GROWTH MODELS FOR LONG TERM FORECASTING OF TIMBER YIELDS

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GROWTH MODELS
FOR
LONG TERM FORECASTING OF
TIMBER YIELDS

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PREFACE

In recent years, the area of forest growth modeling has developed rapidly. Consequently, "Growth Models for Tree and Stand Simulation" was chosen as the theme for two meetings in 1973 of IUFRO Subject Group S4.01 Mensuration, Growth and Yield. Rapid development continues in the application of sophisticated analytical techniques and computing technology to forest growth modeling problems. Progress is also being made in the application of these growth models in forest management decision making; therefore, the leader of IUFRO Subject Group S4.01 decided to convene a meeting with focus on the most recent growth modeling advances and with particular emphasis on applications of growth models in forest management. The meeting was held on the campus of Virginia Polytechnic Institute and State University, Blacksburg, Virginia, USA, in October, 1977. Papers presented at the session in Blacksburg form the contents of this book and are arranged in alphabetical order by last name of the senior author. It should be noted that this volume is a documentation of the material presented at the meeting; we have made no attempt to revise or edit (other than minor format changes) manuscripts submitted by the authors.

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Many individuals contributed to making this IUFRO meeting a success. We wish to acknowledge especially the help of Dr. Otis F. Hall, Head, Department of Forestry and Forest Products, Virginia Polytechnic Institute and State University. In addition to working on the technical papers session held in Blacksburg, Dr. Hall assumed primary responsibility for organizing the field trip to Jacksonville, Florida.

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COVER: Kaingaroa Forest, Rotorua Conservancy, New Zealand
(Photo by J. H. Johns, New Zealand Forest Service)

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PYMOD: A FORECASTING MODEL FOR CONIFER PLANTATIONS IN THE TROPICAL HIGHLANDS OF EASTERN AFRICA

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Summary

Softwood plantations exceeding 200,000 ha exist in Kenya, Uganda, Tanzania and Malawi, mostly comprised of *Pinus patula*, *P. radiata* and *Cupressus lusitanica*. A model to forecast growth and management of this resource is described. Height prediction is by an extended log/reciprocal function. Diameter increment is determined by stand height, tree dominance, and stand basal area relative to maximum basal area. Volumes are determined by a taper function compatible with existing total volume estimators. Scheduling of cutting on compartments is by a volume quota goal satisficing algorithm that may delay thinnings and clear felling and which gives priority to compartments most overdue for cutting. The model is in a FORTRAN computer program called PYMOD.

Introduction

Extensive forest plantations, comprising mainly *Pinus patula*, *Pinus radiata* and *Cupressus lusitanica* have been established in the highland regions of Kenya, Uganda, Tanzania and Malawi at altitudes varying from 1500 m to 3000 m. The total extent of the resource is difficult to gauge, but is now well in excess of 200,000 ha. Apart from a number of sawmills already functioning, there is an existing pulp mill in Kenya, and firm proposals for mills in Malawi and Tanzania. It is clear that the margin between the available increment of the resource, and its actual utilization may be expected to close rapidly over the next decade; this in turn will require a radical change in methods of scheduling harvesting operations to utilize the resource most fully. It also requires more accurate and flexible methods of yield prediction than have hitherto been available.

To assist in this process, the United Kingdom Ministry of Overseas Development has financed a series of research projects related to growth and yield studies in these countries. The present paper is in the nature of an interim report and discussion of a simple growth prediction and scheduling model intended to be made available for operational use in Kenya and Tanzania during 1978 that has been developed under one of these projects.

The emphasis in the model is on conceptual and programming simplicity and robustness; together with a measure of open-endedness in the design which should facilitate the continuing adaptation of the model to differing needs. In order to achieve these aims, a measure of sophistication has had to be abandoned. The stand growth model is, in the terminology of Munro (1974), a distance-independent model whose closest antecedents are the work of Clutter and Allison (1974), and Gibson *et al*. (1970, 1971). The scheduling algorithm is based on simple decision rules applied so as to satisfy a single constraint, *i.e*. harvested volume quota; and has again been evolved from the algorithm used by Gibson *et al*. for regulated cutting in the FORSIM model.

Description of the main features of the model

It is a primary assumption of the present model that each forest compartment is monospecific and even-aged. Each stand is defined in the model by the following variables:-

COMP (3)	A 3-word compartment identity. The first two words are fixed throughout a simulation. The last one may be modified and refers to subunits termed logging units.
SPP	Contains a species code for the stand
AREA	Defines the stand area in hectares
SI	Gives the stand site index, defined as the mean height of the 100 largest d.b.h. stems per hectare (dominant height) at an index age of 15.
PYR	Year of establishment of the stand
HDOM	Stand dominant height
STK	Stocking, as stems/hectare
SV (10)	A vector of ten diameters, representing the 5%, 15%.....85%, 95% points of the cumulative frequency distribution for the stand. Thus, for example, SV (6) would contain a diameter value such that there would be a 55% probability of any tree in the stand being smaller.
YRLOP	The number of years that have elapsed since a required silvicultural operation fell due. Positive values indicate that an operation is overdue, whilst negative values indicate the number of years yet to elapse.
LOPC	Contains a code number for the next operation due on the stand

These variables are generated prior to simulation from two alternative sources. The major source is a data base of compartment summaries produced by a forest inventory program, PIP, one of whose outputs may be a magnetic tape directly compatible for input to PYMOD. The alternative source consists of establishment option cards, which specify planting rates, stockings, species, locations, and site indices, and the years of the simulation over which the option is to be effective.

Also specified as management options prior to the commencement of the simulation are the silvicultural treatments for each species, which are defined by five variables:-

INH	Gives the height at which first thinning is to be performed. If zero, then an unthinned schedule is indicated.
CYC	Gives the minimum period in years that may elapse between thinnings.
FL	Gives the fraction of the stand's stocking to remain after thinning.
RAGE	Gives the minimum permissable rotation to clearfelling
RSTK	Gives the stocking at which the stand is to be restablished after clearfelling.

Another input required is the definitions of up to four merchantable volume classes, given in order of desirability or economic value, and each specified by a minimum diameter and a minimum length. The same merchantable classes are used for all species.

Harvesting is regulated by the specification of total volume quotas for each simulated year for each species. These are input as cards defining years of application, species, and the annual quota. By the use of several cards over sequential dates, irregular yield requirements may be specified. A zero quota suppresses all harvesting operations on the forest.

The simulation phase consists of an annual cycle of harvesting, planting and replanting, and growth. In the harvesting phase all stands available for cutting are sorted according to the YRLOP and LOPC codes, so that all stands overdue for cutting by say n years will have priority over those overdue by n-1 years, and so on. Within these groups, cutting operations scheduled earlier in the life of the stand will have priority over clearfelling. Thus the time of thinning or clearfelling may be delayed, but, when performed, a constant intensity, specified by FL, is used.

The actual thinning simulation is based upon an assumed distribution for thinned stems, defined by the equation:

$$(1)- \qquad r = p^c$$

where r is the probability of any tree remaining after thinning; p is its percentile value in the cumulative distribution, and c is a constant between 0 and 1 which can be shown to depend upon the leave fraction FL. In effect this distribution simply indicates that small trees are more likely to be removed than large ones; that this difference is reduced as thinning intensity increases (until, in the limit at clear felling, all trees are equally likely to be removed); and consequently that a small probability exists of even a large tree being removed. Suitable analysis reveals that given a "leave fraction" L, and a percentile value p_i corresponding to the i'th class of the vector SV, defined as:-

$$(2)- \qquad p_i = (i-1)/10 + 0.05$$

then the percentile value after thinning associated with diameter SV(i), say p_{ai}, will be:-

$$(3)- \qquad p_{ai} = Lp_i^{1/L}$$

Hence from this vector of revised percentiles, it is possible to interpolate a new set of values SV at the standard percentile values p_i. At the same time the difference between the p_{ai} and p_i values for the old stand vector give a frequency distribution for removals. These calculations are performed by subroutine THIN which takes the before thinning values of STK and SV, the thinning intensity FL, and returns the after thinning values and a vector of diameters and frequencies for removed stems which is then used for volume calculations.

Volume estimation involved two aspects. On one hand, existing tree volume equations developed by H.L. Wright of the Commonwealth Forestry Institute, Oxford, have now been in use for about a decade, and have been subject to rigorous validation tests. On the other hand, it was obvious that the required degree of flexibility in volume calculation could only be attained through the use of stem taper functions. A stem taper model was developed that gave a good geometric description of tree profile with an equation of the form:-

$$(4)- \qquad g_r = h_r + b(\quad \cos(2\pi h_r) + 1)$$

where g_r is relative sectional area at a relative height h_r, these two parameters being defined as:-

$$(5)- \qquad g_r = (d/D)^2$$

where d is sectional diameter at height h, and D is d.b.h., and

$$(6)- \qquad h_r = (h_t - h)/(h_t - 1.3)$$

where h_t is tree total height in metres.
The relative height and sectional area terms are derived from Ormerod (1973). Polynomial models of stem profile where studied (Bruce <u>et al</u>., 1968), but equation (4) was found to be simpler and more economical in practice. Volume calculations involve first of all the estimation of tree height h_t from dominant height HDOM and tree diameter for the i'th class SV (i). This done by calculating tree volume from the Wright equations in a subroutine TREVOL. Tree height is solved by assuming a form factor of 0.5, and the taper function is then used to determine the assortment of volumes by an algorithm that considers the minimum diameter and length of each merchantable class. In this way any error in the estimation of volume is limited to its partitioning between classes, with total volumes being consistent with the Wright equations.

For output purposes, volumes are accumulated by species and by thinning operations. Volumes in the different merchantable classes are printed as percentages or printed total volumes in order to emphasise the relatively crude nature of the merchantable volume breakdown.

As each stand is harvested, the accumulated total volume for the current year is compared with the quota for that year. This process may continue either until all available stands have been cut, in which case the quota is not met, or until the complete harvesting of a stand results in an excess over the quota. In this case, a sensitivity trheshold is tested. If the surplus exceeds this threshold then the stand is divided into an uncut original unit and a subunit differentiated by incrementing the logging unit identifier COMP (3). The split-off harvested area is a completely duplicated record from the parent stand and is subsequently treated as an entirely separate entity.

After a thinning operation, the LOPC operation indicator is incremented to the next thinning, and the time interval due to elapse, YRLOP, is set to thinning cycle - CYC (negative, as discussed earlier, because the operation is not yet due). The total time that should elapse before this thinning is compared with rotation length RAGE, and if it exceeds, LOPC is set to 11, indicating that a clearfelling is the next operation. Similarly, after a clearfelling, planting is scheduled as the next operation, in the year following. For stands which could have been harvested but which were not, then the next operation scheduled remains unchanged unless the stand is within 1 year of rotation age, in which case a clearfelling replaces a delayed thinning.

Planting operations are carried out once harvesting is complete, and involve only the resetting of the STK array and the accumulation of the planted area and number of trees required (for nursery planning). The actual diameter distribution for new stands is not generated until the stand reaches a height of 7 metres. At this stage, a Weibull distribution whose parameters are species dependent functions of spacing is used to generate the SV(i) as follows:-

$$(8)- \qquad SV(i) = a + b\ (\ -\ln(\ 1-p_i)^{(1/c)}$$

where a, b, and c are the species and spacing dependent parameters. The Weibull distribution is used in preference to the Beta or skewed Normal distributions as, besides being able to adopt the requisite shapes, it is quite simple to generate the cumulative diameter values. (c.f. Bailey and Dell, 1973; Clutter and Allison, 1974).

At the end of the annual cycle, height and diameter are incremented for each stand. Dominant height is a simple function of age and site index, with coefficients determined for each species:-

$$(9)- \qquad HDOM = \exp(b_o + b_1/AGE + (b_2+b_3/AGE).(\ln SI - c_1)/c_2)$$

and $$c_1 = b_0 + b_1/15$$

$$c_2 = b_2 + b_3/15$$

Diameter class increment is determined in subroutine GROW, using the following sets of relationships:-

(10)- $$GMAX = a\ (1 - \exp(-b.HDOM))^c$$
$$GAMMA = 1 - G/GMAX$$

where GMAX is an estimated maximum basal area for the species determined by a hand drawn curve from permanent plot data as a function of dominant height, and represented numerically by a Chapman-Richards function with species dependent coefficients. GAMMA is an index of stand basal area (G) relative to maximum basal area, defined so that it is zero for stands at the maximum and tends to 1 for open-grown trees.

(11)- $$DINC = a + b_0 \exp\ (HDOM\ (b_1 + b_2.DOM + b_3.GAMMA + b_4.DOM.GAMMA))$$

where a and b_i are species-dependent coefficients, DINC is diameter increment for a given value of SV(i) and DOM is:

(12)- $$DOM = SV(i)/DDOM$$

where DDOM is dominant diameter, determined before-hand as weighted mean of the SV vector with the weights governed by stocking STK. DOM is thus an index of the competitive status of each diameter class. After increments have been added to the SV array, they may be corrected if the resultant basal area exceeds GMAX by a reduction proportional to the excess. Without this correction, unthinned older stands tend to attain to high a basal area.

Simple mortality due to suppression was not found to be a significant occurrence over the range of management represented by the permanent plot data available. Most stands are planted at stockings of 1100 to 1700 stems/ha, and actual survival through the first two or three years tends to be in the range 600 to 1400 stems per hectare. For this reason density dependent mortality is not included. Specialized regional biotic and climatic factors such as deaths from Dothistroma needle blight and windfall may eventually be incorporated as specific subroutines that can optionally be switched on if required. These routines will be developed in response to local needs.

At the end of each annual growth cycle, summarised volumes and planting details are output, as shown in the example. The program then increments the date by one year and repeats the simulation phase. There is no inherent limit at present on the period of time that can be simulated. The program is reasonably fast, operating on an ICL 1906a at a rate of about 0.05 seconds per compartment per year.

Discussion of Model Validity

Space does not permit a detailed consideration of results from model validation studies, but the principal points may be briefly summarized here.

The growth model tends to underestimate yields by a mean factor of approximately 10% of the observed volumes. Over a wide range of stand conditions, 95% confidence limits on volume yield predictions range between 40% unerestimation and 20% overestimation by the model. These figures are relatively crude approximations since, as may be expected, the distribution of errors from the model are far from normal and appear to be several overlapping multinodal distributions. It may prove possible at a later stage to relate typical patterns of deviations from the model to biotic, edaphic or climatic factors in a simple way. Much of the error appears to arise therefore from the very broad regional agglomeration of data in developing the growth functions.

Nonetheless, the model is surprisingly accurate for many stands, and effectively simulates aspects of treatment response, as can be seen from figures (1) and (2). Figure (1) refers to a spacing experiment at Kwira, Tanzania, in *Pinus patula*. This figure shows that volume increment is

Figure 1 : Two Simulations of Thinning Experiment 345 in Tanzania (P.patula) from different starting conditions.

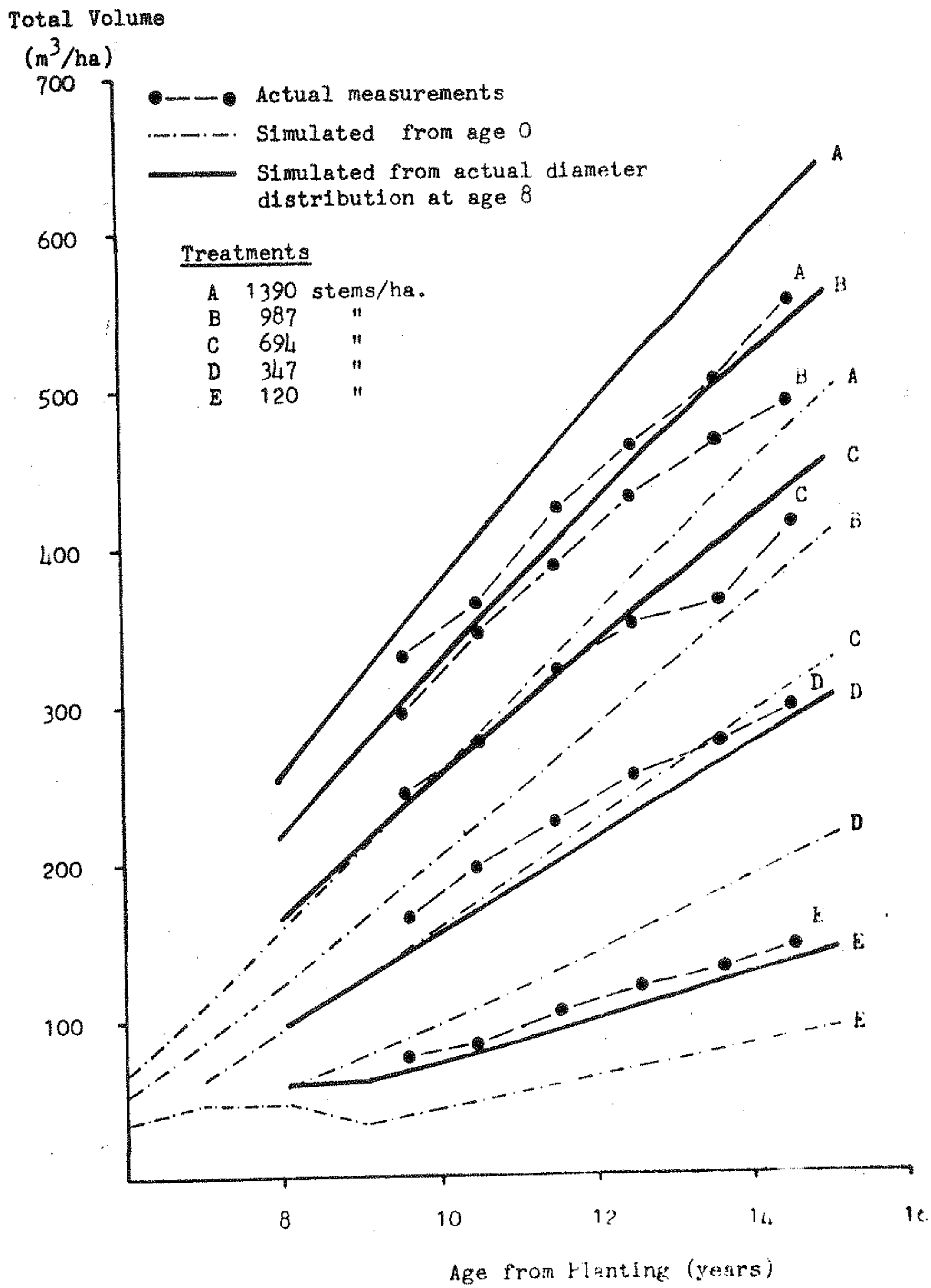
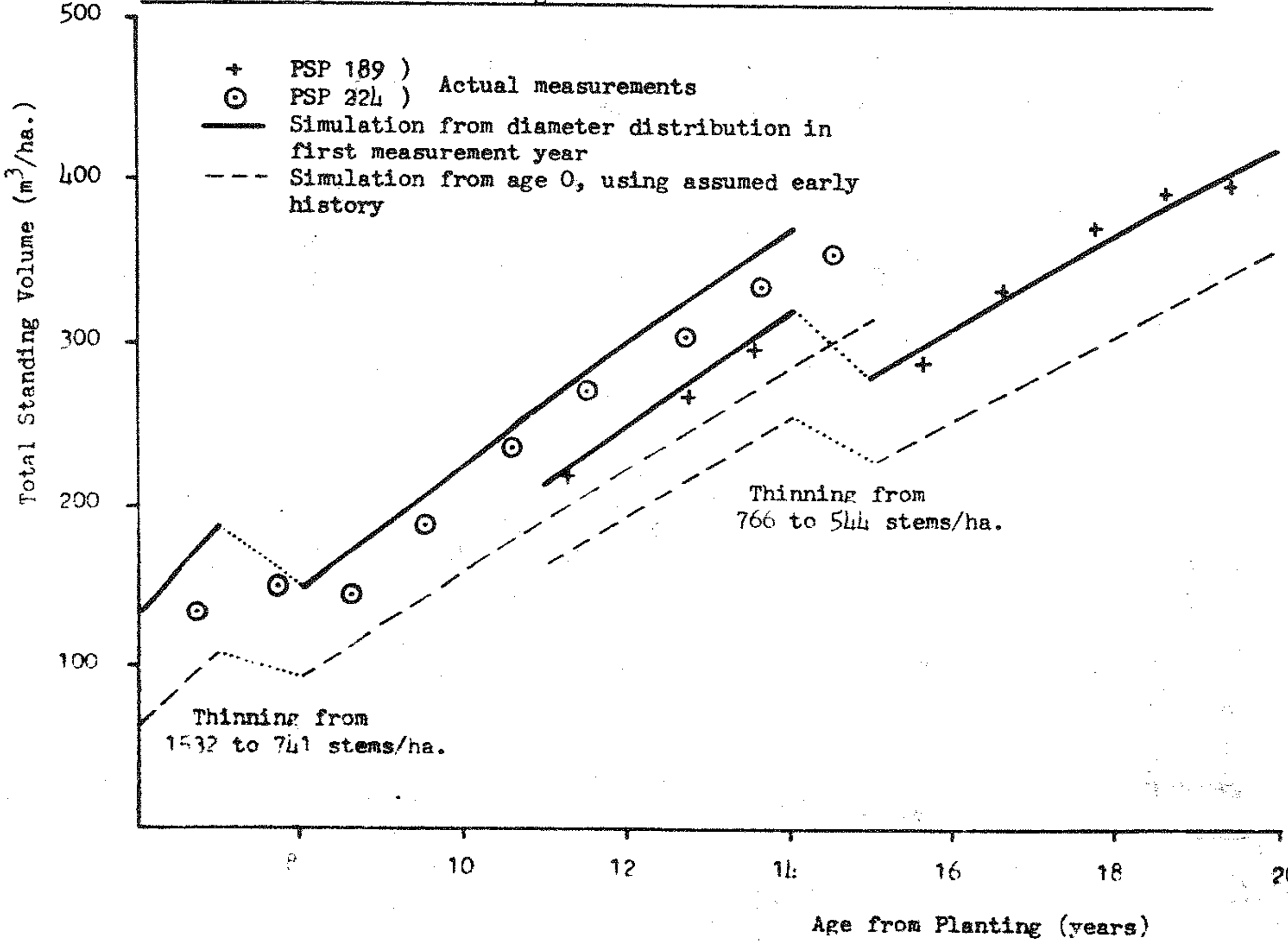


Figure 2 : Simulations of two permanent sample plots in Kenya Cypress stands, showing the effect of thinnings.



accurately represented over a very wide range of spacing treatments.

Figure (2) shows two Cypress permanent sample plots in Kenya which are of different site indices, stockings and ages. Both had a long series (by our standards) of measurements with a central heavy thinning. The simulation implicates an accurate portrayal of thinning intensity in terms of volume and response in terms of subsequent growth rate.

Conclusions: Further Developments

The model as described is an immediately applicable planning tool which fills in an important vacuum in presently available methodology. At the same time, it is likely that any consistent and continuing use of the model is likely to reveal defects in relation to any given local implementation. These will be of two major types:-

(1) The logic of the scheduling algorithm may prove to be inadequate. Two situations in particular can be foreseen:-

- (i) Where management requires the balancing of a number of constraints and production factors simultaneously. Such a situation may be dealt with by linear programming methodology; but the author is currently working on an extension of the simple goal satisfaction method gven here that may also resolve this problem whilst retaining the essential simplicity of the model.
- (ii) Where regional agglomeration of cutting operations are desired Again this may be dealt with through a revision of the satisficing logic to increase weights given to compartments as others in the same area are harvested.

(2) Specific features in the growth model may require modification. This might involve development of local growth functions within the existing framework to improve accuracy; or it might involve the incorporation of subroutines for, for example, proportional losses from fire wind fall, possible on a stochastic basis.

Documentation

Program documentation together with examples of inputs and outputs are available from the author.

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PROBLEMS OF YIELD PROGNOSTICATION IN SLASH PINE PLANTATIONS INFESTED BY FUSIFORM RUST[1]

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SUMMARY

Slash pine plantations cover more than 5.1 million acres (over 2.0 million hectares) or one-third of the commercial forest land base, of the Southern United States coastal plain regions. With an estimated biological potential for annual growth of more than one billion cubic feet (29.3 million cubic meters) these plantations constitute a major source of the area's future wood supply. A good portion of slash pine plantations that have been established during the past decade are growing under intensive management practices which include fertilization, bedding, drainage, and planting of genetically improved stock. However, they are increasingly subjected to epidemics which will have an impact on the expected yield at harvesting age. This paper presents some of the managerial implications introduced by fusiform rust, and suggests a framework to make reliable yield prognoses.

