price, *p*, that does not include taxes. The $1-\tau$ term in the denominator of equation (2.19b) reflects the fact that the result is stated in terms of price. A similar type of result appears in chapter 3, equation (3.10). To see this more formally, differentiate 2.19a with respect to *T* to obtain the conventional harvesting rule:

$$p(1-\tau)f'(T) - rp(1-\tau)f(T) - r\left[p(1-\tau)f(T)e^{-rT} - c\right](1-e^{-rT})^{-1} = 0$$

Now, take derivatives separately with respect to the tax τ and price p to obtain,

for the tax:

$$-p\left[f'(T) - rf(T) - rf(T)e^{-rT}(1-e^{-rT})^{-1}\right] > 0$$

and for the timber price

$$(1-\tau)\left[f'(T)-rf(T)-rf(T)e^{-rT}(1-e^{-rT})^{-1}\right]<0.$$

This should make tax result visible: by multiplying the latter term by $p/(1-\tau)$ we obtain the former.

--Page 32. (1-θ) has been unintentionally dropped from the site productivity tax derivative. The first order condition (1-θ)V_T = 0 indeed simplifies to V_T = 0 as we have in the text, because the term (1-θ) drops out of the first order condition for T, $V_{T\theta}$ = 0 . Thus, as we note the site productivity tax does not affect rotation age.

--Page 39, reference to Faustmann formula in first paragraph. The formula should read: $V=(1-e^{-rT})^{-1}[p_f F(T;q)e^{-rT}-c]$. Also, there is a typo concerning the interest rate impact. Differentiating with respect to *r* and evoking the envelope theorem produces $-Te^{-rT}(1-e^{-rT})^{-2}\left[pf(T)(1-e^{-rT})+(pf(T)e^{-rT}-c)\right]<0$.

Chapter 3[2]

--Page 56, text before equation (3.6). The term $e^{-rT}W$ should equal $e^{-r(T-A)}W$

---Page 56, 8th line after equation (3.6). 'Intersecting from above' should read 'intersecting from below'

--Page 57, equation (3.7b), equation should read: $\frac{dT^H}{dr}=\frac{pf(T)+V+r\frac{dV}{dr}+E+r\frac{dE}{dr}}{W_{TT}}$

--Page 57, line 4 of text concerning the price effect should read $F(T)-rE\geq 0$.

--Page 57, line 1 of text concerning real interest rate effect. The "Faustmann part" should equal $pf(T)+V+r\frac{dV}{dr}$.

--Page 56, line 5 of text concerning real interest rate effect. An exponent -1 is missing in the term $(1-e^{-rT})$ in front of the integral term.

[2] These errors were pointed out by Richard (Bin) Mei.

Page 57, text concerning real interest rate (line 6). A minus one is missing for the last term: it should be $(1-e^{-rT})^{-1}\int_0^T sF(s)e^{-rs}ds$. When collecting the terms, the entire interest rate effect for amenities should be $E + r\frac{dE}{dr} = \left[1 + r - rT - rTe^{-rT}(1-e^{-rT})^{-1}\right]E$, or like we have $\left[1 + r - rT - rT(e^{rT}-1)^{-1}\right]E$. We can re-express it as: $(\left[1+r\right)(e^{rT}-1) - rTe^{rT}\left](e^{rT}-1)^{-1}$ (that is, (1+ *r*) should multiply only the first term). Finally, as demonstrated, the term $E + r\frac{dE}{dr} = 1 + r - rT - rTe^{-rT}(1-e^{-rT})^{-1}$ should be positive.

--page 60, equation (3.10). The derivation here follows along the same lines as for equation (2.19b) on page 31 described for chapter 2 in this errata.

--Page 71, unnumbered equation above equation (3.31a). The discount rate ρ is missing in the exponent of the first RHS term.

--Page 74, equation (3.37). This should read

$$\hat{W}_T = (1-e^{-rT})^{-1}e^{-rT}\{f'(T)(p+q\alpha\beta) - rf(T)[p - q\alpha(1-\beta)] - r\hat{W}\} = 0$$
$$\Leftrightarrow\ f'(T)(p+q\alpha\beta) + rq\alpha(1-\beta)f(T) = rpf(T) + r\hat{W}$$

The LHS represents the marginal benefit (timber harvest, sequestered carbon, and interest on delayed carbon release). The RHS represents the opportunity cost (interest on standing timber and land rent).

--Page 69, bottom of the page. The text should note that the interest rate has a negative effect on the indirect net present revenue function per parcel.

Chapter 4[3]

--Page 80, bottom of box 4.1. Text should have as equations,

$1/Q + 1/(Q-K) = bdt$, and $\int_{Q_0}^{Q(t)}[1/Q + 1/(Q-K)]dQ = \int_0^t bdt$

Notice that an x has replaced the K in these formulas.

--Page 81, line 4 in box 4.1. The correct equation is: $Q(t) = K(1+ce^{-bt})^{-1}$

[3] The third error was pointed out by Selmin Creamer, and all other errors were raised by Richard (Bin) Mei.

--Page 83, equation (4.6b), first order condition is missing '=0' on RHS

--Page 85, equation (4.10). On the RHS, the term $x_{p_1}^{S}$ should equal $x_{p_1}^{1S}$.

--Page 86, text below section title 4.1.4. The text price should read: $\hat{p}_i = p_i(1-\tau_i) - u_i$. Further, in the text below this, c_1 and c_2 should have been written as functions of x_1 and x_2, i.e., $c_1 = \hat{p}_1 x_1 - a_1 - s$, to be consistent. Also, an 'I' subscript is missing on the tax rate here since we define it as 'periodic.'.

--Page 87, line 15 in Box 4.2. Should read $k_1 = 70$ m^3.

--Page 89, equation (4.13a). Should read $x_{\tau}^{1S} = x_{a}^{1S} = 0$.

--Page 91, equation (4.17). This should read: $Rp_1 - (1+g')p_2 = \dfrac{v'(k_1)}{\beta u'(c_2)}$, and under constant prices equation (4.17) would be written as $p(r-g') = \dfrac{v'(k_1)}{\beta u'(c_2)}$.

--Page 101, line 3 below equation (4.27d). The first term in the $\dfrac{\partial V_{t+1}}{\partial b_{t+1}}$ equation should be

$$-u'(c_{1t+1})\tau_b p_{t+1}^{*}.$$

--Page 103, line 4 below equation (4.29b). Should read "because ϕ is less than one."

--Page 106, equation (4.31). the choice variables for this optimization problem should be x_1 and x_2.

--Page 107, equation (4.34). This equation should read: $x^{2S} = Q(1+f) - x^{1S}(1+f) - k(Q - x^{1S})^2/2$.

--Page 92, text above equation (4.18a). When solving for the comparative statics results for current and future harvesting, x_1 and x_2, the first-order conditions (4.16b) and (4.16c) should be referred to in the text. Also, on the top of the next page, we should refer to the Jacobian and not the Hessian matrix.

--Page 93, text just before equation (4.19). the reference Koskela and Ollikainen 1989 should be 1997b in the references section.

--Page 105, line 3 below title for section 4.4.1. The reference should be Amacher and Brazee 1997a.

Note: here is a quick example of the application of comparative statics to current and future harvesting in the basic model of this chapter:

Given first-order conditions:

$$U_x = p_1 - v'(k_1) - (1+g')\beta v'(k_2) = 0$$

$$U_z = p_2 R^{-1} - \beta v'(k_2) = 0$$

The second order derivatives are

$$U_{xx} = v''(k_1) + (1+g')^2 \beta v''(k_2) + g''\beta v'(k_2) < 0$$

$$U_{zz} = \beta v''(k_2) < 0$$

$$U_{xz} = (1+g')\beta v''(k_2) < 0$$

Thus, the determinant of the Jacobian matrix is:

$$\Delta = \begin{vmatrix} U_{xx} & Uzx \\ U_{xz} & U_{zz} \end{vmatrix} = U_{xx}U_{zz} - (U_{zx})^2 > 0$$

Differentiate the first order conditions with respect to p1:

$$U_{xp_1} = 1$$

$$U_{zp_1} = 0$$

Using the determinant we obtain the effect of first period price on current and future harvesting

$$x_{p_1} = \Delta^{-1}\begin{vmatrix} -1 & U_{xz} \\ 0 & U_{zz} \end{vmatrix} = -\Delta^{-1}U_{zz} = -\frac{\beta v''(k_2)}{\Delta}$$

and

$$z_{p_1} = \Delta^{-1}\begin{vmatrix} U_{xx} & -1 \\ U_{zx} & 0 \end{vmatrix} = \Delta^{-1}U_{zx} = \frac{(1+g')\beta v''(k_2)}{\Delta}$$

Chapter 5[4]

--Page 118, equation at bottom of page. The first term in the brackets should be $pf(T)$ instead of $pf'(T)$.