Additional key words: Pest management, _Cronartium fusiforme_, _Pinus elliottii_.

INTRODUCTION

During the last three decades, infestation of pine plantations by fusiform rust has increased exponentially in many parts of the southeastern United States ranging from Maryland to Texas (Fig. 1, 2). The disease is caused by the heteroecious fungus _Cronartium fusiforme_ Hedgc. and Hunt ex. Comm. Water oak (_Quercus nigra_ L.) appears to be one of the most important hosts especially in northern Florida and parts of Georgia. Succulent pine shoots and primary needle tissues are infected during spring time when basidiospores, produced on telial columns on oak leaves, are disseminated by wind to the surrounding areas. Following infection, branch and stem galls are formed. There is no documented evidence that the disease spreads from one pine tree to another. In cases of severely infected trees with stem galls, subsequent wind breakage may cause mortality of varying degrees to pine plantations (Fig. 3, 4). At this stage, no feasible suppression measures can be applied to trees that have already been established.

CONTRIBUTING FACTORS

Recent research findings indicate that there are interactions between site conditions and rust incidence or severity. The available inoculum and the amount of susceptible pine tissue are both proportional to rust infesta-

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[1]Paper presented by Ralph L. Amateis, Graduate Student.

tion in a given area (Hollis and Schmidt, 1977). The former is attributed to the presence of host plants while the latter is a function of cultural treatments and the age of the plantation. The incidence of rust is increasingly pronounced during the first ten years after planting and diminishes from there on (Fig. 5) although early infestation may lead to tree mortality at a later stage. Intensive cultural practices, such as site preparation before planting and subsequent fertilization seem to accelerate the incidence and severity of rust due to the increase on the available succulent pine tissue. There is also speculation that major fresh water drainages, prevailing winds, and relative humidity contribute to the spread of the disease, but no quantitative estimate of their relative merit is available at this stage.

ECONOMIC IMPACT OF FUSIFORM RUST

Reliable estimates of the monetary impact of fusiform rust are not available. Some forest scientists maintain that in Florida and Georgia alone, for example, fusiform rust causes an estimated annual loss of about 100 million cubic feet (2.8 million cubic meters) of growing stock or 280 million board feet of saw timber in southern pines. Others consider this impact rather conservative and argue that an annual stumpage loss of $10 million represents $250 million in finished wood products. Additional losses may occur in wildlife, aesthetics, and outdoor recreation that have not been considered as yet. It appears that objective estimates of the overall economic impact of fusiform rust on slash and loblolly pine will not be available unless the incidence, spread, and severity of the disease through time are well documented.

The effect of fusiform rust on pine plantations varies, aside from site conditions, with the age of plantations, the severity of infestation, and the resistance of species. Extensive mortality may occur both at an early stage of a plantation as well as later due to wind breakages of trees that have developed stem galls. As a result, the expected quantity of wood at harvesting age (25 to 30 years after planting) is reduced. In addition, the quality of pulp produced from infected trees is inferior compared to that obtained from healthy trees. Additional costs are realized in clearing and replanting heavily infected areas, growth losses due to prematurely harvested plantations, and potential sawtimber producing compartments where infected trees have to be removed prior to reaching desired size limits. From the managerial point of view, an array of feasible alternative courses of action is needed to either eliminate or minimize the overall impact of fusiform rust, which is a disturbing force in a forestry operation.

MAJOR ANOMALIES INTRODUCED BY FUSIFORM RUST IN LONG TERM YIELD PROGNOSTICATION

One of the essential inputs of any rational forest management for fiber production is the anticipated yield, however expressed and assessed, at different points in time during the planning horizon. Yield predictions are usually developed on the basis of past recorded evidence and the present growing conditions of a forest. Net yield values may include some natural mortality but seldom destructive forces, like fusiform rust, pine beetles, pitch canker, wildfire, etc., are treated as stochastic elements in yield prognostication. Forest managers need reliable yield estimates under different sets of conditions to evaluate outcomes of alternative strategies and respective probabilities of occurrence (Stage, 1974).

Geneticists are working on various progeny trials to develop fusiform rust resistant trees. However, there are already indications which suggest that trees resistant to rust may be susceptible to pitch canker disease. At this stage, no feasible solution that will eliminate similar problems in the foreseeable future is evident.

EXISTING RULES FOR MANAGERIAL DECISIONS

In the southeastern United States, only in very few cases have attempts been made to develop rules for selecting management strategies that cope effectively with the fusiform rust problem. Table 1 summarizes operational guidelines based on percent of trees infected by fusiform rust (regardless of the severity of the disease), and the number of surviving healthy trees per unit area (Belcher et al, 1977). Far from being perfect or widely accepted, Table 1 is an attempt to set the stage for further ramifications.

It is of interest to note that although thinning is not a widely accepted practice in pine plantations of the Southeast, it is considered a feasible alternative at least in high hazard rust infected areas. In his study, Belcher reported that only in one forest of 19,000 acres (7,600 hectares) which is heavily infected with fusiform rust, about 80,000 cords (287,000 cubic meters) can be salvaged through marked thinnings over a 12 year period. Without thinnings this amount of wood which represents a current stumpage value of $2.2 millions, will become a mortality loss due to fusiform rust. Furthermore, documented low rust incidence in dense natural pine stands has introduced the possibility of returning to natural regeneration, at least in high hazard areas.

THEORETICAL CONSIDERATIONS

As postulated by Shoemaker (1976), a pest control management model encompasses at least three components: (a) Transfer equations, describing dynamic changes in population size through time as a function of other populations and exogenous factors, e.g., environment and management decisions, (b) Objective functions, and (c) Constraints that might be placed on population sizes and management alternatives. Transfer differential or difference equations may be used to predict changes in yield and pests through time as affected by population size (state variables) and managerial decisions (control variables).

If the managerial objective is to maximize net gain, one may proceed as follows:

$$\text{Max } E\,[P(Y) \cdot Y(v) - C(v)], \text{ where}$$

Y = yield

$Y(v)$ = realized yield from employing management practice v

$C(v)$ = cost of implementing management alternative v

P = price

E = expected value.

Contraints on state and control variables depend on specific conditions. Solutions of similar management models can be found through simulation (Leuschner, Matney, Burkhart, 1977) or optimization techniques such as linear or dynamic programming. The major drawback is lack of hard data describing the dynamics and interactive effects of pest populations, the forest plantations, and their site conditions.

CURRENT RESEARCH

The problem of fusiform rust has been studied for a number of years by pathologists, physiologists, and geneticists at the School of Forest Resources and Conservation, University of Florida, and other parts of the Southeast. Our contribution is part of a new effort in the emerged scientific discipline of integrated pest management. The main objective is to develop conceptual and mathematical yield predictive models that will consider the dynamics of stands, the dynamics of the disease, and their interaction with site-specific conditions.

Three test sites of slash pine (*Pinus elliottii* var. *elliottii* [Little]) have been selected in southeast Georgia on the basis of the degree of infestation (light 27%, medium 44%, heavy 70%) and the availability of field observations. Trees are now 14 years old and their breast-high diameters, heights, and conditions (healthy, dead, infected with fusiform rust) of each tree have been recorded 3, 5, and 10 years after planting. The relative position of trees within each stand is specified in terms of X- and Y-rectangular coordinates since they were planted at regular intervals, ranging from 7 x 12 feet (2.1 x 3.7 meters) to 10 x 10 feet (3 x 3 meters).

From the input data, computer generated maps have been developed to study the spatial distribution of healthy, infected, and dead due to fusiform rust trees at different points in time. These maps will also be used to test distant-dependent tree and stand parameters, such as accelerated growth of survivals, and the efficiency of alternative sampling designs for rust impact. With respect to the latter, very seldom, if ever, sampling considerations are considered as an integral part of the decisions that will follow. Projections may be misleading if they are based on estimates of unknown precision, as often is the case.

The need to institute a type of life-table approach to stand dynamics is another consideration of our on-going research. Life tables, a useful tool in biostatistics, have already been used to study insect populations (Waters, 1969). They provide a reliable basis for developing multiperiod transition probabilities of stochastic models for stand projection, such as the one advocated by Penden, Williams, and Frayer (1973).

The inadequacy of past data on rust infection of individual trees has presented problems in deriving meaningful relationships. In subsequent field measurements more detailed observations will be obtained on individual trees and stands (Appendix).

Several hypotheses with long term effect on yields have been postulated that need to be tested:

-The infected trees are evenly distributed among all diameter classes in merchantable stands (Belcher, 1977).

-There is a latitude effect or south-to-north increase in rust incidence (Schmidt, Goddard, and Hollis, 1974; Daniel, 1977), but it is not known whether the rate of change is uniform.

-Major fresh water drainage systems influence rust incidence. It has been reported that plantations within a three mile wide strip along such drainages have three to four times more rust than those located six to seven miles inland (Daniel, 1977). Such postulates bear economic significance and may be used as guidelines in selecting sites for planting superior rust resistant trees in the future.

CONCLUDING REMARKS

The anticipated increase in the demand for wood supply will place additional emphasis on intensive management of pine plantations in the southeastern United States. Cultural treatments that stimulate growth may also increase the chances of pest infestation, such as fusiform rust. Managers need contingency plans that will effectively lead to the best trade-offs under given conditions. Compromises that may result in species substitution, e.g., loblolly pine (*Pinus taeda*) instead of slash pine, and encourage genetic heterogeneity to minimize expected losses in the largely even-age managed pine plantations constitute an integral part of managerial strategies. However, our knowledge of the long term interactive effects of the dynamic system "forest-pest-site" is fragmented and incomplete.

Although standards of utilization and values of various wood products may vary with time, reliable yield estimates should reflect disturbances introduced by forest pests. One should also be cautioned on differences that may exist between yields observed on small research plots and that of larger managed forests (Bruce, 1977). Finally, as it has been suggested by the Pest Control Forest Study Team, National Academy of Sciences (1975), the high productivity of forests under intensive management, coupled with favorable market conditions, should provide needed incentives to include integrated pest management as one of the essential components of the decision making process.

ACKNOWLEDGMENTS

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APPENDIX

Quantification of Fusiform Rust Infection

1. Individual Tree Identification

   Size of tree: diameter, height (total, merchantable 4" top), volume, surface, bark thickness, height to base of live crown.

   Age: time of planting, time of infection after planting.

   Description of tree infection: number of cankers per tree (stem & limb), location.

   stem: height above ground, proportion of stem diameter with canker.

   branch: distance from stem (less than 12 inches, more than 12 inches)

   Size of Canker: length, circumference, diameter, height.

2. Stand Characteristics: volume, surface, basal area, number of trees per unit area, site, age, diameter distributions.

   Cultural treatment: fertilization, site preparation method, thinning, prescribed burning.

   Number of trees with stem cankers versus branch cankers or both. Number of trees with branch cankers which may infect stem.

   Distributions of trees based on condition--healthy, infected (stem or branch), dead.

   Number of infections in dominant and codominant trees.

3. Host Species

   Abundance: volume, surface of leaves, basal area, number of trees per unit area.

   Proximity to pine stand.

   Species present.

4. Climatological data since time of planting.

5. Distance from fresh water drainage.

6. Latitude of stand.

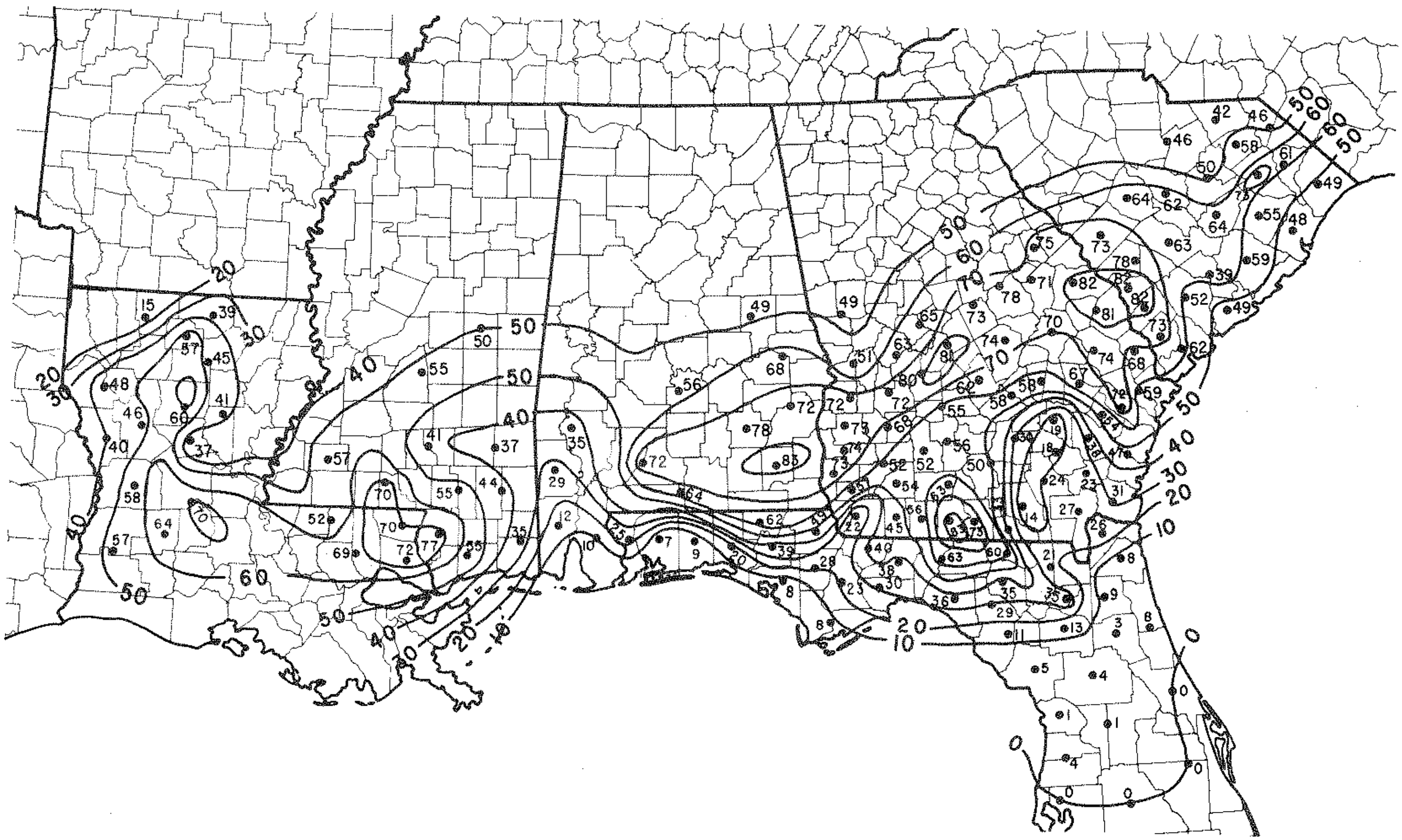


Figure 1.--Percentage of trees infected with fusiform rust in 8- to 12-year-old slash pine plantations (Squillace, 1976).

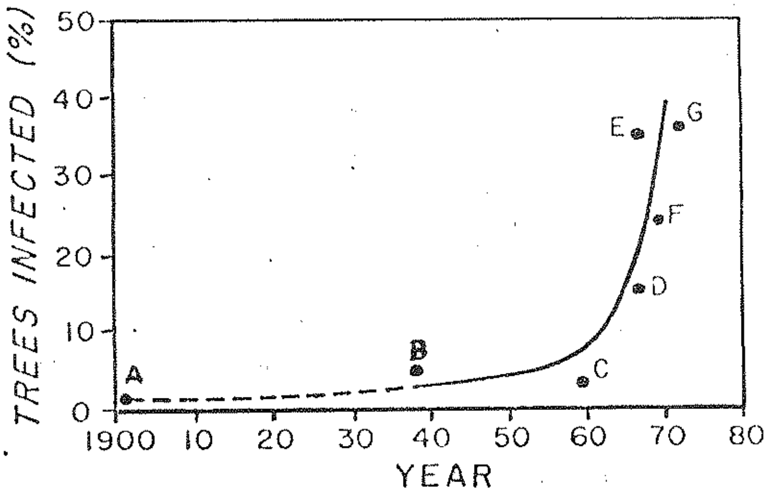


Fig. 2. Incidence of rust in slash pine plantations in north Florida from 1900-1976. Letters indicate source of data (Griggs and Schmidt, 1977).

Fig. 3. Early infection of a young pine with stem fusiform rust gall which often leads to tree mortality.

Fig. 4. Trees with stem galls are highly susceptible to wind damage at an older age.

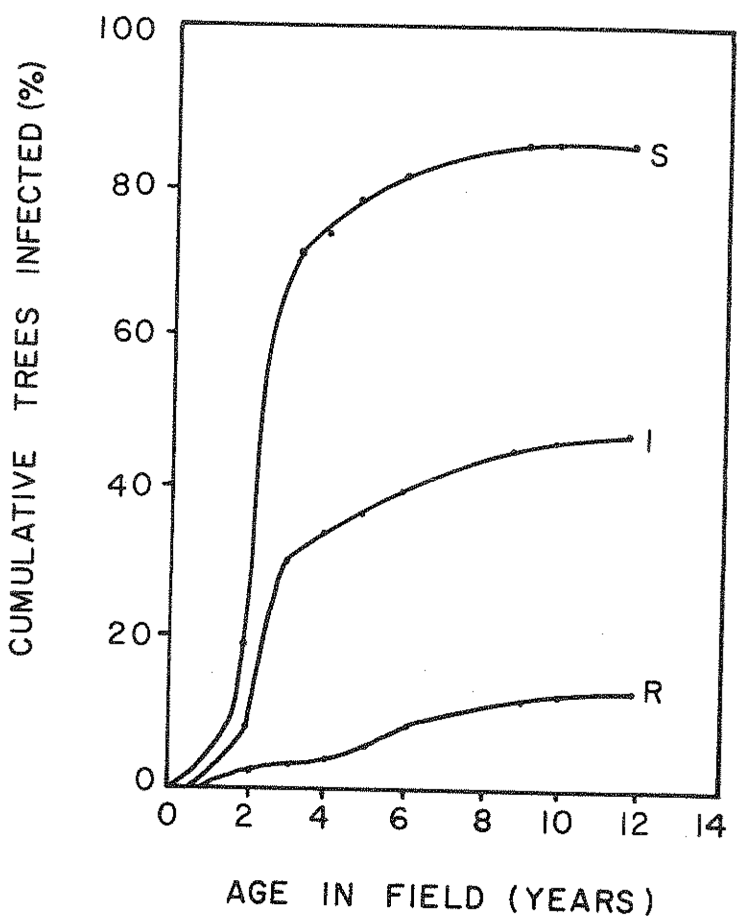
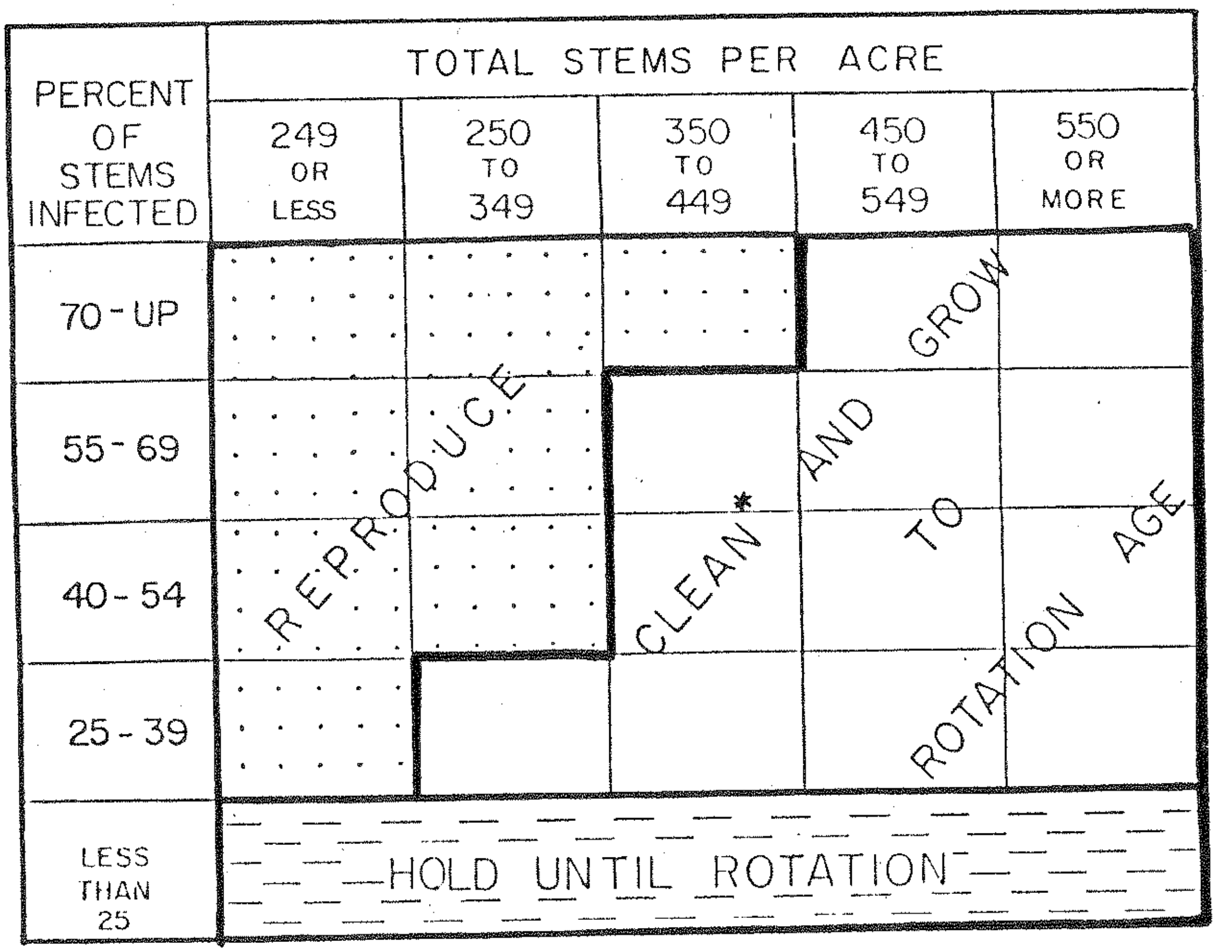


Fig. 5. Disease progress curves for fusiform rust on resistant (R), intermediate (I), and susceptible (S) slash pine in Gulfport, Miss. (Griggs and Schmidt, 1977).

TABLE 1. DECISION CHART FOR MAKING THE CHOICE BETWEEN REMOVING OR THINNING A FUSIFORM RUST INFECTED STAND (BELCHER, 1977).



* REMOVE ALL TREES WITH STEM CANKERS THAT INVOLVE MORE THAN 50% OF THE BOLE DIAMETER OR OCCUR IN A FORK. THIS ASSUMES THE REMOVAL OF EACH 5TH ROW FOR ACCESS.

HUGIN
A SWEDISH RESEARCH PROJECT DEVELOPING METHODS FOR LONG-TERM FORECASTING OF TIMBER YIELDS

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ABSTRACT

This paper describes a 65-man-year research project within the College of Forestry in Sweden with the objectives to develop and evaluate methods for long-term forecasting of timber yield. A specific goal is to develop a computerized simulation system for forecasts on national or regional level, using data from the National Forest Survey as description of the starting situation.

Some requirements and the tentative outlines of this system, the HUGIN system, are presented. Problems concerning the construction of growth simulators and the calculation of the effect of various silvicultural treatments are discussed. It is stressed that it is necessary to use data from inventories <u>and</u> experimental plots as a basis for growth functions. Further the importance is pointed out of making realistic assumptions on the future management of the forests, taking all significant restrictions into consideration. Otherwise there is an obvious risk that the forecasts become too optimistic.

INTRODUCTION

Forestry and forest industry are very important in the Swedish economy. Due to a rapid expansion of the industry the annual cut now equals or even exceeds the annual growth in our forests. The future development of the industry therefore depends to a large extent on how much timber can be produced within the country. From a biological point of view there is a rather big potential for increased timber production through more intensive management of the forests. However, for many reasons this potential cannot be fully utilized in practice. In this situation there is a strong need for reliable methods for realistic long-term forecasts of timber yield and possible cut and for the analysis of

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the effect of different forest management policies, both on national or regional level and on enterprise level.

Forestry research is very well established in Sweden since many decades. We have carried out national forest surveys since the 1920:s and much work has been done in developing methods for long-term forecasts. Nevertheless we think that the existing methods are not good enough for our present need. Yield research has earlier been to little focused on the problem of making growth functions etc, valid for practical forestry. And the results and methods from yield research have not been used as much as would be possible in large scale forecasts.

These are, briefly, the reasons why the research project HUGIN[1)] was started in 1975, with the task to improve our methods for long-term forecasts. The project is performed mainly within the College of Forestry. Financial support is given by the government, some funds and the Swedish Pulp and Paper Association. The project will be finished in 1980.

OBJECTIVES OF THE PROJECT

The objectives of the project can be summarized as follows:
- to develop a computerized system for long-term forecasts of timber yield on national level or for regions within the country, using data from the National Forest Survey 1973 - 77 and the following years as basis for the projections. This system will be referred to here as the HUGIN system
- to develop and evaluate methods for timber yield forecasts to be used in other situations, for instance in long-term planning within forest enterprises. In this case only components (growth simulators etc.), not a complete system, will be developed. Much of this will come out as spin-off effects from the work with the HUGIN system.

A more general objective is, as indicated above, to coordinate som earlier planned research going on and try to direct some of the future research towards this type of applications. Another objective is to contribute with ideas concerning the future design of the National Forest Survey.

In this paper the main emphasis is put on a discussion of the HUGIN system for forecasts on national/regional level. Of course much similar work has been done in other countries. It is not possible to refer to these studies here, due to limited space.

PREVIOUS WORK IN SWEDEN

Here I will give a short review of previous work concerning forecasts on national and regional level. Ever since the first national forest survey in the 1920:s survey data have been used in many investigations for analysis of the actual status of the forest resources and for calculation of potential future timber yield and cut.

1) HUGIN was the name of a wise bird (raven) in Old Swedish mythology.

Before computers were available the methods for such forecasts were rather simple, due to the large amount of calculation work if more sophisticated methods would have been applied. Normally the possible cut was calculated only for the next ten to twenty years together with a rough calculation of the development of the age class distribution and the total growing stock some decades further ahead. Only a few management alternatives could be studied. Some investigations have been made concerning the potential yield in the very long run. Growth forecasts were made by use of series of volume increment percentages or simple yield tables based on inventory data. However, already PETTERSSON (1947) used regression functions derived from yield research plots for growth forecasts for the whole of northern Sweden.

During the last five years a fully computerized dynamic system for long-range forecasts of forest development has been constructed at the Department of Forest Survey within the College of Forestry, (BERGSTRAND & NILSSON, 1976). The system describes the probable development of the growing stock under specified assumptions with regard to future management regimes and levels of cut. The technique used can be considered as simulation, i.e. the technique of linear programming is not used. Data from the National Forest Survey in 1968 - 72 are the basis of the projections as well as of the construction of the growth functions used in the system. The sample plots from the survey are aggregated before the projections into about 100 treatment classes within each of 10 - 15 regions. In the present version of the system forecasts can be made for the next 100 years.

Another system of a similar type has been developed by JONSSON. (See JONSSON, 1977.) In this case the single sample plots are calculation units, and growth is calculated for each single tree.

It should be added that there exist several computer programs in Sweden for similar calculations on enterprise level. In most of these the accuracy of growth forecasts is low.

SOME FEATURES OF SWEDISH FORESTRY

In order to give a better understanding of the problems facing us when making models for timber yield forecasts, I will give a short description of Swedish forestry today.

The total area of productive forest land is 24 million hectares. Out of this 50% are owned by private owners, 25% by companies and 25% are state or other public forests. The present annual net growth and cut are 70 - 75 million m^3 over bark. Although there is a substantial variation in the length of the vegetation period, the climate conditions are rather uniform throughout the country. We have only a few tree species of economic significance. (Norway spruce 46%, Scots pine 37%, birch 13% and other species 4% of the growing stock.)

The most important timber products are pulp wood and saw timber. Average rotation period is 60 - 150 years depending on site quality and climate region. About 30% of the total harvest of timber comes from thinnings. The regeneration of the stands is through clear cutting and planting or by natural regeneration by means of seed trees (pine). At present 180 000 hectares per year are fertilized, mainly mineral soils. This area will probably increase. Some drainage of peat land is performed, which probably also will increase. There is a potential of some millions of hectares of peat land that can be transformed into productive forest land, or where the present yield can be increased. A comprehensive forest tree breeding program is performed. A new tree species, Logdepole pine, is now introduced in a large scale in northern Sweden. This species has shown good survival and rapid growth in young stands and is expected to give a higher yield (25 - 60%) than our own pine.

Insects have caused substantial damage during recent years. Since the use of DDT was forbidden, the pine weevil (Hylobius) has severely damaged our seedling stands. Pine-shoot borers have reduced the growth of our pine stands. Broad-leaved species threaten the pine and spruce in young stands. High labour costs for cleaning and restrictions in the use of herbicides make this an increasing problem.

This exposê is probably enough to give an idea of the features of the forestry system we try to describe in our models.

As mentioned earlier, data from the National Forest Survey will be used in the HUGIN system to describe the starting situation in our forecasts and also for constructing some of the growth functions. Therefore, I will describe in the next section the outlines of this survey.

THE NATIONAL FOREST SURVEY

The National Forest Survey (NFS) continuously provides data for the planning and control of the utilization of the forest resources. The following description refers to the survey 1973 - 77. This design will be used also during the next few years.

The NFS is a systematic sample survey, which covers the whole of Sweden every year. The observation unit is a temporary circular plot with a radius of 10 m. The plots are grouped in clusters, designed as squares with a side length of 1 000 - 1 600 m, each containing 4 - 7 plots.

The main elements of the NFS are
- assessment of site and stand properties, which is the basis for the estimation of areas of various land classes and types of forest
- assessment of growing stock (volume, increment, mortality)
- assessment of the fellings last year (areas and volume), "stump inventory"
- assessment of regeneration status and regeneration activities.

The growing stock is estimated through calipering of all trees which have reached breast height and measurements on sample trees. A bore core from each sample tree is sent to the office, where the width of the annual rings is measured in machines. For the estimation of mortality it is estimated which trees have died or have been wind-thrown during the last three years.

The number of "volume plots" (plots where a complete site and stand description and assessment of growing stock is performed) on forest land is about 10 000 per year. On these plots there are about 200 000 calipered trees and 40 000 sample trees. Thus, a five years material comprises about 50 000 sample plots with one million trees and 200 000 sample trees.

TENTATIVE OUTLINES OF THE HUGIN SYSTEM

Some requirements of the system

The purpose of the HUGIN system is to provide assistance in the planning of long-term timber production. Roughly speaking there are two types of questions to be answered by means of the system:
- How much timber of different kinds can be cut during the following decades? - Such information is necessary for decisions on investments in forest industry.
- What will be the effect on timber yield (and possible cut) of various forest management programs and silvicultural treatments? - This information is needed for decisions on forest management policies and investments in primary production.

We plan so far a dynamic simulation model, which describes the probable development of the forest resources under specified assumptions as concerns future management regimes and desired cut. Thus, the approach is very similar to the one used by BERGSTRAND & NILSSON, 1976.

Some general requirements are listed below:
- The system must be flexible enough to allow studies of a great number of management alternatives.
- The accuracy of growth forecasts should be high, which means as unbiased estimates as possible <u>and</u> correct relations between the effects of different management regimes.
- The forecasts must be realistic, i. e. show what can be achieved in practical forestry. Therefore it must be possible to include various restrictions concerning the utilization of the forests for timber production.
- It must be possible to extend the forecasts at least 100 years.
- The system must be designed in a way which makes it easy to replace different modules by new ones without changing the whole system.
- All productive forest land in the area under study must be included. It must be possible to add new forest area or take away areas that will be used for other purposes than timber production.
- The computer costs for running the system must be acceptable.
- The system must not be too sophisticated and difficult to understand.

The main output will be
- annual yield (increment and mortality) in each decade
- size and composition of the cut in each decade (areas and volumes)
- the actual state of the forest resources at ten year intervals (area distribution, volume and structure of the growing stock).

Besides the stem volume over and under bark the following parameters will be calculated for harvested timber, if desired by the user: the dry matter content in the stem wood, the volume and/or weight of other biomass fractions (stumps & roots, bark, needles/leaves) and an approximative distribution of stem volume on assortments (saw timber and pulp wood). Dry matter content and the amount of "other biomass fractions" will be estimated by multiplying stem volume with appropriate conversion factors.

Suggested design

In order to achieve the desired flexibility of the system and high accuracy in growth forecasts, we plan to use the single sample plots as calculation units. However, this means a large amount of computer work. We are at present examining what the computer costs will be using this approach. If the costs seem to be unacceptable with the computers available today and during the next few years, we will be forced to aggregate sample plots before the projections or include both these options in the system. Perhaps the plot-wise approach is a mere nothing for a computer in the year 1985 or 1988, even with 50 000 sample plots and a million trees. It is an essential task of this project to try to find out how the accuracy of growth forecasts depends on the degree of aggregation and the method for growth simulation.

A general, simplified structure of the HUGIN system is shown in figure 1. The input data describing the starting situation are taken from the National Forest Survey, possibly supplemented with other data that are not normally recorded in the survey.

In phase 1 the sample plots to be included are selected. Some basic calculations and editing of input data and possibly aggregation of plots are performed. Much of this work must not be repeated in every run of the program.

Phase 2 includes decisions on treatments of the various plots/ groups of plots, projection of the state of the forest five years ahead by use of growth simulators and calculation of the cut (including dry matter content, assortments etc. of the harvested timber, if desired). This loop is repeated as many periods as wanted, for instance 100 years. The treatments of the stands will probably be carried out at ten years intervals, which is normal in this type of applications in Sweden. However, it must be checked that this approximation does not lead to biased results.

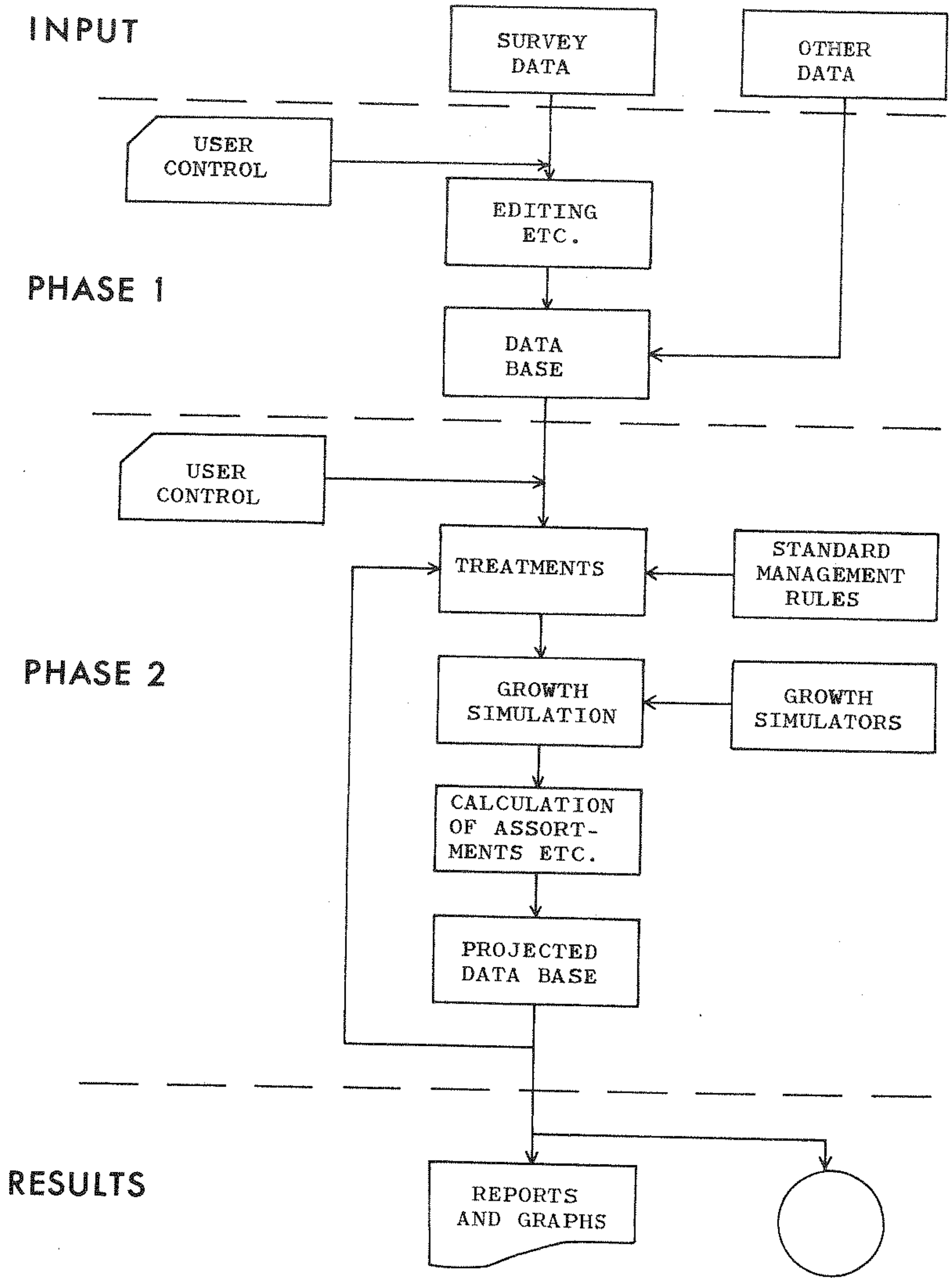

Figure 1. Tentative general structure of the HUGIN system.

The user of the system can direct the calculations by means of control cards. The system will be flexible so that the user can either specify in detail the management alternative to be studied, or apply prepared standard rules for the treatment of various types of forest, or use a combination of the two options

The results will be presented in tables and graphs and will also be stored on disk or magnetic tape for further analysis.

No economic criteria will be included in the system. The economy of various forest operations must be considered in other ways when specifying a management alternative.

The HUGIN system will be deterministic. However, some random component must perhaps be used to get realistic diameter distributions and a realistic picture of mortality.

ORGANIZATION AND COSTS

The HUGIN project involves researchers from several departments of the college. The main work is done within the Department of Forest Survey (which is responsible of the National Forest Survey) and the Department of Forest Yield Research.

The organization of the project is shown in figure 2. The project committee has members from the college and from organizations outside the college. The research work is divided into four sections:

Establishment of stands. Deals with the problems of growth forecasts for young stands (less than 8 m height) and prediction of the result when using various methods for establishment of new stands.

Growth in established stands. Deals with the problems of forecasting growth of stands higher than 8 m and prediction of the effect of silvicultural treatments, for instance thinning and fertilization. Problems concerning estimation of site quality are also handled within this section.

Management regimes. This section studies how to specify management regimes in the HUGIN system in a realistic way.

System construction. This section works with the design, programming and testing of the HUGIN system.

Some associate sub-projects will deliver information on the effect of fertilization etc. These sub-projects are not directly managed by the HUGIN project, but they are coordinated with the HUGIN, so that the information delivered will fit in with the growth models. The mutual exchange of experience between research disciplines is an essential objective of the HUGIN project.

The total work planned until now corresponds to about 65 man-years, the associated sub-projects excluded. Out of this 50

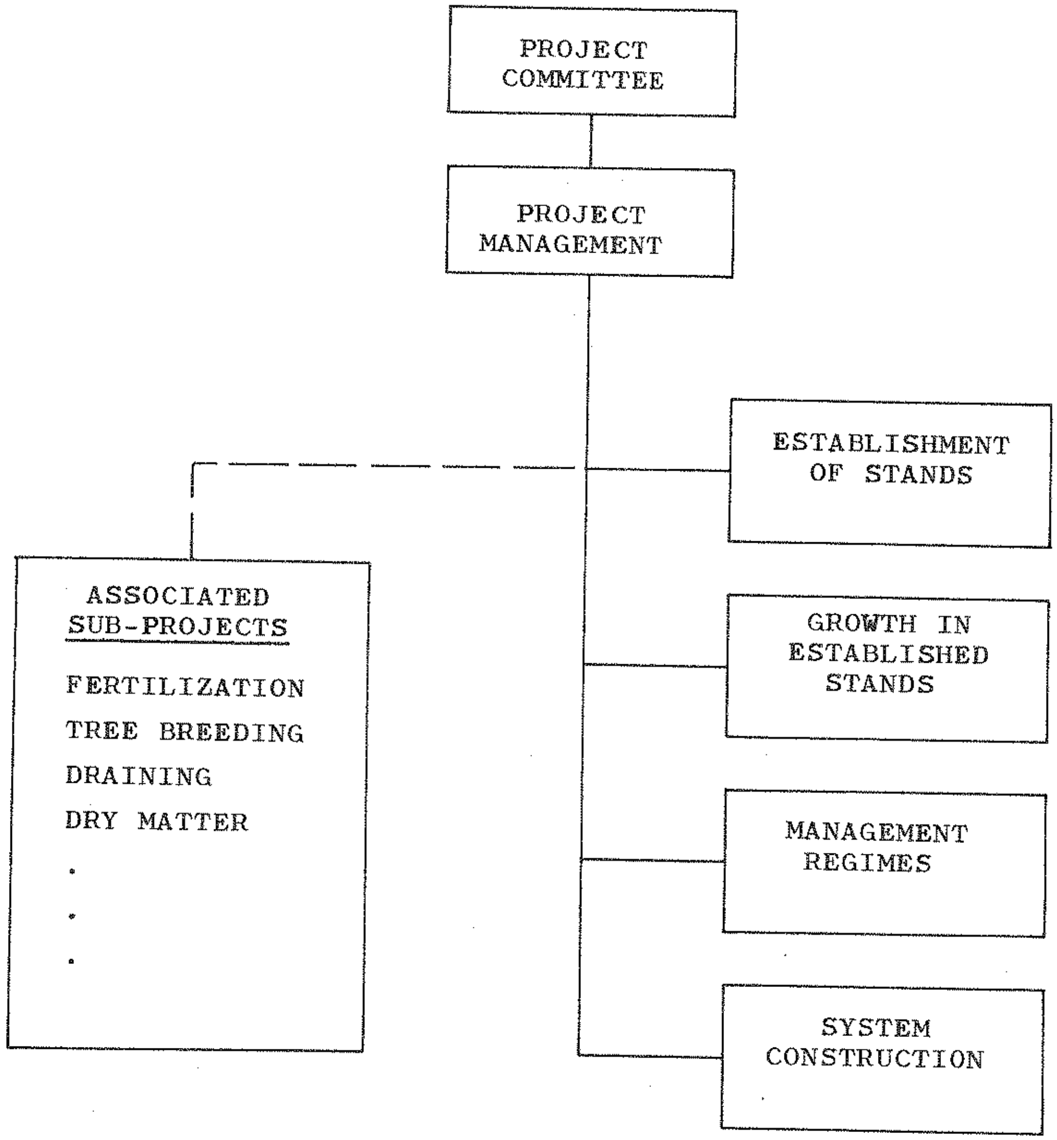

Figure 2. The organization of the HUGIN project.

years are within the sections Establishment of stands and Growth in established stands, some part of it being field work for collection of new data. The total costs, including extra costs within the associated sub-projects, amount to 10 million Swedish Crowns, i.e. 2.1 million US $. It must be stressed again, that some of this research was initiated before the start of the HUGIN project and would have been carried out anyway.

PROBLEMS TO BE STUDIED AND SUGGESTED SOLUTIONS

General

In this section some problems studied in the HUGIN project will be mentioned. Suggested approaches or solutions to some of these problems are also presented, but a detailed discussion is not possible here. The discussion is focused on application in the HUGIN system for national/regional forecasts. A well-known general problem is that growth data from experimental plots almost always represent well-managed homogeneous stands on homogeneous sites. Hence, they are not typical for large areas of practical forests. The way to solve this problem is an intelligent combination of information from inventories and experimental plots. But, how to do it? That is the crucial point.

Another universal problem is the lack of data for new species, new treatments etc., because there are no stands old enough to study.

Establishment of stands

The problem of predicting the development of seedling stands and young stands is not an easy one. High and irregular mortality, especially the very first years after establishment, and competition from weeds and not desired tree species are difficult to model and quantify.

What we do is briefly:
- Study the state in very young stands established with various methods on various types of sites in practical forestry, using existing regeneration survey data.
- Study the development of stands by means of existing data from experimental plots, where repeated measurements have been performed.
- Collect new material in practical stands of the age 12 - 30 years, where stand history is known, using probability sampling in order to get a material representative for the whole forest area of Sweden.

The following approach has been suggested for prediction of the development of young stands. The development of the number of trees and the height of the trees are predicted. When the dominant height is about 8 m, the height distribution of the trees is transformed into a diameter distribution, from which basal area can be estimated. The predictions are made separately for pine, spruce and broad-leaved species.

The average time needed to get an acceptable stand when using natural regeneration is an important question. Another one is to what extent the appearance of naturally regenerated seedlings can improve a stand where the plantation has been unsuccessful. Recent investigations have confirmed that the future yield in our forests will depend very much on how successful forest owners are in establishing new stands.

Growth in established stands

Site classification

Site index (h_{100}) will be estimated either by means of height development curves or by means of site properties as described by HÄGGLUND, 1977.

Growth models

Within the project some methods for large scale growth forecasts will be evaluated, methods suitable for national/regional forecasts as well as methods to be applied by forest enterprises. The models to be studied are dynamic growth simulators. The two main approaches are stand simulators and single tree models. Since information on spatial distribution (stem charts) is very seldom available, only distance-independent models are considered.

One requirement of the HUGIN system is to predict not only total yield but also the structure of the forests and of the harvested timber. The importance of this requirement depends on the management problem under study. An obvious advantage of single-tree models is that the diameter distribution is obtained automatically. However, stand functions are probably less expensive to run in the computer. But, if substantial computer time is needed in the last mentioned case to derive diameter distributions, the time consumtion will perhaps be of the same order for both methods. This is under study at present.

Data for construction of growth functions

The choice of data basis for constructing growth functions is at least as important as the choice of growth model. Our two main sources for growth functions are the National Forest Survey (NFS) and the so called Great Yield Inventory (GYI). The GYI was carried out during the years 1940 - 65 and comprises 2 075 temporary plots with a size of 0.05 - 0.20 hectares. Data from the GYI have already been used for a number of growth functions. Although the GYI plots were located in practical stands, they represent a more homogeneous population than the average forest (homogeneous sites, homogeneous stands without severe damages etc.).

Our tentative approach is to use data from the NFS to get the level of growth, because this data set originates from the same population of forest sites as it will be applied on. However, stand history is very badly recorded in the NFS. Therefore the effect of thinning must be taken from the GYI or permanent experimental plots. How this integration of information from the various data sets or calibration to pratical level will be done is not yet fully clear to us. Data from permanent plots will be used mainly for test of growth functions and for studies of the effect of thinnings.

New growth functions

The following new growth functions are now being developed within the project:
- stand growth functions based on NFS data
- single tree functions from NFS data (further development of JONSSON's model)
- stand growth functions for mixed stands based on GYI data
- stand growth functions for Pinus contorta based on temporary and permanent yield study plots in Sweden and Finland

In an independent project stand growth functions for pine are being constructed from GYI data.

Which models and functions will be used in the HUGIN system?

The question will remain for another couple of years. The decision will depend on the accuracy of the various methods, their ability to show treatment effects and the computer costs at the applications. Possibly a few different alternatives can be used, to be applied in different types of forest. A hypothesis is that functions from NFS data are the most suitable for existing middle-aged and old forests, which have only a few decades left to clear-cutting. (Notice that the functions then will be applied on the same set of sample plots as they are derived from.) For existing young forests and especially for stands generated in the model after clear-cutting of the existing stands, possibly functions from GYI data, but calibrated to practical level, will be the best alternative. One reason is that such functions can describe thinning effects in a better way, which is more important to study in stands that will be left to grow for many decades. It is likely that the single tree approach will be more successful than stand functions in inhomogeneous stands (uneven-aged, many species). In homogeneous stands the two methods will perhaps have about the same accuracy.

Effects of thinnings

The thinning effect, as recorded in existing experimental plots, can be considered as "ideal" effect. In practical forestry the selection of trees is not done as well as in experimental plots, strip roads are often used, trees get damaged at logging operations etc. The effect of thinnings in real forests is therefore less than the "ideal" one. Some reduction is necessary in our models, but unfortunately there is still a lack of data suitable for studies of this problem.

Mortality

Recent studies of data from the NFS have shown that the mortality during the last ten years has been at least 10 per cent of gross increment, the main part being windthrow and damage by snow. This is more than was known earlier, but on the other hand severe damages of many kinds have been extremely frequent. The mortality simulator we are ging to develop will be founded mainly on the mortality data from the NFS. A total number of about 75 000 temporary sample plots from six years' survey are now available with estimated mortality during six overlapping periods of three years, together covering the period 1968 - 77.