--Page 128, equation (5.18a), The notation x and z should be replaced by x_1 and x_2.

--Page 129, text just below (5.20b). The equation reference is wrong, it should read "…future harvesting condition in (5.20b)."

-- P129, in the text below equation 5.21. The text should read: "Moreover, $p_1 - R^{-1}p_2(1+g')$ is larger in absolute value…".

--Page 132, text below equation (5.27). Amacher et al. (2002) in references should be referenced here.

--P138, equation (5.36b). The equation should have *m* as subscripts, i.e.,

$$G_m = \alpha(U_T - X_T)T_m^H - \alpha/(1-e^{-rT}).$$

--Page 139, second equation in the text below equation (5.38). The subscripts should equal *m*, i.e.,

$$G_m = \tau(Y_T - X_T)T_m^H - \tau/(1-e^{-rT}).$$

[4] Errors here were pointed out by Richard (Bin) Mei.

--Page 142, equation (A5.1b), the second term in brackets on the RHS of this equation should equal

$$\tau_b \, pb_{\tau_b} .$$

--Page 143, equation A5.3b. The last two terms on the RHS should be $+ \, p\varepsilon$.

-- Page 143, equation A5.4b. The terms in the first bracket of the RHS of this equation should be

$[v(k_{1t}) + \beta v(k_{2t}) + \phi(v(k_{1t+1}) + \beta v(k_{2t+1}))]$.

--P144, equation A5.7b. The last term in brackets on the RHS should be $p\varepsilon$.

Chapter 6[5]

--Page 158, text below equation (6.17). Should read $-c''(Q) < 0$".

--Page 160, line 3 in proposition 6.2, should read "… differs from the non-corruption case…".

--Page 169, text below equation (5.22b). Should read $d\alpha / dn = \alpha_n + \alpha_\theta \theta'$.

--Page 166, equation (6.20b). A note for clarification: As discussed in the text, this is Reed's (1984) interpretation of the Faustmann model under risk of catastrophic loss. Reed modeled fire loss, but we model timber trespass and loss of rents from it. Reed defined Net Present Value slightly differently because he assumed that initial planting costs are zero, and then planting is paid for at the second rotation and beyond. In chapter 6 there is no difference in marginal cost of planting after trees are stolen, but in chapter 10 there is a difference in planting marginal cost if a fire arrives compared to planting if a fire does not arrive, and thus the formula in chapter 6 and 10 are slightly different to reflect this.

[5] Errors here were pointed out by Richard (Bin) Mei.

Chapter 8 (no known errors)

Chapter 10

--Page 272. In the two equations at the bottom of the page, the last term in these equations has the wrong subscript, that is, *J* should be defined as: $J = E\left\{\sum_{n=1}^{\infty}(e^{-rX_1}e^{-rX_2}\ldots e^{-rX_{n-1}})\right\}E(e^{-rX_n}Y_n)$.

--Page 273, equation (10.5). The upper limit of first integral should be ∞ instead of *X*.

--page 274, equations 10.8a-10.8b. There is no typo here, but note that the nonzero term $\frac{r+\lambda}{r[e^{(r+\lambda)T}-1]}$ has been deleted to facilitate discussion. A similar simplification was made to the first order conditions for (10.12a) – (10.12b).

Chapter 11

--Page 309-310. In the comparative statics derivatives, 'c' should be replaced bt c1 and 'μ' by 'α'

--Page 311. Equation 11.22, $EX_X(e^{-rT})$ should read $E_X(e^{-rT})$ in the numerator.

--Page 314, Second paragraph. The drift parameter 'μ' should be replaced by the symbol 'α'.

Chapter 12

--Page 328. The text at the bottom of the page should state $g[V(t)]$ is growth in the forest stock.

--Page 331. In line seven from the top of the page, the term defining land rent *a* on the RHS .

Should have $f_h(.)$ in place of $f_V(.)$.

Mathematics Appendix

--Page 358, line 5 in first paragraph. The first order condition should read: $V_x(x;\theta) + \lambda = 0$ and $\lambda x = 0$.