Ingrowth

The problem of predicting ingrowth of trees is usually not a very big one in our situation (except for the youngest stands), because in the NFS all trees that have reached breast height are calipered. However, since increment is not recorded for trees with diameter less than five cm, we have some problem of making growth functions for small trees.

Adjustment to average weather conditions

Before the constructing of growth functions the recorded increment is adjusted to the level corresponding to average weather conditions. This is done by means of annual ring indices based on sample trees from the NFS according to a method developed by JONSSON, 1969. A problem when preparing the indices is to separate the effect of weather from that of thinnings, fertilization, damages etc. More than one million hectares of forest land have been fertilized at least once. In order to be able to separate the effect of fertilization from the natural growth level and the annual ring indices we have been forced to investigate which plots have been fertilized. This was done through questionnaires and visits at the forest owners concerned. (Notify that it is impossible to determine in the field with an acceptable certainty whether a stand is fertilized or not.)

Effect of fertilization and drainage

As regards the effect of fertilization we still have very few data from repeated fertilization during periods longer than 15 years. Further there is still a rather great uncertainty concerning the effect of practical fertilization compared to that in experiments. An associated sub-project deals with the question of predicting productivity of peat land after drainage and drainage combined with fertilization.

Effect of forest tree breeding

Forest tree breeding in Sweden is directed mainly towards better survival of young trees and/or higher yield of the stands. A fundamental problem when modelling the growth of genetically improved trees is whether the superiority in yield at low ages will remain during the whole rotation. Much information from provenance trials etc. concerns the growth of single trees ("single tree plots"). The question is whether the superiority in growth will be as high in dense stands with hard competition.

Some statistical problems

A number of statistical problems are studied as regards the construction and application of growth functions as well as an analysis of the total accuracy of forecasts in the HUGIN system. The following examples could be mentioned:

- the size and effect of errors in independent variables
- the effect of correlation between variables in inventory data, for instance between site class and age, and how to separate the effect of these variables when used as independent variables in growth functions
- the influence of plot size. Growth functions are often based on data from plots of a another size than the size of the plots or stands they will be applied on. The consequences of this are examined.

The "superforest" - does it exist?

A question very little studied (at least in Sweden) is to what extent the effect of various yieldincreasing measures can be added (or multiplied). Let us take an example. If we establish a stand using the best available technique, if we use the "best" species, for instance lodgepole pine now being introduced in northern Sweden, if the plants are genetically improved and carefully selected in the nursery and the stand is intensively managed with weed and pest control, optimal cleaning, thinning and repeated fertilization - what will be the result? - Certainly there is an upper limit (far) below the yield level that could be calculated from all separate effects. The example is perhaps extreme, but evidently the problem exists even in the case of more moderate management of the forests. We must be aware of the problem, not to make unrealistic forecasts.

Management regimes

The HUGIN system will be used mainly for forecasts including all categories of forest owners. The various owners have varying goals and manage their forests in different ways with varying intensity. We must bare in mind that all the forest owners will never manage the forests in a way that is optimal from the society's point of view. There are also a number of restrictions which influence the possibilities to produce and harvest timber. It often takes a long time until new ideas and new knowledge are applied in a large scale in practical forestry.

In the HUGIN project we are studying how to simulate the practical forest management and how to include the effect of significant restrictions. Although this problem is very important, it cannot be discussed here in detail.

System construction

In a simulation model like the one we are going to develop the following steps seem to be the most time-consuming in the computer:

- initial calculations and editing of data, which however must

not be completely repeated every time
- the selection of units (sample plots) to be treated in a certain way during the various periods
- the projection of the state of the forests by means of growth simulators.

As mentioned above we are at present studying the computer time needed when using some various approaches for growth simulation. Since we have no results to report from this study, the problem will not be discussed further.

FINAL REMARKS

It is obvious to everyone that it is impossible to solve all problems concerning forecasts of timber yield in 4 - 5 years. But we think that an integrated and concentrated effort like the HUGIN project can achieve quite a lot. It will be necessary to continue the work even after 1980, in order to further improve the system and include new knowledge.

A further development is to use the increasing knowledge of casual relationships between growth and primary factors such as insolation, water supply and nutrition. This approach will certainly be useful, for instance for the modelling of the effect of repeated fertilization and for prediction of the effect of acid precipitation.

The next generation of the HUGIN system could be models using information from the permanent sample plots, that we hope will be established within the National Forest Survey. Such plots would give us valuable information on among other things mortality, the development of young stands and on how the forests are managed. But, if permanent plots are introduced, say from the year 1982, the information from the first remeasurement after five years will not appear until in the end of the 1980:s.

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MULTIPLE BENEFITS FROM FORESTS: A SOLUTION TO A MOST PUZZLING PROBLEM

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Demands for forest goods and services are rapidly expanding. This variety of demands is stimulating the development of multiple kinds of management such as wildlife, timber, energy, watershed, wilderness, road, and many others. These new kinds of management for new benefits are creating conflicts in management actions on the same piece of land. Often, two or three benefits can be matched one-to-one with an equal number of management actions. However, when many benefits are considered, the complexities of designing a whole management system become enormous. How to harmonize management actions to yield a desired combination of multiple benefits is perhaps the most puzzling and intractable problem for forestry in this century.

This report describes a system for managing the forest by actions harmonized to produce multiple benefits. The system is cybernetic in that it is guided toward a goal by feedback processes. It is based on a computer model of the managed forest, in which the harvest of timber is regulated to guide the forest toward a steady-state distribution of habitats yielding a constant annual flow of harvested timber and other benefits. This report describes a new structure for forest management systems, the underlying bionomic theories, and how the system can be applied to a forestry situation. The details of the model are described in another paper (Boyce 1977).

NEW STRUCTURE FOR FOREST MANAGEMENT SYSTEMS

Control and communication in biological systems are possible because of structure--the way the component parts are connected. There are many kinds of structure; the type that is of interest here forms information feedback loops that direct the behavior of the system to achieve or maintain some goal, such as keeping the level of blood sugar within the limits for life. These are negative feedback loops with goals. A negative feedback process is one in which a decisionmaking process regulates the system by comparing the present conditions with a standard or goal and making adjustments to achieve steady state. Feedback loops make the system aware of its own performance, using outputs to regulate inputs.

Other kinds of structure may have no feedback loops, or may have feedback which is positive rather than negative. Forests are examples of groups of organisms structured as communities without negative feedback loops and with positive feedback systems. People, acting as managers, can interact with positive feedback systems, such as a forest, to form negative feedback systems which can be brought to a steady state. At steady state these systems can provide certain benefits in perpetuity.

A clock is a good example of a positive feedback system, which when managed by a person becomes a negative feedback system and provides time as a benefit. Energy from the spring moves the hands forward, providing, as an output, the display of time. But the input is not affected by the output; the clock has no mechanism for sensing its own performance and taking corrective action in accordance with a goal. If the clock is running fast, it will deviate increasingly from the standard as time passes. Past states do not control future states, even though the states (positions of the hands) are repetitive. The clock becomes part of a negative feedback process when its owner adjusts the speed control device on its back. He compares the clock's performance with a standard and decides to take corrective action by retarding the speed control. Later, he evaluates the performance again and takes further action, probably making a smaller correction. The system oscillates and gradually reaches the goal.

If forests are to provide multiple benefits in perpetuity, managers and forests must become negative feedback systems with a goal and the potential for steady state. And, obviously, a structure for forest management must be compatible with the natural forest. Thus it is necessary to have theories, validated by potential falsification (Popper 1962; Becht 1974), that describe the structure of the biological systems.

Bionomic Theories

Four bionomic (ecological) theories serve to interrelate empirical data and to reduce complexity. As a model of the dynamics of control and communication in a forest, the theories underlie the cybernetic structure for management.

Theory of Individualistic Systems

Each living organism and its environment forms an individualistic system with negative feedback loops guiding behavior in accordance with the goal of survival. Behavior is directed by decision mechanisms. These mechanisms are genetically and environmentally determined and are the physiological, anatomical, and morphological structure of the individualistic system. Each individualistic system senses and reacts to its own state. Past actions influence future actions to achieve the goal of survival.

Mortality as well as behavior results from the dynamics of individualistic systems. Death occurs when a feedback loop fails to maintain one or more essential functions, such as respiration, circulation, food consumption, hydration, and physical integrity, within the limits for survival. Failure in one or more feedback loops may result from changes in the environment, or failure in the physiological, anatomical, and morphological structure of the individualistic system. It is this mortality, resulting from the dynamics of individualistic systems, that organizes survivors into forest communities.

The theory for individualistic systems would be falsified by discovery that any individualistic system's existence, mortality, or behavior is for the exclusive achievement of a goal for the forest community.

Theory of Community Organization

The mortality of individualistic systems organizes survivors into communities without goals. Large numbers of propagules of plants and animals attempt survival in a forest. Most propagules, which are individualistic systems, die by their own dynamics. This mortality organizes the survivors into successive states. These successive states have dynamics similar to those of the watch hands previously described, but there is a major difference. Watch hands exactly repeat former states, whereas biological communities do not. If the genotypes and environments are similar to combinations in preceding forests, individualistic systems will be similar but not identical. Thus the successive states of organization only appear to be repetitive. These apparently repetitive states of organization are well known as "stages of succession."

Forest communities are organizationally unstable because the surviving individualistic systems are joined without community goals and without feedback loops. Lacking feedback loops the community cannot sense its own stage of succession and cannot bring past actions to influence future actions to achieve a goal for the community. The community has no decision mechanisms to direct the life, death, reproduction, and replacement of mortal individualistic systems. The forest community is an aggregation of survivors. No species is essential for community organization. For example, the removal of American chestnut (<u>Castanea dentata</u> (March.) Barkh.) did not kill the forest in the same way that girdling kills a tree. The chestnut was replaced by other individualistic systems. In a similar way, successive generations of individualistic systems are genetically and environmentally different from preceding generations. Mortal individualistic systems are replaced by different individualistic systems, not as a carousel (Moore 1975) but never again to be exactly repeated. In the absence of community sensing and decision mechanisms to direct all individuals toward a community goal, forests are perpetuated through geologic time without goals and with the dynamics of systems without feedback loops.

The theory for organization of forests would be falsified by discovery of a control mechanism that senses states of community organization and directs the mortality of individualistic systems and the behavior of survivors to achieve organizational goals for the forest community.

Theory of Material and Energy Flows

The flows of energy, nutrients, carbon dioxide, oxygen, water, and organic materials are unidirectional and have the dynamics of systems without goals and without negative feedback loops. The flows of energy and materials through the landscape are delayed by the kinds, numbers, and dynamics of individualistic systems in the forests. Rates of flow are modified by changes in the states of community organization. Cycling is an additional period of delay that is determined by the dynamics of individuals. The significance of this theory is that the delays in flows of energy and materials are brought to a steady state when the distribution of states of community organization is brought to a steady state.

The theory would be falsified by discovery of sensing and decision mechanisms in the community that regulate the inflows, internal status, and outflows of energy and materials to achieve organizational goals for the forest community.

Theory of Multiple Benefits

The kinds and proportions of states or organization (habitats) determine the kinds and proportions of human benefits available from a forest. Many variables define the kinds of habitats of a forest area: plant and animal species, age and size of the dominant plants, amount of shrubs and herbaceous plants, depth of the litter, volume of accumulated wood, amount and proportional distribution of nutrients, physical structure of the soil, biomass, temperature, rainfall, and the flow of nutrients and water. An infinite variety of habitats in a forest results in an infinite variety of potential human benefits, which include scenic values, hunting successes, hiking experiences, timber, streamflow, and places to live for plants and animals. These benefits are available from the states of the forest in the same way that time is available from a watch according to the location of the hands. The stages of succession in a forest are conveniently referred to as "habitats." In these habitats, one may expect to find certain kinds and combinations of individualistic systems and thus certain benefits. It is the proportional distribution of kinds of habitats that provides a particular combination of benefits at a given time.

The theory for multiple benefits would be falsified by discovery that all potential benefits are simultaneously available from a single state of community organization.

A New Management Structure

The bionomic theories reveal a logical structure for management. The new management structure is to convert the goalless, non-feedback system for organization of forest communities to a system with negative feedback loops and with a goal of bringing certain proportional distributions of habitats to steady state. The goal is determined by the selection of one of an infinite number of biologically possible combinations of multiple benefits. The selected combination of benefits determines the proportional distribution of habitats, a single goal, toward which management actions are to be directed. With a single goal, the complexities generated by attempts to match benefits and actions on a one-to-one basis are reduced to a single set of harmonious actions.

The way to achieve a distribution of habitats is to regulate the diversion of older habitats to young habitats. If one uses two or more rotation periods, habitats can be proportionately distributed to provide various combinations of benefits in perpetuity. If two rotation periods are superimposed (fig. 1) so that no habitat is allocated to a rotation period until the time for harvest of stands from the shorter rotation, then a large number of different combinations of habitats and benefits are available for consideration.

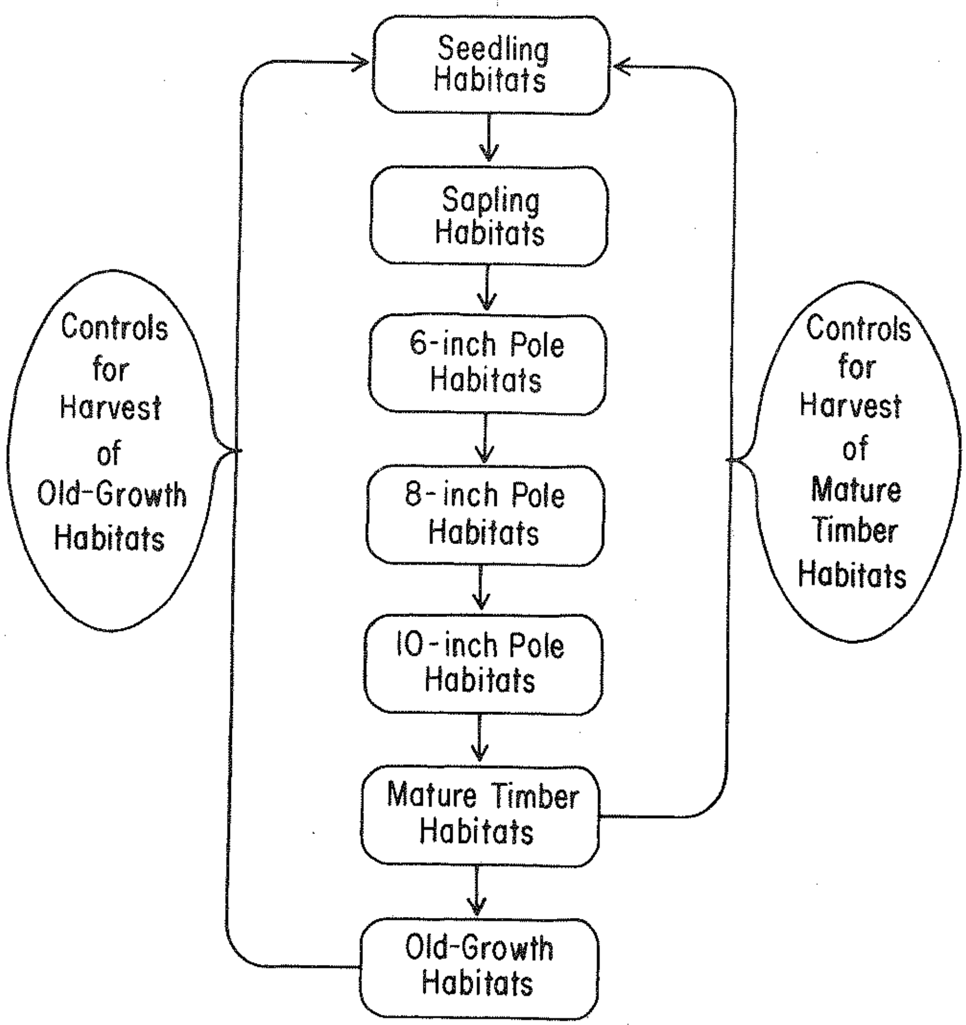

Figure 1.--A simplification of the core model which projects the distribution of habitats from different patterns of harvest.

Habitats may be delineated in various ways. In the example provided in this paper, five habitats are recognized:

Seedling habitats.--Stands with the median diameter of dominant trees less than 1 inch (2.5 cm).

Sapling habitats.--Stands with half of the dominant and codominant trees between 1 and 5 inches d.d.h. (2.5 to 12.4 cm). A few scattered large trees are admitted.

Pole habitats.--Stands with half of the dominant and codominant trees between 6 and 11 inches d.b.h. (12.5 to 27.7 cm). Pole habitats are classified by 2-inch (5 cm) diameter classes as pole-6, pole-8, and pole-10 habitats (fig. 1).

Mature timber habitats.--Stands with half of the dominant and codominant trees between 11 and 16 inches d.b.h. (27.8 to 40.6 cm).

Old-growth habitats.--Stands with half of the dominant and codominant trees larger than 16 inches d.b.h. (40.7 cm).

A dynamic model projects the distribution of these habitats resulting from different patterns of harvest, or modes of management. Three variables compose a mode of management: the fraction of the forest which is to rotate through the old-growth stage, as opposed to being harvested as younger mature timber; the ages of harvest for old-growth timber and mature timber; and the size of openings created when timber is removed.

Some old-growth timber is needed in an area being managed for multiple benefits. Old growth furnishes superior habitat for many kinds of wildlife, it has obvious esthetic value, and, when finally harvested, it yields large saw logs which are essential for certain uses. Delaying the harvest of mature timber beyond the period for maximum timber values can increase mast and the availability of this kind of habitat for many plants and animals.

The size of openings is an important variable in the mode of management. Large openings lead to efficient harvest of timber and less movement of sediment because fewer roads are needed in proportion to the area harvested. Different sizes of openings provide different kinds of habitats for plants and animals. And, the sizes of stands for the older habitats are determined by the size of openings at harvest. Size of openings does not affect the rate of harvest but does affect the number and distribution of openings for a given rate of harvest.

The new management structure is comparable to that of a watch (the forest) and a person (the forest manager). The management goal is to keep the hands of the watch (habitats of the forest) proportionately distributed to display accurate time. There is an infinite number of "accurate" times, such as eastern standard time and others, as there is an infinite number of combinations of forest benefits. The selection of an accurate time (a combination of benefits) determines the proportional distribution of the hands on the watch (the habitats in the forest).

The biologically possible combinations of benefits can be computed with the dynamic model of the managed forest. One examines different modes of management by varying the coefficients for the proportion of the total area permitted to succeed through old-growth habitats, the age of harvest of the old-growth timber, the age of harvest of the mature timber, and the size of openings formed by harvesting. The rates and directions of succession can be predicted for short intervals such as a decade and with acceptable accuracy for many years. Inventories taken each decade can be used in a corrective feedback loop to keep the model of management actions congruent with the dynamics of the forests.

AN EXAMPLE

Once the theories and the structure for management are stated, a number of mathematical methods can be used to model the system for management. The techniques of industrial dynamics and the DYNAMO compiler (Forrester 1961; Pugh 1976) are used in the example provided here.

From the bionomic theories the availability of benefits depends on the state of physical organization of the forest--the proportions which are covered by seedlings, saplings, pole timber, mature timber, and old growth. Algorithms, or statements of relationship, must be constructed to express how a particular benefit depends on the distribution of habitats. The algorithms are written as nonlinear supplementary equations. An algorithm may be added, deleted, or changed without affecting the core model or the other algorithms; thus this part of the structure is quite flexible and may be readily adapted to different circumstances or modified by new knowledge from research or inventories.

The particular relationships assumed between habitats and benefits must always be regarded as tentative and subject to revision. We can derive algorithms from current practical and scientific knowledge and make adjustments with new information. For example, a potential timber index was developed from yield, stand, and volume tables for even-aged, upland oak forests (Schnur 1937). Similar computations were made for the habitats of various animals and plants, for the expected scenic values, for the movement of sediment, and for other benefits and impacts. These algorithms are described in a previous report (Boyce 1977). Any number of benefits for which adequate data are available may be used.

All benefits and impacts were scaled 0 to 1. The value 1 represents the production of a particular benefit if management favored that benefit over all others. For impacts the value 1 represents the undesirable limit. Zero is the minimum for benefits and the minimum for impacts. For example, the potential timber index is the ratio of maximum harvest to actual harvest. The indices for bluebird, ovenbird, deer, squirrel, and pileated woodpecker are ratios of the potential to the actual amount of habitat for these animals. The index called "sediment" is the ratio of the geologic to the actual movement of sediment and is related primarily to the amount of roads constructed or used. "Ugly" is a subjective indicator of the closeup, disorderly appearance of recently harvested areas.

The example is 6,396 acres (2,588 ha) of deciduous forest on National Forest lands in Buncombe County, North Carolina. Three of the infinite number of modes of management are illustrated. The "fast" mode provides timber and bluebird habitat near the maximum and minimizes the habitat types for squirrels and pileated woodpeckers (fig. 2). For the fast mode only 1 percent of the area is permitted to succeed through old-growth habitats, 99 percent of the area is harvested at about age 85 years as mature timber, and harvest openings are limited to about 25 acres (10.1 ha).

The "salvage" mode provides maximum habitat for pileated woodpeckers and minimizes the habitat for bluebirds (fig. 3). Sediment remains at the natural or geologic level for the first 70 years because there are few trees near the 300-year age for harvest and few roads are constructed. After 70 years, the construction of roads to harvest 300-year-old timber increases the movement of sediment. Coefficients for the salvage mode are to permit 99 percent of the area to succeed through old-growth habitats, harvest the old growth when about 300 years old, and limit harvest openings to about 1 acre. More roads are required to harvest trees in 1-acre (.4 ha) openings than to harvest the equivalent area in 25-acre (10.1 ha) openings. This relationship explains the relatively high rate of sediment movement as old growth is harvested. "Ugly" has a low index because openings are so small.

One intermediate mode called "moderate" illustrates another of the infinite number of choices (fig. 4). This combination of benefits and impacts may be acceptable to most people because no benefits have a 0 index and no impact indices approach 1. The moderate mode permits 30 percent of the area to succeed through old-growth habitats, and old growth is harvested at about 250 years. The 70 percent of the area succeeding through mature timber habitats is harvested at age 95 years. Harvest openings for old growth are limited to 2 acres (.8 ha), and those for mature timber to 25 acres (10.1 ha).

Displays such as figures 2, 3, and 4 permit interested parties to make a rational choice among combinations of benefits. The infinite number of potential choices provides alternatives not previously defined or considered. The selection of a combination of benefits determines both the goal and the actions for management. The goal is to achieve and maintain the particular distribution of habitats that will provide the selected combination of benefits. The only action considered here is to harvest old-growth and mature timber at the rates and with the sizes of openings specified to achieve the goal. Thinning, fertilization, and other actions to enhance habitats may be included in the analyses (Boyce 1977).

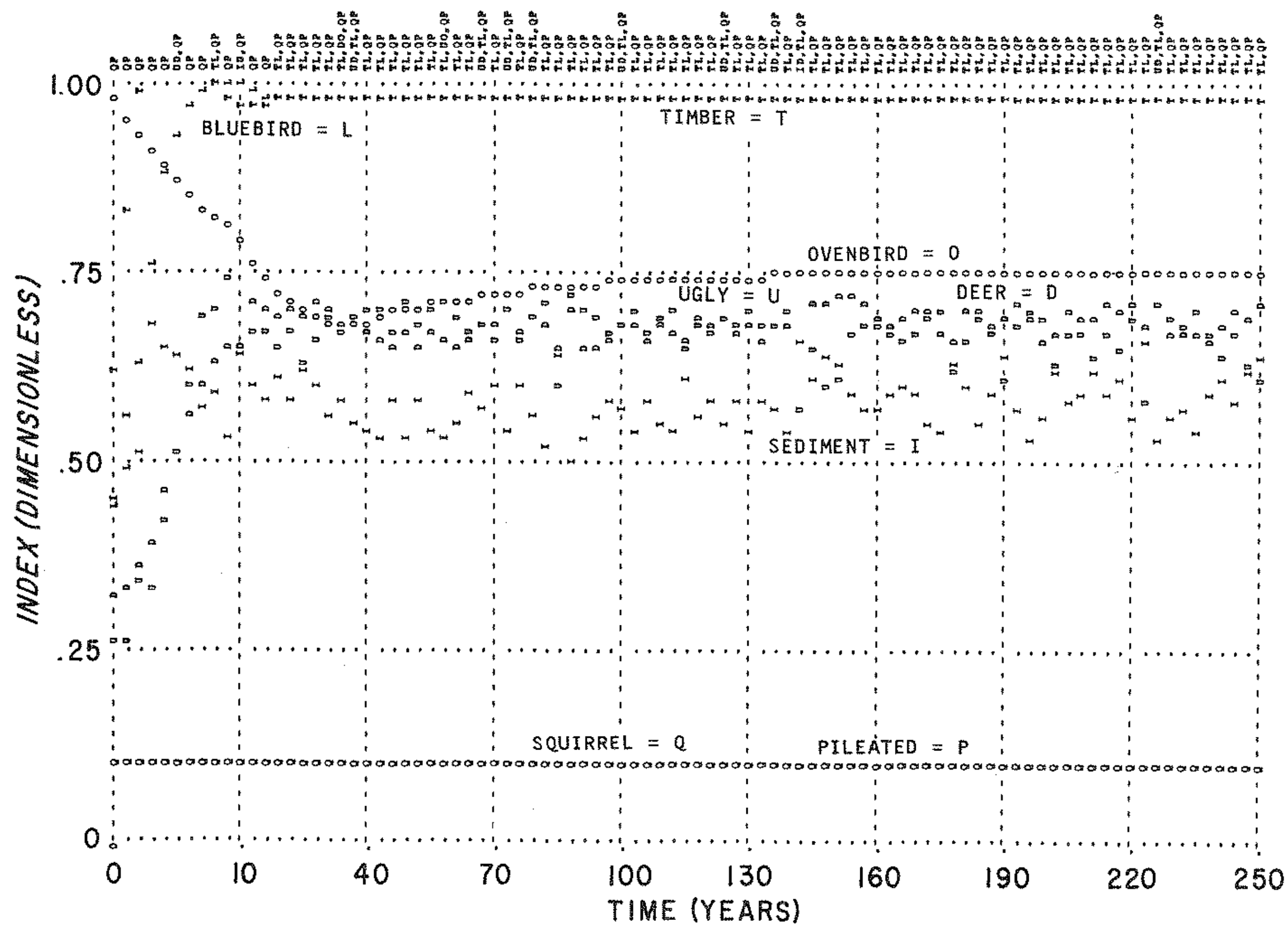

Figure 2.—A computer plot, enhanced for illustration, showing the behavior of benefit and impact indices for the "fast" mode of management as generated through time by the model. Indices are plotted annually for the first 10 years, then at 3-year intervals. Letter groups at the top of the graph indicate where the indices coincide. Only the first letter of each group is plotted.

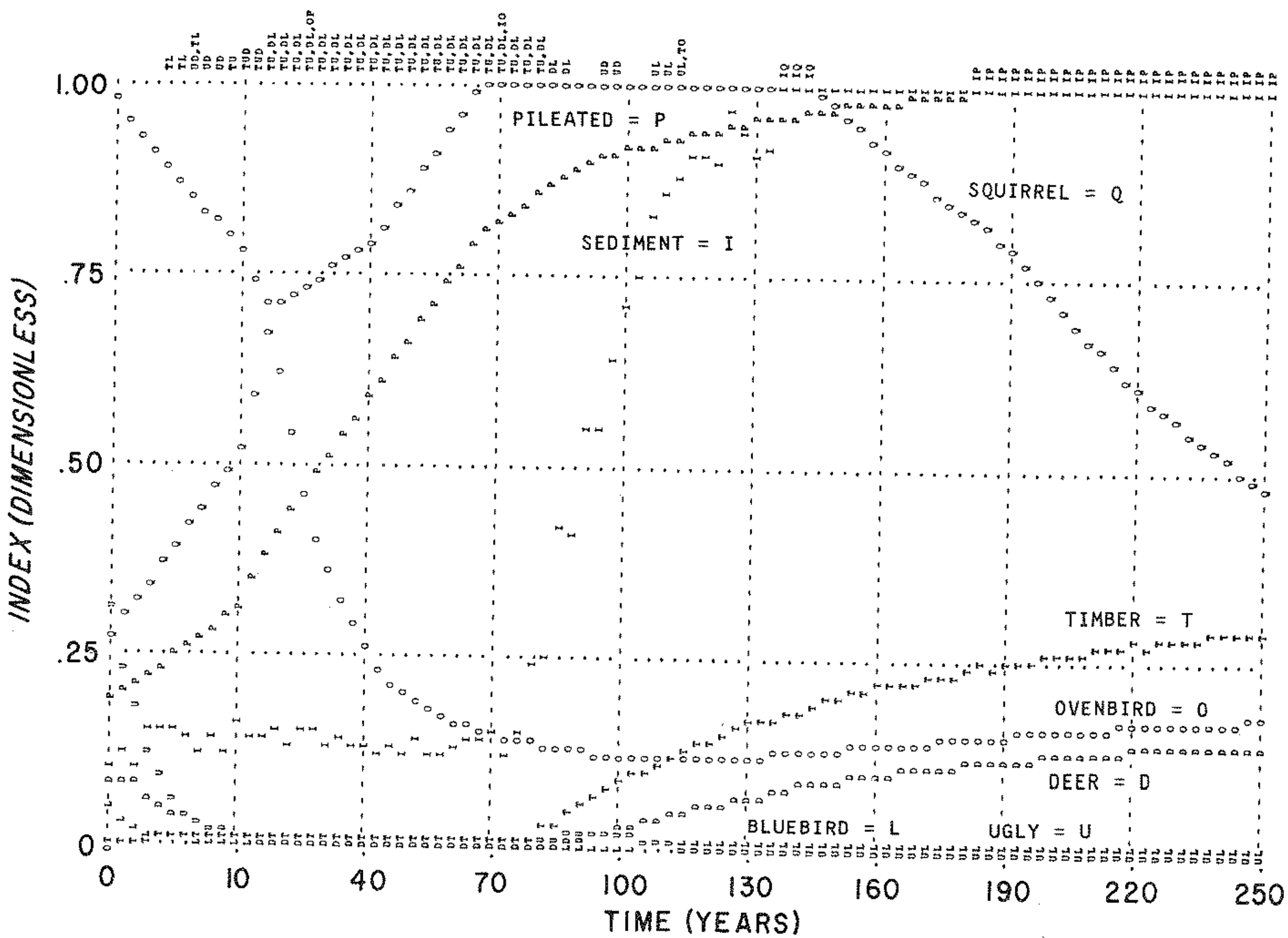

Figure 3.—A computer plot, enhanced for illustration, showing the behavior of benefit and impact indices for the "salvage" mode of management as generated through time by the model. Indices are plotted annually for the first 10 years, then at 3-year intervals. Letter groups at the top of the graph indicate where the indices coincide. Only the first letter of each group is plotted.

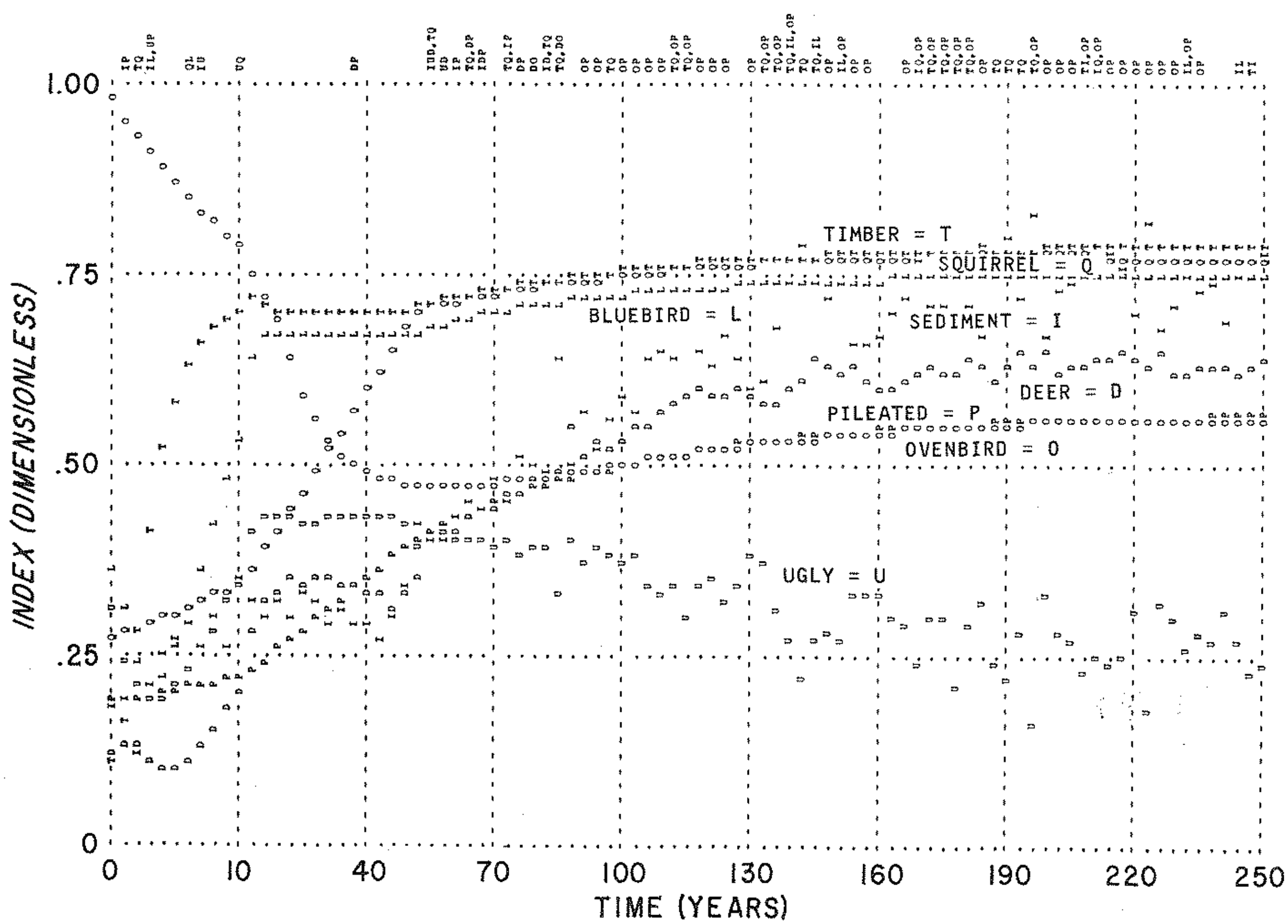

Figure 4.--A computer plot, enhanced for illustration, showing the behavior of benefit and impact indices for the "moderate" mode of management as generated through time by the model. Indices are plotted annually for the first 10 years, then at 3-year intervals. Letter groups at the top of the graph indicate where the indices coincide. Only the first letter of each group is plotted.

The model computes the management actions for the next decade in relation to the current inventory and the goal. For the example of the "moderate" mode of management, about 210 years are required to bring the distribution of habitats to the equilibrium distribution that will provide the desired combination of benefits for a long time (fig. 3). The management actions for the next decade are to harvest no old-growth habitats and limit the harvest of mature timber. The model directs the 10-year rates of timber harvest to bring the distribution of habitats to the goal (table 1).

Table 1.--The rates of harvest of mature timber habitats for the next decade, the current inventory of the distribution of habitats, expected change for the next decade, and the goal for the "moderate" example

Years from inventory	Rate of mature timber harvest	Distribution of habitats				
		Old growth	Mature	Pole	Sapling	Seedling
	Acres (ha)	Percent				
Current inventory		2.9	5.7	68.7	19.6	3.1
1	29 (11.7)	2.9	6.9	68.0	19.6	2.6
2	40 (16.2)	3.0	7.9	67.3	19.6	2.2
3	47 (19.0)	3.1	8.8	66.6	19.5	2.0
4	48 (19.4)	3.3	9.5	65.8	19.3	2.1
5	49 (19.8)	3.4	10.1	65.1	19.1	2.3
6	49 (19.8)	3.5	10.5	64.6	18.9	2.5
7	50 (20.2)	3.6	11.0	63.9	18.7	2.8
8	50 (20.2)	3.8	11.4	63.1	18.6	3.1
9	51 (20.6)	3.9	11.8	62.5	18.5	3.3
10	51 (20.6)	4.0	12.1	62.0	18.5	3.4
210 (goal)		18.0	10.5	41.9	25.4	4.2

Deviations from the management actions can be detected in the 10-year inventories. The inventories are used to correct harvest rates for unexpected changes that may result from changes in markets, road construction, wildfires, land use changes, insects, diseases, weather, and other effects. The inventory is the sensing mechanism of a negative feedback loop that functions each decade and keeps the management system congruent with the forest.

CONCLUSIONS

Described here is a new structure for forest management systems, the underlying bionomic theories, and how the system can be applied to a forestry situation.

The system harmonizes forest management actions for multiple benefits, gives all interested parties an opportunity to participate in the selection of biologically possible combinations of benefits, and provides alternatives for management previously not available. No one benefit is given preference over others and no benefits are arbitrarily treated as constrained to the production of others. Management control is achieved with a dynamic model that is kept congruent with the forest through periodic inventories.

Keywords: Forest management, bionomic theories, dynamic model, multiple benefits.

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YIELD OF SITE-PREPARED SLASH PINE PLANTATIONS IN THE LOWER COASTAL PLAIN OF GEORGIA AND FLORIDA

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SUMMARY

This paper describes a system for estimating growth and yield of unthinned slash pine plantations established following intensive site preparation. The procedure was developed from field data collected on 487 plots in the lower coastal plain of Georgia and Florida. Predictor variables used by the system are plantation age, average height of dominants and codominants, and number of surviving stems per acre at the prediction age. The basic predictions produced by the system are (1) number of stems per acre by one-inch dbh classes, and (2) average tree heights by one-inch dbh classes. Per acre volume and weight predictions to various merchantability limits can be easily calculated from this basic stand structure information. Some comparisons with predictions from previously developed yield tables are discussed.

Keywords: Growth and yield, Weibull distribution, yield tables, diameter distribution.

INTRODUCTION

The lower coastal plain of southeastern Georgia and northern Florida contains one of the world's great concentrations of forest industry. The approximate boundaries of this area (which is often referred to as "the flatwoods") are shown in Figures 1 and 2. Some 9.8 million acres of commercial forest land are contained within the Georgia-Florida flatwoods of which 4.1 million acres are owned by or are under lease to forest industry (McClure, 1970; Bellamy, 1971; Knight, 1971).

The most recent figures available (Knight and McClure, 1971; Knight and McClure, 1974) indicate that this region contains at least 2.2 million acres of pine plantations with 1.5 million acres of this either owned or managed by forest industry. As time passes, the area of plantations will continue to increase and the industrial wood supply will become increasingly dependent on plantation-grown trees.

Prior to about 1955, almost all plantations in the flatwoods were planted on old-fields or established in so-called "rough-woods" situations which usually involved simply burning the site following harvest of a natural stand followed by subsequent planting. Rough-woods plantings were generally attempted only when competing understory vegetation was light but the results of such plantings were still often unsuccessful. Numerous yield tables are available for old-field plantations in the flatwoods (Coile and Schumacher, 1964; Bennett and Clutter, 1968; Bennett, 1970). Yield estimates for rough-woods plantations have been published by Coile and Schumacher (1964).

During the 1950's forest industries in the flatwoods began to expand their plantation acreage through the use of mechanical site preparation followed by planting. Today, some form of mechanical site preparation and planting is the standard method of regeneration for forest industry lands in the flatwoods. Because of the relatively recent introduction of mechanical site preparation as a silvicultural tool, yield tables for site-prepared plantations have not been previously available. This paper describes a procedure for predicting yields of site-prepared slash pine (*Pinus elliottii* Engelm.) plantations in the Georgia-Florida flatwoods.

THE DATA BASE

During the summer of 1975, University of Georgia field crews, working in the Georgia-Florida flatwoods, measured 417 temporary sample plots in unthinned, site-prepared, slash pine plantations made available by co-opera-

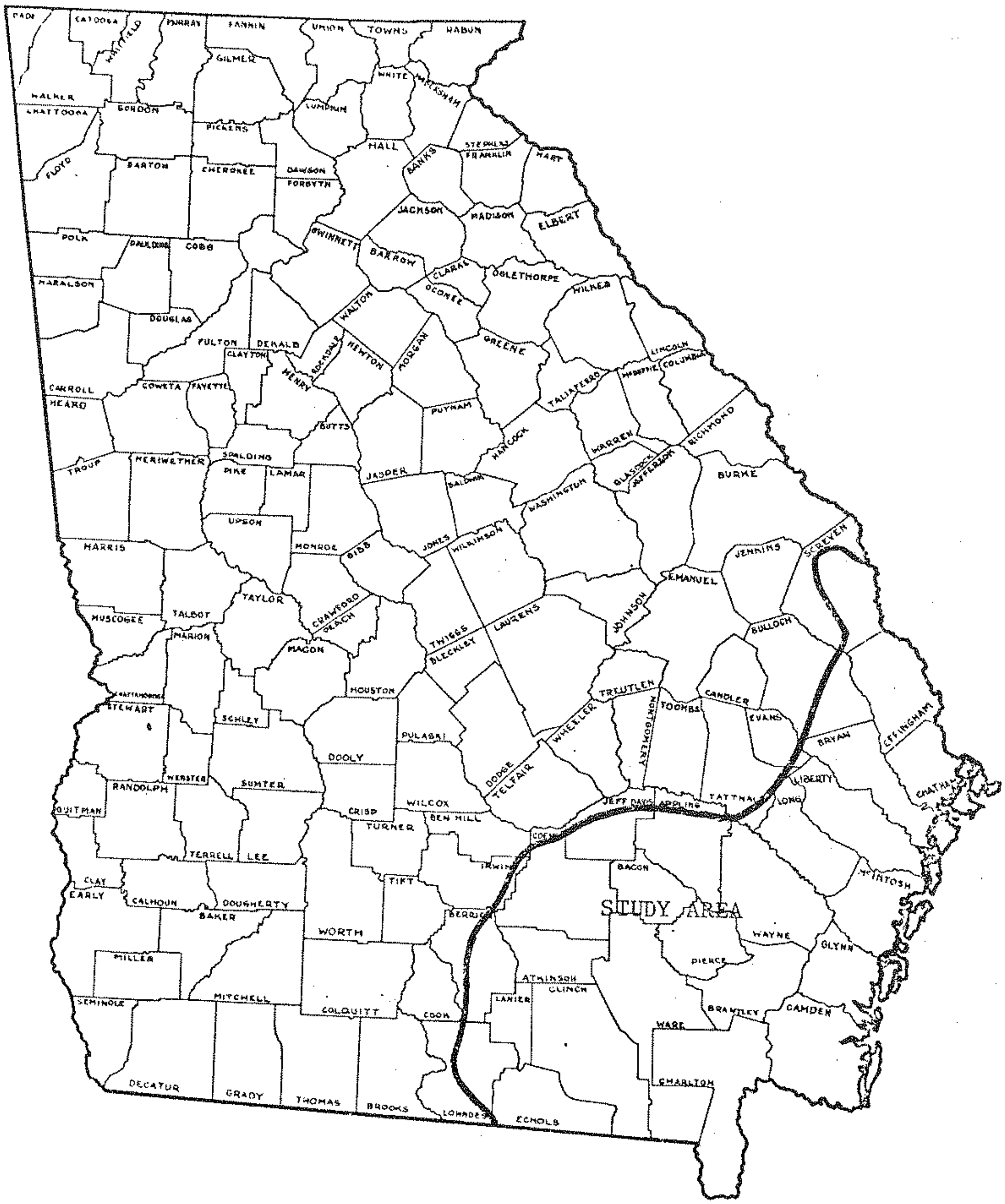

Figure 1. Location of study area in Georgia.

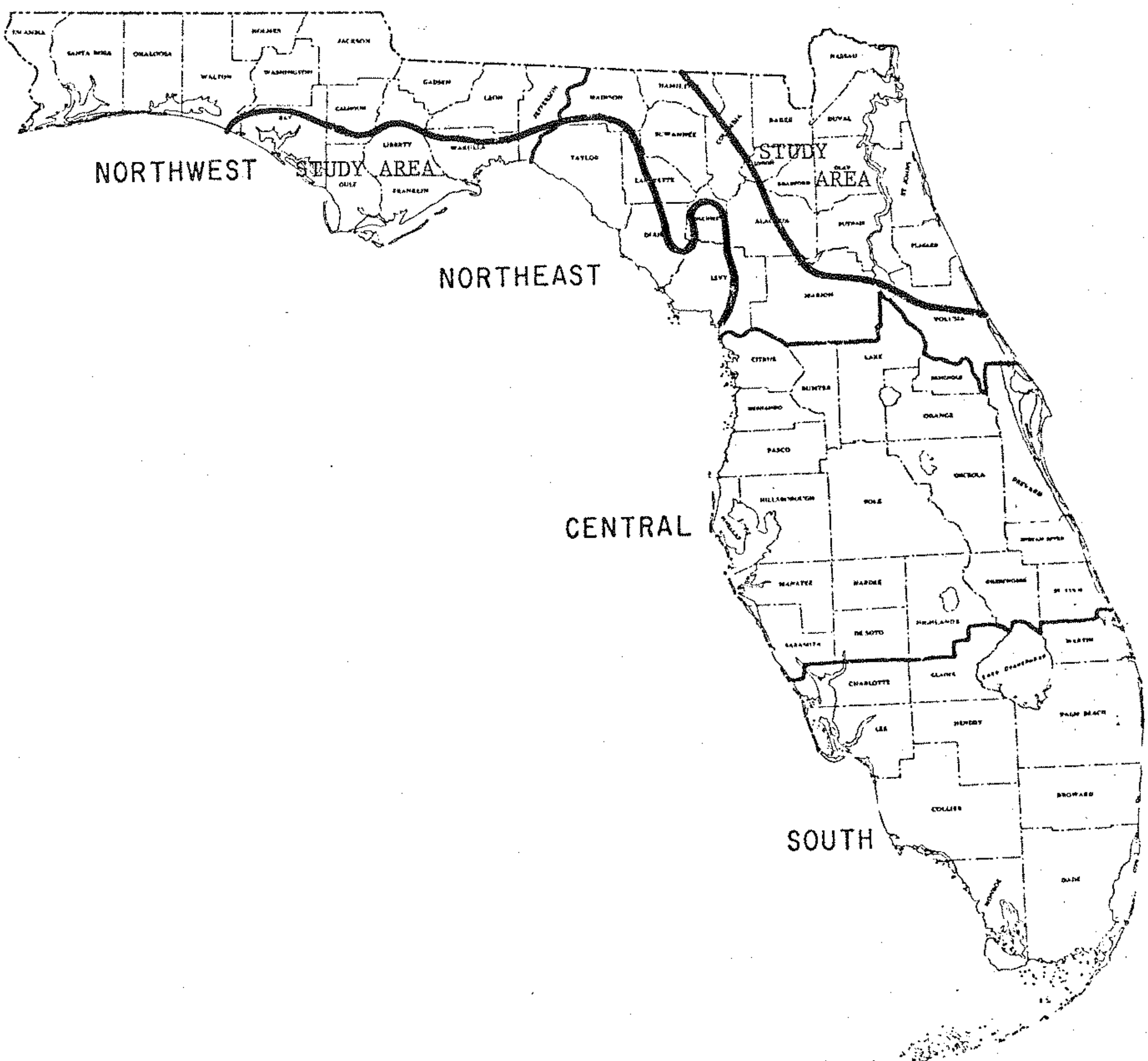

Figure 2. Location of study area in Florida.

ating pulp and paper companies.[1/] This initial data base was subsequently augmented by the inclusion of 70 additional plots that had been previously measured by cooperating companies. Out of the total of 487 plots, 391 were "select" plots purposively located within sample stands to cover areas of uniform (not maximum) density while 96 "random" plots were established at randomly established locations within sample plantations. All sample plots were rectangular in shape and were laid out to include approximately 64 original planting spacings (usually in an 8 x 8 configuration). On each plot the following measurements were taken:

1. Plantation age
2. Length and width of plot (in feet)
3. Number of rows and planting spaces
4. Diameter breast height (o.b.) for each tree on the plot
5. Diameter, height, and crown class for at least two sample trees in each occupied diameter class.

From the field measurements, number of trees per acre by diameter class and average height of the dominant and codominant sample trees were determined for each of the sample plots. Diameter-height regressions of the form

$$\ln(H_{ij})^{2/} = K_{0i} + K_{1i}\left(\frac{1}{D_{ij}}\right) \tag{1}$$

were fitted on a plot-by-plot basis where H_{ij} = height of the jth sample tree on the ith plot, D_{ij} = diameter of the jth sample tree on the ith plot, and K_{0i} and K_{1i} are the sample regression coefficients determined for the ith plot. Individual tree volumes were computed with the volume equation (Bennett et al., 1959)

$$V = 0.002706\ D^2H - 1.045389 \tag{2}$$

where V = cubic-foot volume (outside bark) to a 4.0 inch top (outside bark)
D = observed diameter at breast height (inches), and
H = observed total tree height (feet).

Equations (1) and (2) were used to compute individual tree volumes by diameter classes on a plot-by-plot basis. These volumes were multiplied by

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[1/] Brunswick Pulp Land Co., Container Corporation of America, Continental Can Co., Inc., Gilman Paper Co., Hudson Pulp and Paper Corp., ITT Rayonier, Inc., Owens-Illinois, St. Joe Paper Co., St. Regis Paper Co., Union Camp Corp., Buckeye Cellulose Corp., and the International Paper Co. all made suitable areas available for study and provided historical data concerning site preparation methods applied prior to stand establishment. The first ten companies named have also provided financial support for data analysis activities through their participation in the University of Georgia Plantation Management Research Co-operative.

[2/] The notation "ln" indicates a logarithm to the base e.

the observed number of trees in each diameter class and converted to a per acre basis to give the observed yield [3/] of the plot.

Additional details concerning methods of data collection and the characteristics of the data set have been presented by Belcher. [4/]

PREDICTING PLANTATION STRUCTURE AND YIELD

Diameter Distributions

Methods for estimating stand yields from predicted diameter distributions were originally described by Clutter and Bennett (1965) and have been subsequently applied in various modified forms by a number of other research workers. The predictions developed here are based upon use of the Weibull distribution (Bailey and Dell, 1973) as the underlying model for diameter distributions in the sample plantations.

The probability density function for the Weibull distribution is

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} e^{\{-(\frac{x-a}{b})^{c}\}} \qquad (a \leq x < \infty) \qquad (3)$$

$$= 0 \text{ elsewhere,}$$

where $0 \leq a \leq \infty$, $b > 0$, and $c > 0$. The cumulative distribution function is obtained by integrating equation (3) giving

$$F(x) = 1 - e^{\{-(\frac{x-a}{b})^{c}\}} \qquad (a \leq x < \infty) \qquad (4)$$

$$= 0 \text{ elsewhere.}$$

The proportion of the distribution lying between any two specific values of x say x_1 and x_2 $(x_1 \leq x_2)$ is defined as

$$P(x_1, x_2) = F(x_2) - F(x_1)$$

3/ All yields used in this study are outside-bark cubic-foot volumes per acre to a 4.0-inch top (o.b.) for trees 4.6 inches DBH and greater.

4/ Belcher, David M. 1977. Yield of slash pine plantations in the Georgia and North Florida Coastal Plain. Unpublished M.S. thesis. University of Georgia.

or

$$P(x_1, x_2) = e^{\{-(\frac{x_1 - a}{b})^c\}} - e^{\{-(\frac{x_2 - a}{b})^c\}}. \qquad (5)$$

Equation (3) was fitted to the observed diameter distribution data from each of the 391 select plots. Estimates of the parameters a, b, and c were calculated for each plot using a maximum likelihood algorithm developed by Harter and Moore (1965) as incorporated in a FORTRAN language fitting program produced by Bailey (1974). Standard multiple regression procedures were then used to develop equations for predicting the estimated Weibull parameters ($\hat{a}$, $\hat{b}$, and $\hat{c}$) as functions of plantation age (A), average height of dominants and codominants (H_d), and number of surviving stems per acre at the prediction age (N). In most prediction situations, H_d values would be obtained by referencing an appropriate site index equation. The resulting equations are:

$$\hat{a} = 8.04979 - 0.1015176A + 0.131034H_d - 3.04792 \ln(H_d) \qquad (6)$$

$$r^2 = .107 \qquad s_{y.x} = 1.044$$

$$\hat{b} = -3.84157 + 0.05628A + 481.30737/N + 1.91111 \ln(H_d) \qquad (7)$$

$$r^2 = .357 \qquad s_{y.x} = 1.114$$

$$\hat{c} = 3.6261 + 9.4599/A \qquad (8)$$

$$r^2 = .020 \qquad s_{y.x} = 1.091$$

Although the r-square values for these equations are relatively low, use of the equations produces better estimates of the observed diameter distributions than the use of constant values for a, b, and c. For a given set of values for A, H_d, and N, the predicted diameter distribution is generated by first solving equations (6), (7), and (8) for a, b, and c and then solving, for each diameter class containing enough trees to be practically significant, the equation

$$n_i = NP(i - .5, i + .5) \qquad (9)$$

where n_i = number of trees per acre in diameter class "i", and $P(i - .5, i + .5)$ is the distribution proportion function defined in equation (5).

As a practical check on the utility of this technique, differences in observed and predicted mean diameters for each plot were tabulated. Table (1) shows that predicted mean diameter differed from observed mean diameter by less than 0.5 inches in 84.7 percent of the observations and by less than 1.0 inches in 99.0 percent of the observations. As a further check, frequencies of differences in observed minus predicted basal area were calculated and are presented in Table (2). Deviations of less than 10 square feet are present in 62.1 percent of the sample plots while 97.4 percent of the differences are less than 25 square feet.

Table (1) -- Frequency of observed minus predicted mean diameters for 391 sample plots.

Difference in mean diameters (in.)	Frequency	Relative Frequency (%)
less than -1.0	1	0.2
-1.0 to -0.5	25	6.4
-0.5 to 0.5	331	84.7
0.5 to 1.0	31	7.9
greater than 1.0	3	0.8
	391	100.0

Table (2) -- Frequency of observed minus predicted basal area for 391 sample plots.

Difference in Basal Area (sq. ft.)	Frequency	Relative Frequency (%)
less than -25	6	1.5
-25 to -20	6	1.5
-20 to -15	19	4.9
-15 to -10	44	11.3
-10 to -5	46	11.8
-5 to 5	139	35.5
5 to 10	58	14.8
10 to 15	36	9.2
15 to 20	23	5.9
20 to 25	10	2.6
greater than 25	4	1.0
	391	100.0

Height-Diameter Relationships

Estimation of per acre volumes or weights from the predicted diameter distribution requires some procedure for predicting average tree heights by diameter classes. As a first step toward developing this capability, the regression model

$$\ln(H_{ij}/H_{di}) = \hat{W}_{0i} + \hat{W}_{1i} \ln[F(d_{ij})] \tag{10}$$

was separately fitted to sample tree data from each of 326 select sample plots

where H_{ij} = total height of jth sample tree on the ith sample plot,

H_{di} = average height of dominants and codominants on the ith sample plot,

d_{ij} = diameter of the jth sample tree on the ith sample plot,

$\hat{W}_{0i}$, $\hat{W}_{1i}$ = sample regression coefficients estimated from the data, and

$F(D_{ij})$ = Weibull cumulative distribution function value for $x = d_{ij}$ (see equation (4)).

Equations to predict $\hat{W}_0$ and $\hat{W}_1$ from A, H_d, and N were then developed using standard regression procedures. The final equations obtained are:

$$\hat{W}_0 = 0.492376 - 0.80449/A - 0.10039 \ln(H_d) \tag{11}$$

$$r^2 = .196 \qquad s_{y.x} = .02898$$

and

$$\hat{W}_1 = 2.52167 + 0.00010503N + .009147H_d - 3.5611/A - 0.686789 \ln(H_d) \tag{12}$$

$$r^2 = .209 \qquad s_{y.x} = .08124$$

For any given set of values for A, H_d, and N, a height-diameter relationship is obtained by solving equations (11) and (12), and then substituting the $\hat{W}_0$ and $\hat{W}_1$ values into an algebraically rearranged form of equation (10) to obtain

$$H = H_d\, e^{\hat{W}_0}\, [F(D)]^{\hat{W}_1} \tag{13}$$

where D is tree DBH and H is the corresponding estimated total height. Although the r^2 values for equations (11) and (12) are relatively low, equation (13) explain 87.6 percent of the variability in sample tree heights. When equation (13) is used in conjunction with an estimated diameter distribution, the D values used are the mid-point diameters of the occupied classes with the exception of the diameter class containing the Weibull location parameter (a). In this case, the diameter class height is computed using a diameter mid-way between a and the upper limit of the class.

Calculating Predicted Per Acre Yields

The procedure for estimating yield for any combination of age, number of trees per acre, and average height of dominants and codominants is as follows:

1. Determine the expected diameter distribution in terms of number of trees per acre in each diameter class using equation (9).

2. For each occupied diameter class calculate a predicted height using equation (13).

3. Use the midpoint of each diameter class and its associated height in an appropriate volume (or weight) equation to calculate expected volume (or weight) per tree for each diameter class.

4. Multiply expected volumes (or weights) per tree by the corresponding diameter class frequencies.

5. Sum the values obtained in step 4 to obtain the final predicted yield.

Table (3) shows an example of this technique for a sample plot with A = 25 years, N = 400 trees per acre and H_d = 60 feet.

As a preliminary test of how well the model fits the data on which it was based, a comparison was made between observed and predicted yields for all 391 select sample plots by subtracting the predicted yields from the observed yields and computing the standard deviation of the differences. The appropriate t-test shows the mean difference of 36 cubic feet per acre to be significantly different from zero at the 5% significance level. Although this average overrun of actual yield in relation to predicted yield is small in practical terms, (less than 3 percent of the average observed yield) users may wish to adjust estimates produced by the previously described system by adding 36 cubic feet per acre to all predicted yields.

An additional comparison was made by tabulating the percent deviations over the observed yield classes. Percent deviation was determined by subtracting the predicted yield from the observed yield, dividing this difference by the observed yield, and multiplying by 100. Table (4) shows the distribution of 390 sample plots by percent deviation and 1000 cubic feet per acre volume classes. (One sample plot was excluded because it had zero observed volume). Out of 390 plots, 64 percent showed deviations of less than 20 percent in volume and 87 percent deviated by less than 50 percent in volume. It should be noted that the higher percentage deviations (13 plots had deviations less than or equal to -100%) are associated with relatively low observed volumes.

Comparison With Random Plot Yields

The model previously described is based on data from purposively selected sample plots with uniform stocking and no evidence of damage. Thus it is an estimator of potential yield of slash pine plantations in the study area. In order to evaluate the ability of the model to predict average yields attained

$\hat{a} = 0.8033$ $\hat{b} = 6.5934$ $\hat{c} = 4.0045$

$\hat{W}_0 = 0.0492$ $\hat{W}_1 = 0.1581$

Dbh Class (inches)	Trees/ Acre	Midpoint F(D)	Height (feet)	Volume/ Tree (cu. ft.)	Volume/ Class (cu. ft./acre)
2	1.7	.0011	21.4	0.0	0.0
3	9.3	.0122	31.4	0.0	0.0
4	26.5	.0536	39.7	0.0	0.0
5	53.1	.1511	46.7	2.11	112.5
6	80.1	.3199	52.6	4.08	327.0
7	91.2	.5416	57.2	6.54	596.3
8	75.6	.7583	60.3	9.40	710.9
9	43.1	.9084	62.1	12.57	541.0
10	15.6	.9774	62.8	15.95	249.2
11	3.3	.9968	63.0	19.58	64.7
12	.4	.9998	63.0	23.50	8.6
Total	400.0				2610.2

Table (3). Example application of the yield prediction technique for $A = 25$, $H_d = 60$, and $N = 400$.

Table (4) -- Frequencies of percent deviation of observed minus predicted volumes by volume class for 390 sample plots.

Percent Deviation	500	1500	2500	3500	4500	5500	Total
≤ -95%	13						13
-90	5						5
-80	3						3
-70	5						5
-60	4	1					5
-50	4	1	1				6
-40	10	4		1			15
-30	13	10					23
-20	10	15	2				27
-10	18	24	9	3			54
0	15	30	15	3	1		64
10	10	28	12	5	3		58
20	18	21	8	6		1	54
30	8	11	3		1		23
40	9						9
50	2	5					7
60	2	2					4
70	4						4
80	6						6
90	3						3
≥ 95	2						2
Total	164	152	50	18	5	1	390

under current conditions, predicted values for volume per acre were compared with observed values for the 96 sample plots which were randomly selected in the study area. On the average, observed yields were 66 cubic feet per acre less than predicted yields, a difference which is statistically significant at the 5 percent level. Hence, when the model is used to estimate potential yield of healthy, uniformly stocked plantations, an average underestimate of 36 cubic feet can be expected, but when the model is applied to estimate currently attainable actual yield of existing plantations, an average overestimate of 66 cubic feet per acre can be expected.

COMPARATIVE YIELDS OF OLD-FIELD, SITE-PREPARED, AND ROUGH-WOODS PLANTATIONS

The availability of yield estimates for site-prepared slash pine plantations invites comparison of these estimates with predicted yields previously published for old-field and rough-woods plantations of the same species grown in the same geographical area. Comparable rough-woods and old-field estimates have been tabulated by Coile and Schumacher (1964). A second set of appropriate old-field predictions has been prepared by Bennett and Clutter (1968). (The latter figures have also been presented in a modified format by Bennett (1970)).

Yield estimates predicted by the method described in this paper together with comparable estimates from the sources given above are presented in Table (5) for various combinations of plantation age, site index, and number of surviving stems per acre. Dominant heights corresponding to various combinations of age and site index were calculated for the system herein described using a previously unpublished site index equation developed by the senior author algebraically rearranged into the form

$$\ln(H) = 4.4548 - 17.6098/A + e^{9.2679\left(\frac{1}{A}-\frac{1}{25}\right)}\,[\ln(S) - 3.7504] \qquad (14)$$

where S = site index (base age 25). The entries in Table (5) for site index 70 include densities of 200, 300 and 400 stems per acre rather than 300, 400 and 500 stems per acre since it is doubtful that a stand density as high as 500 stems per acre could be achieved at age 25 or 30 on site index 70 land.

Inspection of the results shown in Table (5) shows that the expected yields for site-prepared plantations are almost always greater than the corresponding expected yields for rough-woods plantations. There are some cells where the rough-woods values are slightly greater than the associated site-prepared yields, but it would be easy to overinterpret the importance of these outcomes in view of the very small sample size involved in the Coile and Schumacher rough-woods estimates. It is of interest to note that agreement between the two sets of old-field estimates is quite good for site indices 50 and 60. For site index 70, the Coile and Schumacher expected yields exceed corresponding values from Bennett and Clutter by 300 to 500 cubic feet per acre. It is, I believe, quite possible that reasonable men might disagree as to which of these two sets of estimates for site index 70 is most realistic.

Table (5) -- Comparable yield estimates for old-field, site-prepared, and rough-woods slash pine plantations.

SITE INDEX 50			
Type	Estimated Yield (cubic feet/acre)		
Age 20	N=300	N=400	N=500
Rough-woods (Coile and Schumacher)	932	988	1016
Site-prepared	817	889	967
Old-field (Coile and Schumacher)	1185	1284	1351
Old-field (Bennett and Clutter)	1294	1372	1417
Age 25	N=300	N=400	N=500
Rough-woods (Coile and Schumacher)	1263	1365	1431
Site-prepared	1247	1406	1571
Old-field (Coile and Schumacher)	1587	1744	1860
Old-field (Bennett and Clutter)	1779	1927	2030
Age 30	N=300	N=400	N=500
Rough-woods (Coile and Schumacher)	1538	1679	1777
Site-prepared	1615	1853	2097
Old-field (Coile and Schumacher)	1919	2125	2282
Old-field (Bennett and Clutter)	2171	2382	2534

Table (5) -- Continued

SITE INDEX 60			
Type	Estimated Yield (cubic feet/acre)		
Age 20	N=300	N=400	N=500
Rough-woods (Coile and Schumacher)	1703	1872	1997
Site-prepared	1562	1793	2030
Old-field (Coile and Schumacher)	2109	2349	2536
Old-field (Bennett and Clutter)	2069	2291	2464
Age 25	N=300	N=400	N=500
Rough-woods (Coile and Schumacher)	2259	2511	2706
Site-prepared	2228	2610	2998
Old-field (Coile and Schumacher)	2777	3118	3392
Old-field (Bennett and Clutter)	2759	3098	3373
Age 30	N=300	N=400	N=500
Rough-woods (Coile and Schumacher)	2717	3037	3291
Site-prepared	2784	3296	3813
Old-field (Coile and Schumacher)	3325	3750	4097
Old-field (Bennett and Clutter)	3315	3754	4113

Table (5) -- Continued

SITE INDEX 70			
Type	Estimated Yield (cubic feet/acre)		
Age 20	N=200	N=300	N=400
Rough-woods (Coile and Schumacher)	2306	2737	3066
Site-prepared	2169	2629	3109
Old-field (Coile and Schumacher)	2792	3343	3776
Old-field (Bennett and Clutter)	2495	3012	3421
Age 25	N=200	N=300	N=400
Rough-woods (Coile and Schumacher)	2998	3587	4045
Site-prepared	2878	3563	4269
Old-field (Coile and Schumacher)	3617	4358	4947
Old-field (Bennett and Clutter)	3225	3948	4530
Age 30	N=200	N=300	N=400
Rough-woods (Coile and Schumacher)	3564	4282	4848
Site-prepared	3455	4328	5222
Old-field (Coile and Schumacher)	4291	5186	5905
Old-field (Bennett and Clutter)	3803	4698	5426

When the expected yields for site-prepared stands are compared with the Coile and Schumacher old-field yields, it can be shown that the site-prepared values range from a minimum of 69 percent of the corresponding old-field value to a maximum of 93 percent. Lower values of this ratio tend to be associated with low densities, younger ages, and lower sites while large values of the ratio tend to occur with high densities, older ages, and higher sites. Overall, an average of 80 to 85 percent is reasonably representative. When site-prepared yields are similarly compared to corresponding old-field predictions from Bennett and Clutter, the site-prepared values range from 63 to 96 percent of the old-field estimates. The trend of the percentages in relation to age, site index, and stand density is similar to that observed for the Coile and Schumacher percentages.

The final point should be made that the per acre yield estimates presented in this paper are tentative interim figures since old-field site index curves and old-field volume tables were used in calculating the estimates. Site index curves and volume tables for site-prepared stands are currently being prepared and final yield tables will be published upon their completion. It is, however, expected that the final yield estimates for site-prepared stands will be very similar to the interim figures presented in this paper.

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SINGLE-TREE COMPETITION MODELS, PREDICTING STAND DEVELOPMENT AFTER CLEANING

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1. Introduction

With the purpose to analyse the possibilities in mechanizing the work of cleaning (precommercial thinning), the project "Mechanized cleaning" was started. The study at hand deals with stand development after different cleaning patterns. These patterns can be divided into the following groups;

- Selective cleaning aiming at a remaining stand with evenly distributed trees of good quality and large diameter.
- Strip cleaning, where trees are removed only within corridors running through the stand.
- Combinations of strip and selective cleaning.

The patterns are shown in figure 1.

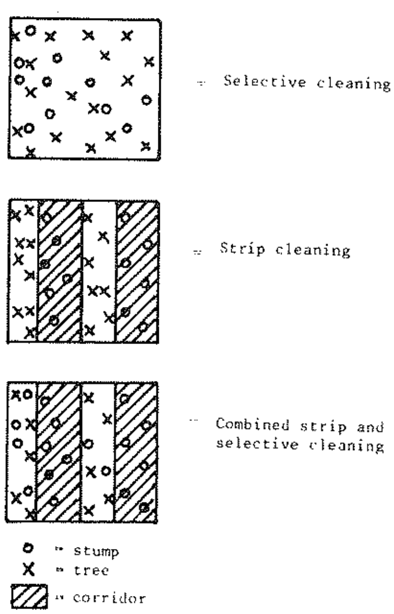

Figure 1. Selective, strip and combined strip and selective cleaning.

2. Increment analysis of single trees

The increment model to be developed should respond for competition from neighbouring trees at various distances. An effect of uneven competitor distribution as for trees at the edge of the corridor should also be considered in the model.

The study started with measurements on sample plots in stands, that earlier for some reasons, had been subject to cleanings of various types mentioned above. The material consists of about 300 trees of Norway Spruce (Picea abies) and Scots Pine (Pinus silvestris) on 17 plots cleaned at the age of about 6 years at breast height. The plots were measured 5 to 12 years after the cleaning. The site index class was H_{100} = T20-T24 m. The arithmetic mean height of the trees varied between 1.3 m and 5.8 m at the time of cleaning. The number of stems per hectare before cleaning were 2 700 to 8 500.

2.1. Basic increment model

Effects of cleaning on increment are dependent on effects from other increment factors, such as annual variation of the weather, site index class, competition before cleaning etc. Such relationships are considered in the functions, using the following principal expression;

$$i_d\ (t) = f_u\ (\text{uncleaned state}) \cdot f_c\ (\text{cleaning})$$

where

i_d (t) is the rate of increment at the moment t

f_u ( ) is the function of the effects of the uncleaned state

f_c ( ) is the function of the cleaning effect

The above expression was then developed into the following basic model transformed to suite the regression analysis.

$$\log \frac{i_d\ (0,t_p)}{\text{clim}\ (0,t_p)} = a + b_1 \cdot \text{DIAMO} + b_2 \frac{1}{\text{AGE} + 10} + b_3 \cdot \text{SITE} -$$

$$- \text{COMP} + \text{CLEAN} + \varepsilon$$

where

the dependent variable is the logarithm of diameter increment during the period 0 to year t_p. Observed diameter increment was adjusted each year, using a series of local climatic indices.

a and b are regression parameters.

DIAMO is the diameter at breast height under bark in mm of the tree studied at the beginning of the period.

AGE Influence from age class has been taken into account by the inverted mean age (AGE) at breast height of the trees on the plot and added by ten.

SITE Site index is a difficult factor to determine in these young stands. The method chosen in this study is based on the length of the top shoots above breast height. SITE is calculated as the arithmetic mean height in meters (m) of the trees on the plot at 10 years age at breast height.

COMP, the competition effect, and CLEAN, the cleaning effect are described according to each competition model.

ε is the error around the function.

Differences of increment between Pine and Spruce was regarded just by grouping the material after tree species.

2.2. Competition models

Different competition models were tested on the material through multiple regression analysis. The analysis was divided into three model series, all estimating the effect of changes in the close environment on the growth of individual trees. In series No. 1 the influence on increment of competition and removal of competition at varying distances was tested. In this connection the effect of one-sided competition also was analysed. This can be seen in figure No. 2, where the competitors are grouped according to the distances and to the positions in relation to openings around the trees studied.

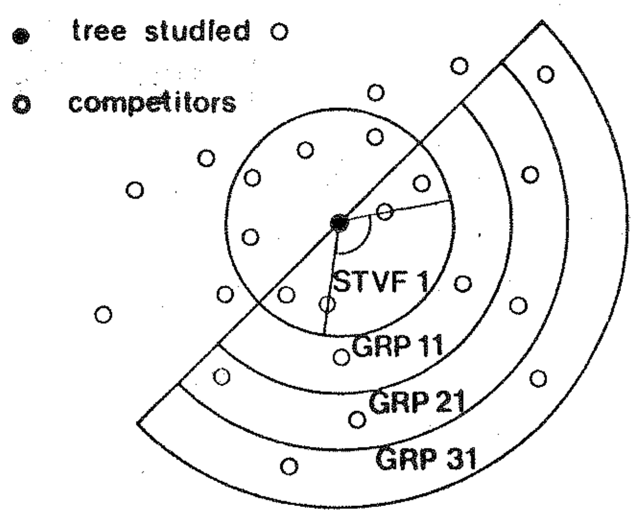

Figure 2. Competition model to test influence on diameter increment of the greatest free angle (STVFI) and of competition from trees at different distances (GRP11, GRP21, GRP31) outside the angle.

In series No. 2 a number of models were tested, previously studied by others. These models often include the competition situation by using previously fixed algorithms to describe e.g. the influence of the size of the trees. An example of this type of competition model is illustrated in figure No. 3.

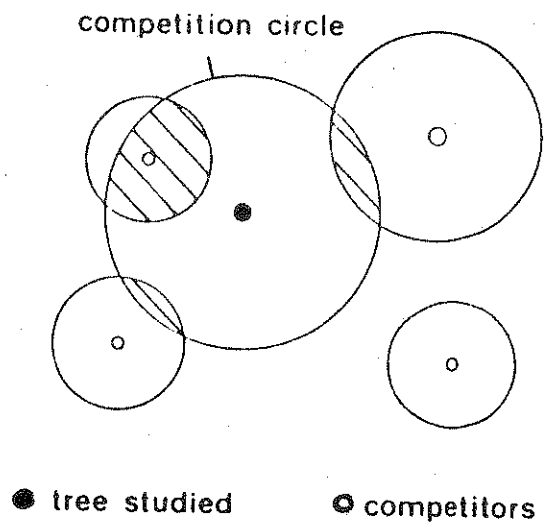

Figure 3. Model where the competition index is expressed as the common area of competition circles of the tree studied and its competitors. The diameters of the circles are in this case proportional to the diameters of the trees in question.

The influence of the level of increment before the cleaning on the increment reaction after cleaning, was taken into account, especially in the third model series.

2.3. Results of the increment analysis

The analysis showed that height of the tree studied related to mean height of the competitors within 5 m in all models was an important factor. Furthermore, it was found that the effects caused by competing trees disappear at a distance of 3 m from the tree studied. The influence decreases over distance in a way that is shown by the relations of the columns in figure No. 4, as well as by the following selected part of a regression equation for Pine.

$$^{10}\log i_d = \cdots - 0{,}69^{***}\,GF1 - 0{,}08^{*}\,GF2 - 0{,}06^{*}\,GF3 \cdots + 0{,}65^{***}\,GR1 + 0{,}11^{**}\,GR2 + 0{,}03\,GR3 \cdots$$

where

$^{10}\log i_d$ is the logarithm of the increment of diameter

GFk is the basal area of competitors before cleaning within the distance classes k=1 0-1 m, k=2 1-2 m, k=3 2-3 m

GRk is the basal area of competitors cleaned within the distance classes k=1 0-1 m, k=2 1-2 m, k=3 2-3 m

* symbols the level of significance of the coefficient estimated at the analysis

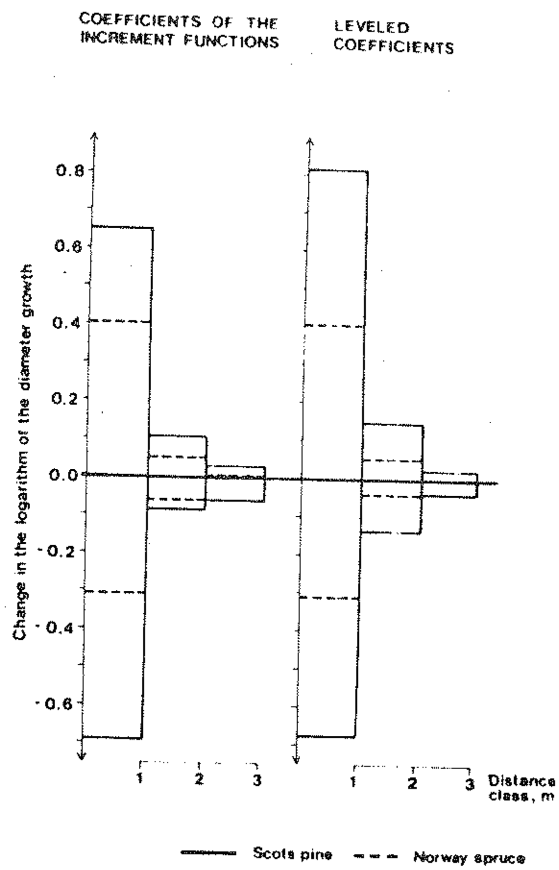

Figure 4. Influence of competitors before cleaning (columns under the line) and of competitors removed (columns above the line) on the logarithm of the diameter increment of the trees studied. The columns to the left show regression coefficients from the analysis and those to the right show the coefficients leveled according to the model:

$$\text{coefficient}_i = a + b\frac{1}{(\text{DIST}_i)^2}$$

where i is distance class
a and b are constants and
DIST is the distance between the competitors in distance classes and the tree studied.

There was no significant effect on the increment of one-sided competition from the neighbouring trees after the cleaning. On the other hand, there was an effect on the increment during the first five years of one-sided competition which was prevalent before the stand was cleaned. There was also indications that trees further away than 3 metres influenced the growth of the tree studied through the free angles (sectors without competitors). This effect showed up during the period 5-10 years after cleaning. It ought to be pointed out that the results apply to the increment situation of individual trees.

Thus the individual trees do not react much different to strip cleaning than to selective cleaning provided the intensity is the same. Intensity is here defined as in the function in page No. 6 . The competing basal area before cleaning, however, is important, as well as the intensity of the cleaning viz. the number of stems removed and their size, and also the distance between the tree studied and its competitors.

3. Simulation

The next step was to use one of the models studied above in simulated treatments of cleaning stands. The types of stand treatments studied were conventional selective cleaning, strip cleaning (corridors) and combinations of these two types. The factors varied were the number of stems per hectare before and after cleaning, the width of corridors and the distances between these.

The treatments above were simulated on parts of four young stands at cleaning age. The simulations were repeated in order to reduce the variation of the results, depending on e.g. spatial grouping or uneven distribution of tree species or dimensions in the stands. After each repetition 10 years of increment in height, diameter, basal area and volume were calculated for every single tree. The calculations were based on a chosen model where influence of the competitors was described by the basal area at breast height of trees grouped in circle rings around the tree studied. Influence of the height of the tree studied was related to arithmetic mean height of the neighbouring trees within 5 metres.

Natural thinning caused by crowding of trees was calculated from a function, derived from studies of planted spacing experiments.

The results give stand development without taking into account effects of insect damage, damage caused by transport vehicles or of nutrition losses caused by removal of whole trees from the stand.

4. Results of simulation studies (see figure No. 5 and No. 6)

- With the same number of remaining stems per hectare, total growth per hectare of stem volume after cleaning is higher after selective than after strip cleaning methods. One example is shown in figure No. 5 after cleaning in a Pine stand of 10 000 stems per hectare.

- With the same remaining stem volume, total volume growth is on the other hand higher after strip cleaning (many small trees) than after selective (a few big trees).

- When strip cleaning to the same remaining volume, the effect of changing the width of corridors (and the width of strips proportionally) in the interval 2-4 m is very small.

- When cleaning heavier by leaving less wide strips at constant width of corridors, the result will be a positive effect on diameter increment, for investigated intervals of strip width (these changes between plots) - but negative on the volume growth per hectare.

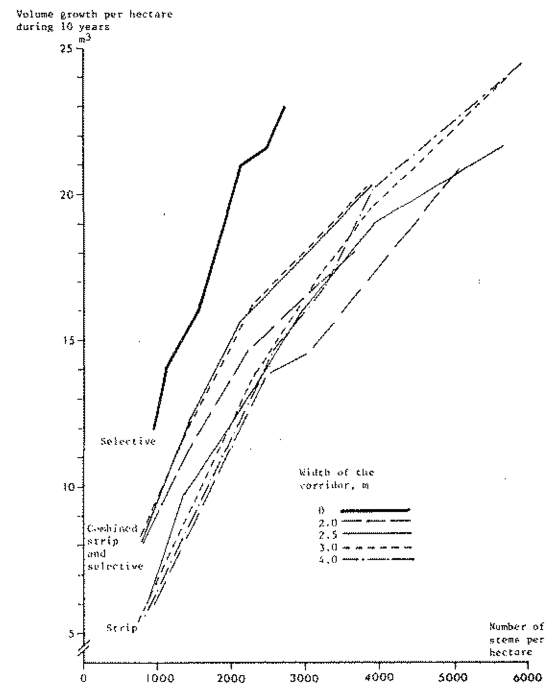

Figure 5. Total volume increment of stems under bark 10 years after cleaning, plotted over remaining number of stems per hectare with the methods selective, strip and combined strip and selective cleaning. When cleaning with corridors the widths of those are varied between 2.0 and 4.0 m.

- When cleaning heavier by widening corridors from 2 m to 4 m at constant width of strips, the negative effect on volume increment is rather small.

- The highest total volume growth as well as the highest natural thinning will be the result when not cleaning the stand.

- Natural thinning is high when strip cleaning and increases rapidly with an increase in remaining volume. When selective cleaning, natural thinning is low and occurs only at a relatively high remaining volume.

- Selective cleaning produces stands with large arithmetic mean diameter. This is caused mainly by the removing of small trees and to a smaller extent by the growth during the subsequent 10-year period. Diameter is larger after heavy treatments of high selectivity. As an example, cleaning to 2 400 stems per hectare will give a diameter of the median stem 10 years after cleaning of 66 mm with selective, 57 mm with combined strip and selective and 47 mm with strip cleaning.

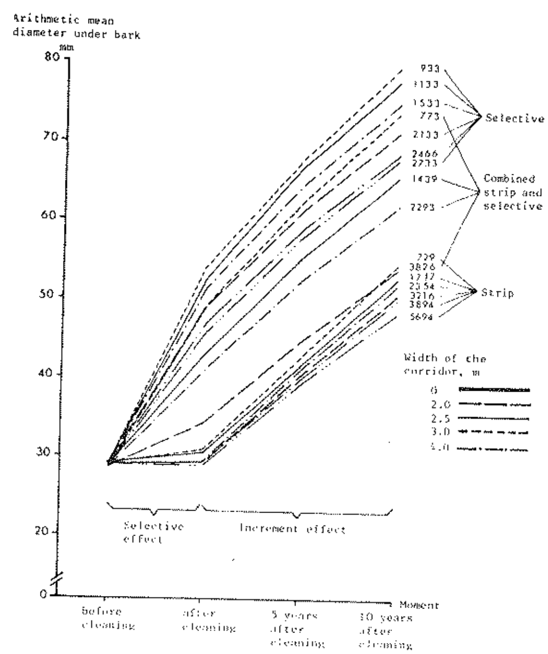

Figure 6. The arithmetic mean diameter with different methods of cleaning to varying number of stems at the four points in time: before and after cleaning and after periods of increment of 5 and 10 years. When cleaning with corridors the width of these are 2.5 m.

- The effect of the selection of trees on the increment of diameter is high when the number of trees before cleaning is high. It is low when the trees are grouped distributed or when cutting corridors in combination with selective cleaning.

- Strip cleanings do not affect the diameter distributions. The increment however is unevenly proportioned between diameter classes. Selective methods will result in high and narrow distribution curves and an evenly proportioned increment between stems.

The results are confirmed by comparisons with the development during 5 years in strip cleaning experiments, as well as by comparisons with some other investigations.

The investigation is reported in an English summary (stencil of about 30 pages) with the same name as this paper. Two reports in Swedish with English figure texts contain the whole work as well as the references. The reports are published by the Department of Operational Efficiency, Royal College of Forestry, Sweden as "Research Notes" No.:

99/1976 Competition models for individual trees after cleaning. Lennart Eriksson.

108/1977 Simulation of stand development after cleaning. Lennart Eriksson.

THE GRID METHOD - A WAY TO GENERALIZE SAMPLE TREE DATA

by

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ABSTRACT

An alternative to more common methods of forecasting forest growth is to construct "whole stand" growth functions based on inventory data. This approach gives a correct representation of the population under study, and, at the same time, has the simplicity and other advantages of the "whole stand" concept. However, an evident problem is to estimate growth per unit area and other variables on inventory plots. The problem arises from the fact that the number of sample trees per plot is usually too low to permit "within plot" calculation of growth per unit area etc. In this paper, some methods to solve that problem are dealt with. The main principle of all methods discussed is to calculate growth per unit area etc for each plot by means of sample tree data from several plots -- in other words sample trees are "borrowed" from other plots. The method finally chosen - the so-called grid method - is described in detail. The outcome from a case study is reported.

BACKGROUND

As reported in the paper by GÖTE BENGTSSON, we are now at The College of Forestry in Sweden, working with a large scale, long-term timber yield forecasting model. This work is named the HUGIN-project. One essential part of this project is to develop "whole stand" growth functions, based on inventory data. These functions will describe growth per unit area under different circumstances. We think this "whole stand" type of growth functions might be an alternative to the more common approaches

- single-tree growth functions based on inventory data (see for example BENGT JONSSONs paper) or
- whole stand growth functions based on yield research data and reduced to a "practical" level (see for example KILKKI & PÖKÄLÄ, 1975).

These two approaches are also dealt with in the HUGIN project and we hope to be able to make interesting comparisons.

When making whole stand growth functions from Swedish inventory data (the National Forest Survey) the first and most evident problem is to estimate growth per unit area on the sample plots. The difficulty is that the plots are small (circular, radius 10 m) and growth is measured only on a few sample trees (on average 3 - 5 trees per plot including two dominant height trees). Besides growth also age, height, diameter, species and so on are recorded for the sample trees. Every tree on a plot is a so-called caliper tree which means that diameter and species are recorded. With these notations a sample tree is also a caliper tree.

Obviously the number of sample trees per plot is too small to permit plotwise calculation of growth, volume etc per unit area in the traditional yield research way. Therefore an investigation was made, the aim of which was to develop and analyze new methods for the generalization of sample tree data, which could solve our problem. We did not have to start from scratch, as such a method - the grid method - had already been developed though not thoroughly analyzed with respect to the HUGIN problem. This method has earlier been used by research leader GÖTE BENGTSSON for estimating volume, and by mr K-G BERGSTRAND, professor N-E NILSSON and professor BENGT JONSSON in connection with single tree growth functions for timber forecasts.

In our work with the grid method in the HUGIN project, mr SÖREN HOLM made the statistical analysis of the method while mr ANDERS MÅRTENSSON programmed the method for computer.

SOME ALTERNATIVE METHODS

To be able to formulate our problem more precisely and to describe som alternative methods, we have to introduce some symbols.

X : A vector with variables, the values of which are known both for sample trees and caliper trees. The components in X might refer to the tree (diameter, species) or to the plot (site index, stand density etc). A component in X is denoted x_k

Y : A vector with variables which are known for sample trees only. We denote a component in Y as y_k

i : An index for sample tree no, i = 1, 2 ....., n where n is the total number of sample trees in the data set used

j : An index for caliper tree no, j = 1, 2 ....., m where m is the total number of caliper trees in the data set used

A : A vector with regression coefficients.

What we want to do is to estimate a vector Y for each caliper tree. For this operation, the information in the vector X is used. The estimated Y:s shall be summarized to plotwise sums (or means). From these "plot data", regression functions describing growth per unit area shall be constructed. We want the estimation of the Y:s to be such that the growth functions become reasonably unbiased and precise.

For performing the estimation of Y-vectors, at least three different methods might be considered. The regression method means that every component of Y is described separatly as a function of X. The functions are constructed according to the principle of least squares and might be written as

$$y_k = f(A,X)$$

This approach is fairly similar to the standard methods of yield research. However, here we use data from many plots to estimate A, while the standard methods usually only make use of "within-plot" data.

For the distance method a "distance", D, in terms of X is defined between a sample tree and a caliper tree. The "distance" is written as a function

$$D_{ij} = g(X(i),\ X(j))$$

For example, D_{ij}, might be computed as

$$D_{ij} = \sum_k (w_k\ (x_k(i) - x_k(j))^2)$$

where w_k is a weight for x_k.

Each caliper tree is compared to each sample tree. A certain caliper tree is allotted that vector Y which originates from the sample tree minimizing D_{ij}.

A third alternative is the grid method which might be regarded as a special case of the distance method (the distance is either 1 or 0). Groups of trees, so-called cells, are defined in terms of the different x_k. For example, one cell might contain all spruces with a diameter of 15 - 20 cm, growing on plots where site index is 20 - 25 m and so on. The cells together form a multidimensional grid. The caliper trees are allotted Y-vectors from sample trees in the same cell.

This allottment is performed in steps. As a first step, the grid is fairly densely spaced. Then there are of course many cells with caliper trees where there are no sample trees. After making those allottments which are possible, the cells are made bigger step by step until all caliper trees are allotted Y-vectors.

The three methods have been judged to have the following properties.

The regression method probably gives good estimates of each y_k separately. However, as the different y_k:s are estimated separately (multidimensional regression techniques are not considered), there is an evident risk of creating trees with unreasonable combinations of properties. Further, the regression method will strongly decrease the variation in the data set, which is undesirable. We therefore judge the regression method as not suitable for our purposes.

The distance method is theoretically attractive, but demands in our case some hundreds of billions of comparisons between caliper trees and sample trees. Even if this problem might be solved technically, it is difficult to find proper values for different w_k. Therefore, like our precursors, we have chosen to work with the grid method. Whether this method will give acceptable results or not mainly depends on the construction of the successive grids. Dense grids, opened in small steps, will give good results but high costs. Very open grids will be cheap, but will, in extreme cases, give similar results to those obtained by a random allottment of Y:s.

SOME FEATURES OF THE GRID METHOD

Below the variant of the grid method used in our work with the HUGIN model is described. Also some statistical aspects of the method will be discussed.

As mentioned above, the grid method is used for allotting a vector Y to each caliper tree. Y is taken from sample trees and comprises the following components:

Bark thickness
height
height to first live branch
eventual damage
age at breast height
diameter growth the last five years and the five years before
volume growth the last five years

When analyzing the output from the grid method, the components height, age and (especially) diameter growth the last five years are considered as most important.

At the practical application of the grid method, the data set from the National Forest Survey is first divided into a few large groups, so-called supercells, defined by geographical regions. The main reason for introducing the supercells is that the sample trees are selected with varying sampling intensity in different regions. Within each supercell, the sampling intensity within 5-dm diameter classes is constant. The maximum size of each supercell is determined by the size of the core store of the computer used. The supercells are treated separately.

Within the supercells, grids are successively formulated in terms of the X-vector components. Some important such components are listed below.

Tree variables:

- species
- diameter at breast height. The 5-cm diameter class is kept as a cell border through all grids. This is because the sampling intensity varies between 5-cm classes

Plot variables:

- stand age
- site index
- basal area per ha
- species composition
- latitude and altitude
- forest type
- earlier thinnings
- occurrence of fertilization

and others

In total, we use 12 X-variables, each on maximally 15 levels. Some of the X-variables are combinations of original variables.

The first grid is very densely spaced and defined by species, diameter class and plot no. Thus, at this step, all allottments are made "within plots". In this way we are able to ensure that all caliper trees which are also sample trees get their "own" Y-vectors.

After this "within plots" step, the grid is opened up, step by step, making the cells bigger. As soon as a cell with caliper trees also contains sample trees, allottment of Y:s is made, and the caliper trees are together with their Y-vectors stored on a result file.

Technically, the grid method is done in such a way that the cell-identifying X-variables are -for each step - coded in a few computer words. The caliper and sample trees are stored on two separate files which are sorted according to the coded cell-identifications. After sorting, the two files are matched and an allottment of Y-vectors is made when caliper and sample trees have the same cell-identifications. If there is more than one tree of each type with the same identification, special rolling routines are used. If there are still caliper trees without Y:s left after the complete matching, a new, more open grid is formulated. Thus, the cell-identifications must be recoded and the files resorted. In an especially simple case the recoding is done by simply dropping the last variable in the identification code. Here, no resorting is needed.

An evident problem in the procedure described here is how to judge the quality of the results. Such judgements are necessary, for example when choosing between alternative grid designs. Below, some statistical aspects of the grid method are discussed. The discussion is founded on investigations made by mr SÖREN HOLM.

It is evident that the grid method will produce some bias at plot level. There will be a tendency that "extreme" caliper trees are allotted Y-vectors from sample trees which are not so "extreme". In other words, there will be a bias towards more central parts of the data set. Further, plots with many sample trees will tend to be over-represented. It is important to design the successive grids so that the bias on plot level is not transmitted to the growth functions. As mentioned above, the outcome of the performance of the grid method will be a data set which could be used for constructing functions for growth per unit area.

For a more quantitative statistical analysis of the grid method, several methods could be used. HOLM has developed a model, by means of which it is possible to deduce formulas for different interesting variances, standard errors etc. As the model is fairly complicated, this way of working is difficult. A simpler, but somewhat less informative way of judging the results is to perform the allottment procedure with the sample trees temporary acting as caliper trees. (SS-allottment: Sample tree to Sample tree). In this way the allotted Y-vectors can be compared to the measured, "true" ones. For each sample tree, a measure of deviation, b, can be calculated according to the formula

$$b_k(i) = \tilde{y}_k(i) - y_k(i)$$

where

i : index for (sample) tree no

$\tilde{y}_k$: allotted value of the k:th component of the Y-vector

y_k: "true", i e measured, value of the k:th component of the Y-vector

The deviations b might be aggregated in different ways, for example by plot and/or over different components in the X-vector. In this way, bias of different kinds can be detected. A "good" grid design will give small values both of average b and of the variance of b.

COMPUTER ASPECTS

The grid method does pose some computer problems, mainly because of the large amount of data which needs to be used. In order to avoid unacceptably high costs, we have to design our computer programs very carefully. We use a fairly big and fast computer, a CD 6600 with a core store of 80 000 words, each of 60 bits. We will not describe the programs in detail here, but merely give an idea of the main principle of the processing.

First, the data from the National Forest Survey is primarly processed. At this moment volumes, site indices etc are estimated. The resulting file is used to create two binary work files, one for the caliper trees and one for the sample trees. The cell-identifications are coded according to the principles described above. The files are matched, and the caliper trees are allotted Y-vectors. The cycle is repeated in a step by step procedure until there are no caliper trees without Y-vectors left. Figure 1 shows schematically the structure of the processing.

The programs used are written in FORTRAN IV code. As the programs involve many machine-specific operations such as shifting and masking, they probably cannot be used on computers other than big CD computers without extensive modifications.

A CASE STUDY

The grid method has been preliminarily tested within the HUGIN project by means of a data set comprising 2 392 plots from the National Forest Survey. The numbers of caliper and sample trees were 43 391 and 5 111 respectively. The X-vector contained the following variables

	number of values
tree diameter class	9
tree species	5
species composition of the stand	10
stand age class	10
site index	8
relative density	10
cutting class	10
field and bottom layers	8
form of stand (even-aged, two-storeyed etc)	8
earlier cuttings	6

The allottment of Y-vectors to caliper trees was performed in six steps, denoted 0 to 5. Step 0 was an allottment within plots. At step 1, all X-variables had their original codes. This means that the grid at step 1 contained about 1.4 billion cells. The grid was successively opened in steps 2 - 5 and contained at step 5 only 9 cells, corresponding to the same number of diameter classes (see table 1).

Figure 2 shows the number of caliper trees allotted Y-vectors in the different steps. As can be seen, more than 40% of the caliper trees got their Y:s in step 0. The dense grid in step 1 gave a low number of allottments (4%). However, these are probably fairly accurate. Most of the remaining allottments were made in steps 2 and 3. The very open grids in step 4 and 5 needed to be used for less than 0.5% of the total number of caliper trees.

Figure 3 shows the number of times each sample tree was used in steps 1 to 5. On average, each tree was used 5 times, but the distribution is skewed. Most sample trees were used less than 5 times, but a few trees were used very often (up to 60 times). In the National Forest Survey, tree sampling intensity does increase with increasing diameter. As can be seen in figure 4, this leads to small sample trees being used more often than large.

One essential part of the case study was the "SS-allottment",

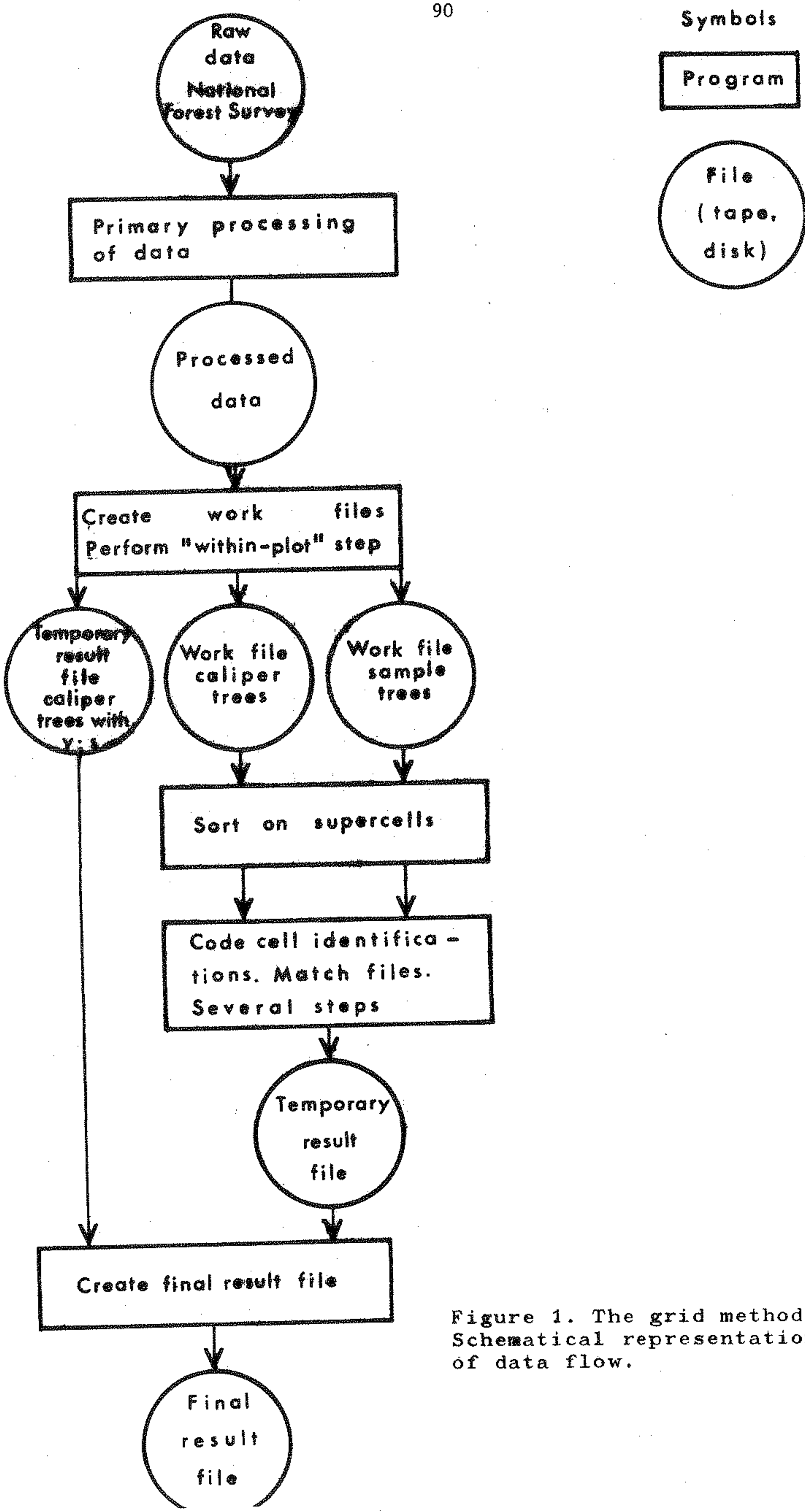

Figure 1. The grid method. Schematical representation of data flow.

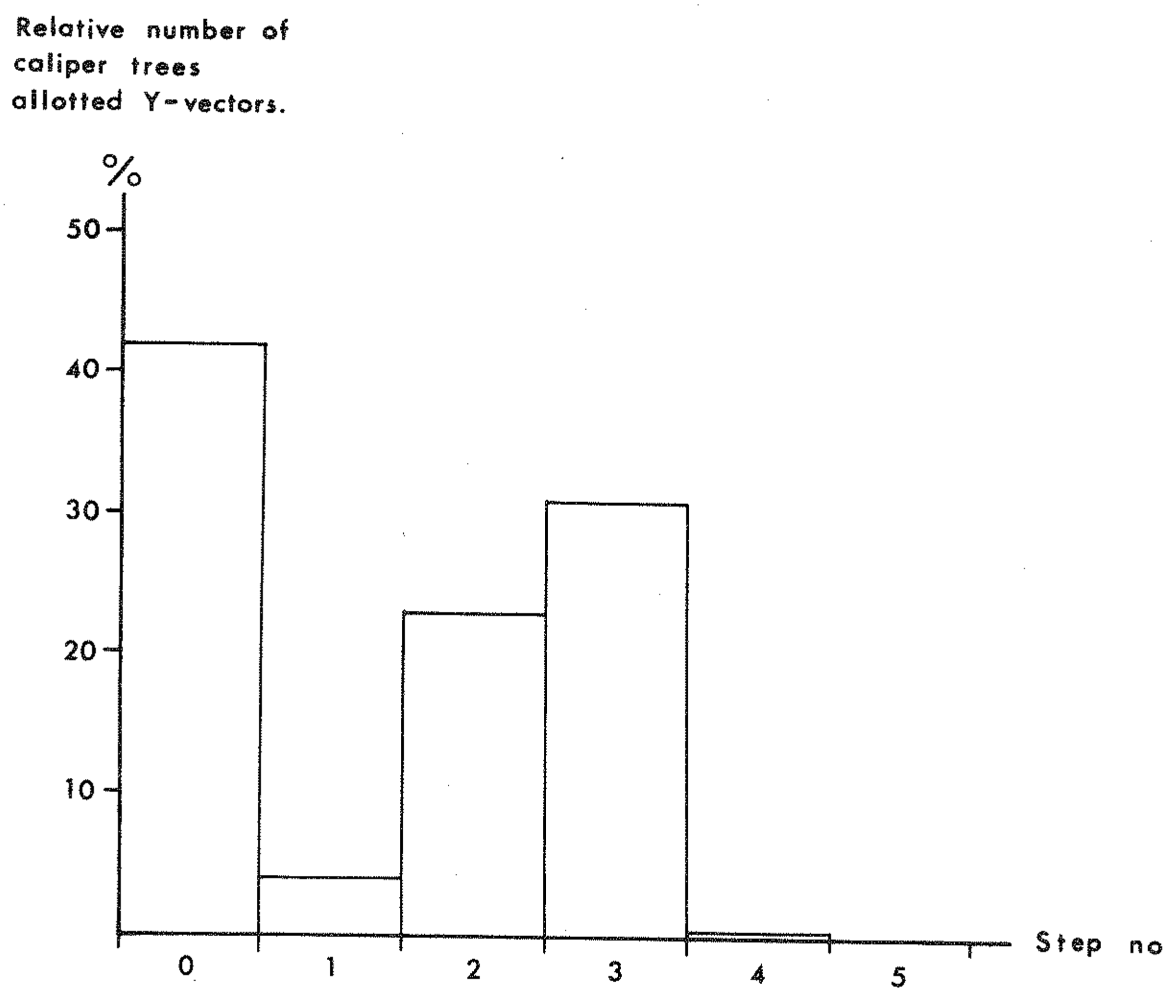

Figure 2. Relative number of caliper trees allotted Y-vectors in different steps.

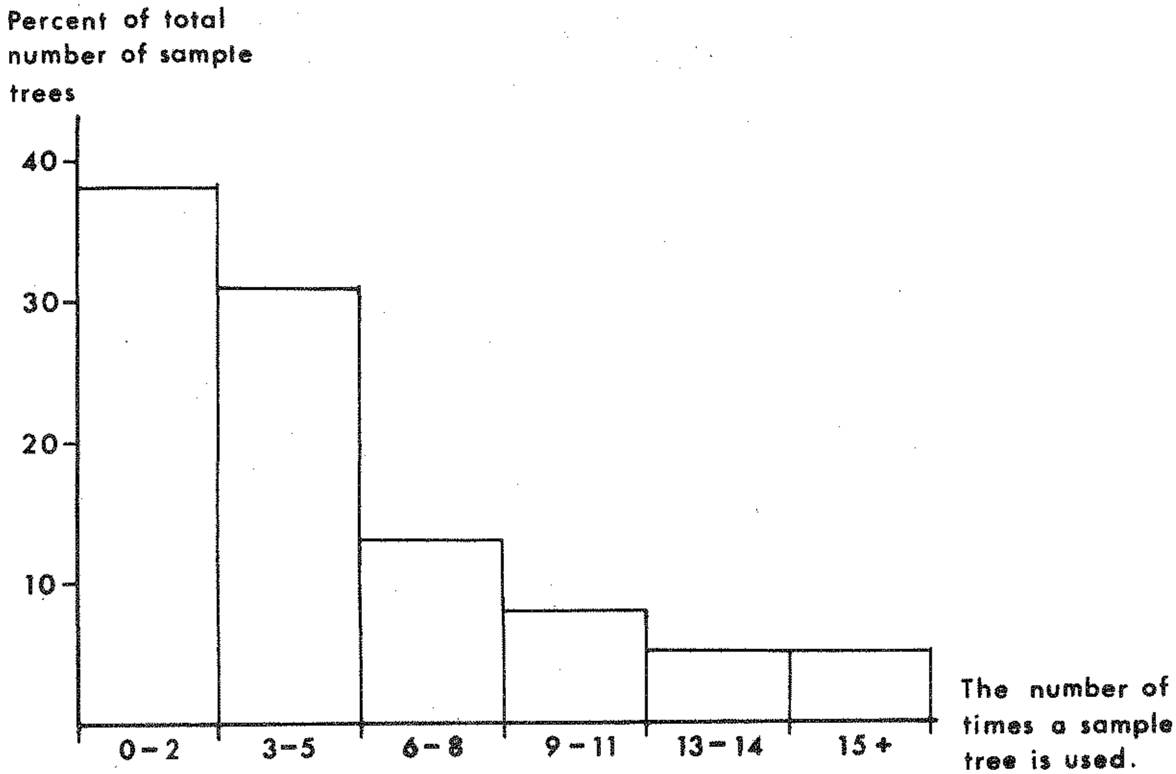

Figure 3. The number of times a sample tree is used. On average each tree is used five times. The figure does not include step 0.

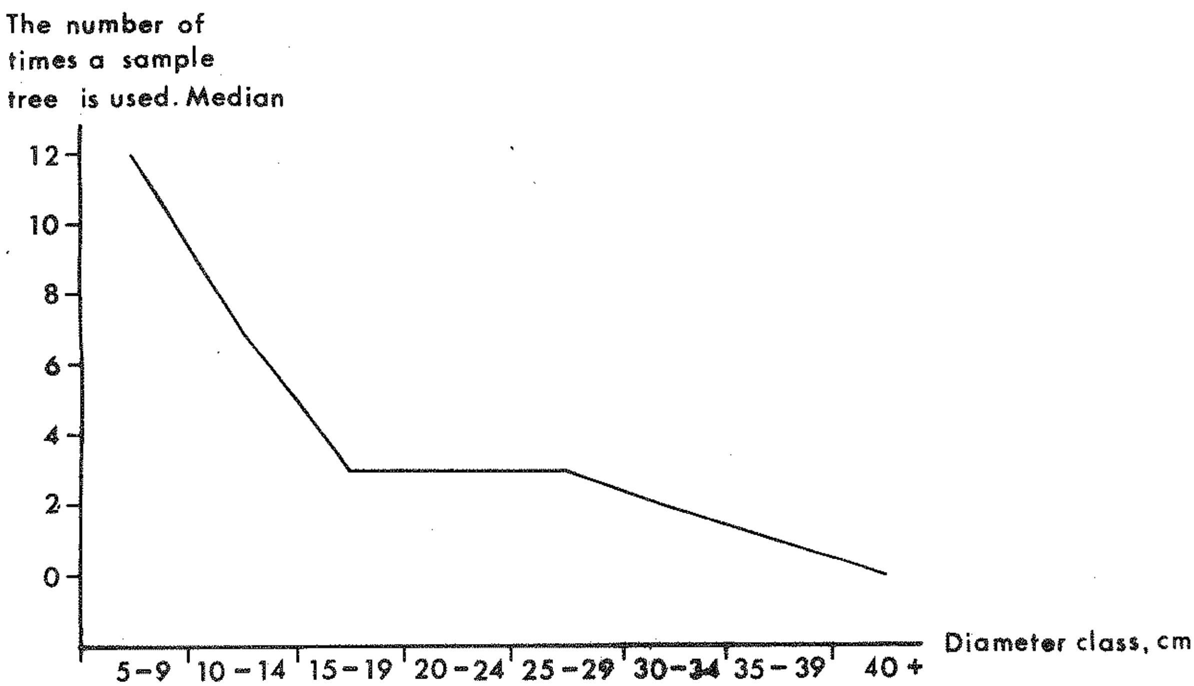

Figure 4. The number of times a sample tree is used (median) over diameter class.

Table 1. The grid design at the case study

	Number of levels at step no				
	1	2	3	4	5
Diameter class	9	9	9	9	9
Species (tree)	5	5	5	3	1
Species (stand)	10	4	2	1	1
Stand age class	10	4	2	2	1
Site index	8	3	2	2	1
Relative density	10	3	2	2	1
Cutting class	10	3	2	1	1
Field and bottom layers	8	4	3	2	1
Form of stand	8	4	2	1	1
Earlier cuttings	6	3	2	1	1
Number of cells	$1.38 \cdot 10^9$	933 120	17 280	432	9

performed according to the principles described above. Most results from this operation relate to the allottment of the Y-component "five years diameter growth".

In general, the standard deviation of differences between measured and allotted growth is much greater than the mean difference. As expected, there is a general tendency for bias towards the more central parts of the data set. For example, very small trees are, on average, allotted values of growth which are too high while large trees are given values which are too low.

To summarize, our most important conclusions from the case study were

- age and diameter class are the two most important X-variables, but many other variables are of interest
- it is very important to keep trees (and plots) treated (thinning, fertilization) in different ways in different cells
- the number of X-variables and the number of steps must be increased in order to reduce bias and increase precision. This will increase the costs for computer time, but not as much as was preliminarily estimated.
- the SS-allottment seems to be a powerful tool for evaluating different grid designs.

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EVALUATION OF SITE QUALITY IN CONNECTION WITH A MODEL FOR LARGE SCALE FORECASTING OF TIMBER YIELDS

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ABSTRACT

In Sweden, a site quality evaluation system is under development. Those parts of this system which are auxiliary for assessing site index[1] have been incorporated in a model for long term forecasting of forest yield, the so-called HUGIN[2] model.

Two methods for estimating site index are used in the HUGIN model and thus described in this paper. Site index curves, showing the relationships between age and dominant height for different levels of site productivity, are used for estimating site index on plots where the stands are even-aged, undamaged, not too young, mainly of one species etc. Such stands are reflecting site productivity in an unbiased way. On the rest of the plots, where the stand does not fulfil those restrictions mentioned, site index is estimated directly from site factors. This is performed by means of a set of functions, derived from National Forest Survey data.

In a dynamic forecasting model, the species on the plots might change due to measures taken by man or nature. This leads to some problems as estimated site indices are primarly linked to the species existing at the time of plot measurement. To overcome these difficulties, functions converting site index from one species to another have been constructed and implemented in the HUGIN model.

Studies of the accuracy of different methods of site index estimation show that site index curves should be used as soon as the stand fulfils those restrictions mentioned above.

INTRODUCTION

One of the most important parts of most large scale/long term forecasting models in forestry is the prognosis of future forest growth. The magnitude of this growth is - to a large extent -

1) dominant height at a fixed reference age
2) as reported in the paper of Göte Bengtsson, the HUGIN model is now under development

dependent on the quality of the forest sites in the area under study. Hence, forest site quality must be estimated in a reliable way if the model is to give a correct prognosis of yield. In this paper is reported how site quality is evaluated in the Swedish HUGIN forecasting model.

In the HUGIN model, site quality is expressed through site index, i e the dominant height of a stand at a prefixed reference age. The "stand" which site index refers to is "ideal", i e even-aged, undamaged, of one species and so on. Numerous studies indicate that site index is a fairly efficient measure of site quality in growth functions.

Estimating site index involves several operations such as data collection, data processing, the use of different functional relationsships and so on. All those routines and functions needed to estimate site quality together form a site quality evaluation system, a SQ-system. If such a SQ-system is to give correct estimates of site index, it must fulfil certain requirements. Some important such requirements are listed below.

- site index must reflect the properties of site and may not include uncontrolled effects of stand history. This is the reason why we state that site index refers to an "ideal" stand.

- site index shall be estimated without large scale bias and (under this restriction) with as high precision as possible

- the estimation of site index must be strictly objective

- every site (plot, stand) on forest ground must be assigned an estimate of site index

- the relationships between site indices for different species must be known to such an extent that every site (plot, stand) can be assigned a site index corresponding to any species which might be of commercial interest on that particular site

- it must be possible to computer-code all parts of the SQ-system involving the manipulation of data.

The requirements listed lead to some important conclusions. That site index always shall refer to an "ideal" stand implies two alternative designs of the SQ-system. One is to develop a method for site index estimation in which the existing stand is not used at all. The second possibility is to use the existing stand when it is in acceptable condition, i e is reasonably similar to the "ideal" stand. When the stand is not acceptable, some stand-independent method must be used. We have chosen the second type of SQ-system, mainly because this approach seems to give the highest overall precision. Hence, we have to design at least two methods for site index estimation, one for acceptable stands and one for areas where the stand cannot be used.

THE SQ - SYSTEM

The SQ-system used in the HUGIN model is a part of a more complete system for site quality evaluation. In the complete system, site productivity is measured in terms of potential yield (mean annual increment at the time of culmination). For the HUGIN model, we only use those parts of the system which concern site index estimation. These parts are schematically illustrated in figure 1.

In stands acceptable as indicators of site productivity, site index is estimated from measurements of dominant height and age by means of site index curves. Such curves give fairly accurate estimates of site index, but the indices obtained are primarly linked to the dominating species of the stand. Therefore, functional relationships between different species growing on similar sites are constructed. These relations might be used for converting site index from one species to another.

If there is no stand, or if the stand is not acceptable, site index curves cannot be used for unbiased estimation of site index. In these cases we use functions expressing the relations between site index and site factors. In this way we get less accurate but also less biased estimates of site index than with site index curves. Functions of this type can be developed for different species. Hence, in this case, it is not necessary to use the "change-of-species" facility. In the following, the different parts of the SQ-system used in the HUGIN model will be described more in detail

SITE INDEX CURVES

Site index curves show the relationships between stand age and stand height at different levels of site quality. Underlying the curves is often some mathematical function, the parameters of which are estimated from height/age data by means of some statistical procedure. Site index is defined as height at some prefixed reference age. The curves are used for assessing site index in stands acceptable as indicators of site productivity.

The site index curves belonging to the Swedish SQ-system have the following properties. Most of the curves show the development of dominant height over age at breast height. Dominant height is defined as the arithmetic mean height of the 100 (by diameter) largest trees per hectare. The most common definition of site index is dominant height at a total age of 100 years, h_{100}. Figure 2 shows a typical set of curves, those for Scots pine. Today we have curves for Pinus silvestris, Pinus contorta var latifolia, Picea abies, Betula verrucosa, Quercus robur and Fagus silvatica (HÄGGLUND, 1974).

The principles used for the construction of the Swedish site index curves is reported in HÄGGLUND, 1972. A brief summary follows below.

An important principal basis for the construction of the curves

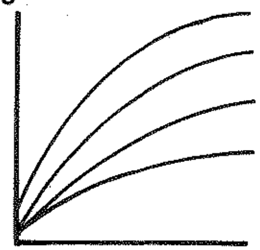
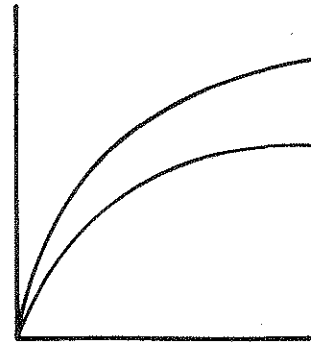
Condition of existing stand	Field measurements	Primary estimating of site index	Site index for another species	Result
Acceptable as indicator of site productivity 	Dominant height Age	Site index curves 	Site index pine 	Site index pine " " spruce " " birch etc
Not acceptable as indicator of site productivity 	Site factors (latitude altitude moisture vegetation soil depth etc)	Functonal relationships site index= =f(site factors)		

Figure 1. The part of the SQ-system used in the HUGIN model.

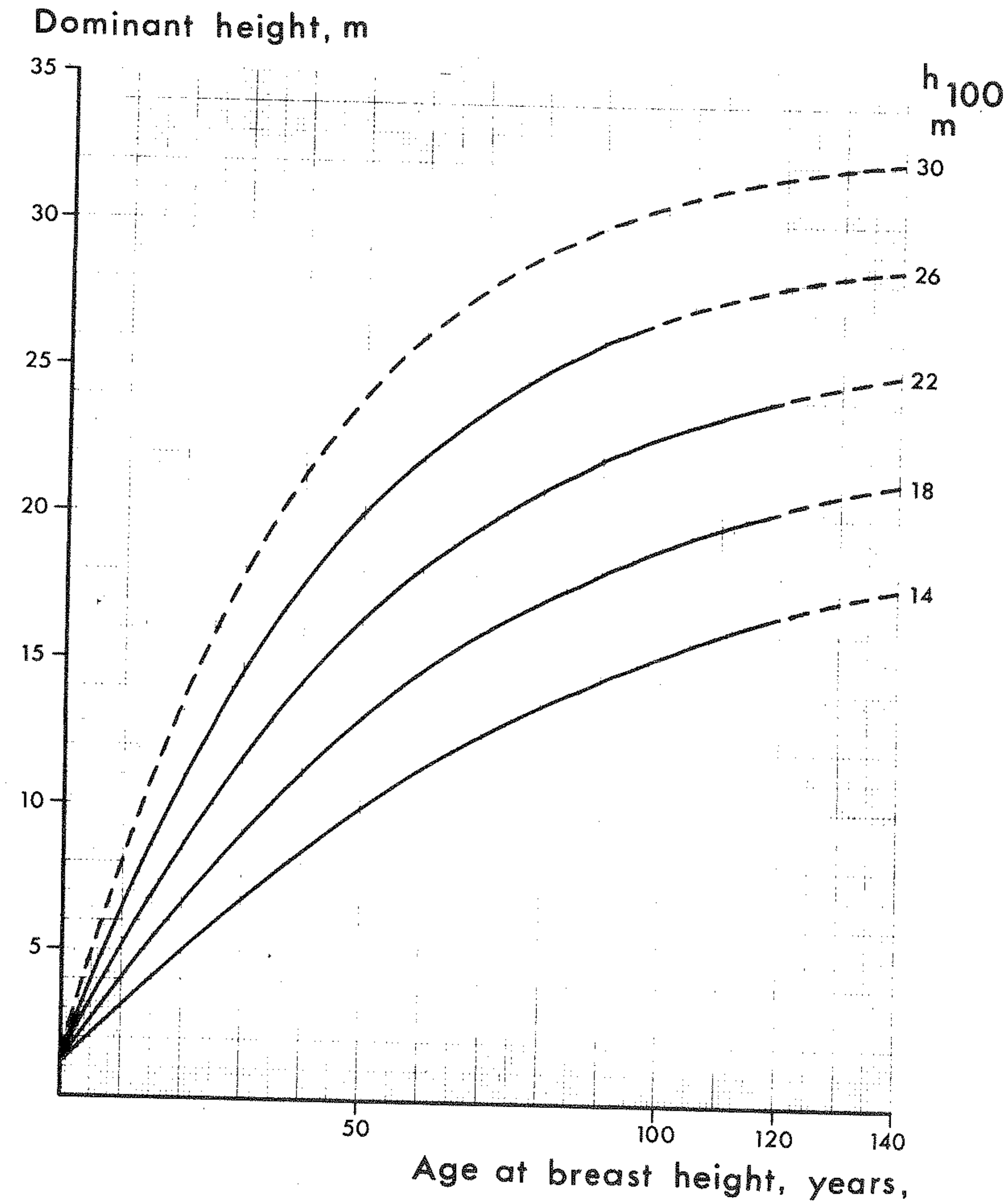


Figure 2. Site index curves for Scots pine in Sweden.

is the definition of "true" site index for a stand. Assume that we have estimated site index for a stand a large number of times. The estimates refer to different ages, uniformly distributed during the whole rotation. "True" site index is the arithmetic mean of these estimated indices. At the construction of site index curves we want to minimize the variance about "true" site index in some given data set. At the same time we have restrictions, saying that two points (h_1, t_0) and (h_2, t_0), where h_1 and h_2 are two different heights and t_0 an age, must be placed on different curves. Further, the curves may not intersect.

Preliminary studies indicate that a fit of a function

$$h = f(t, A) \qquad (1)$$

h : height

t : age

A : stand (site) parameter

to a set of observed heights and ages will give a set of curves satisfactorily fulfilling the requirements implied by the definition of "true" site index. This is the technique used to construct the Swedish site index curves.

Data for the Swedish curves consists of true time series of observations of height and age. Data originates either from repeated measurements of permanent plots or from stem analysis of felled trees. In the latter case, the felled trees are among the 100 largest per ha at the time of felling. However, it is not certain that these trees have been dominants throughout the whole life of the stand. In other words, the development of "dominant height trees" might differ from the development of dominant height. We call these differences "ranking effects" (see figure 3). Studies of the ranking effects by means of permanent plot data show that these effects are of such magnitude that they have practical importance, at least for Norway spruce. Therefore, we have corrected stem analysis data for ranking effects.

The model used for describing dominant height development is principally of type (1). It is derived from the so-called Chapman-Richards function (CHAPMAN, 1961; RICHARDS, 1959) and is, in our version, given the following form.

$$h(i, t) = A(i) \cdot (1 - e^{-kt})^{1/(1-m)} \qquad (2)$$

$$k = b_{11} + b_{12} \cdot A(i)^{b_{13}}$$

$$m = b_{21} + b_{22} \cdot A(i)^{b_{23}}$$

where

h(i, t) = dominant height in stand no i at age t years.

A, k, m = parameters to be estimated. A is an asymptote and a "site parameter". k and m are functions in A.

b = parameters to be estimated in the relations between k, m and A.

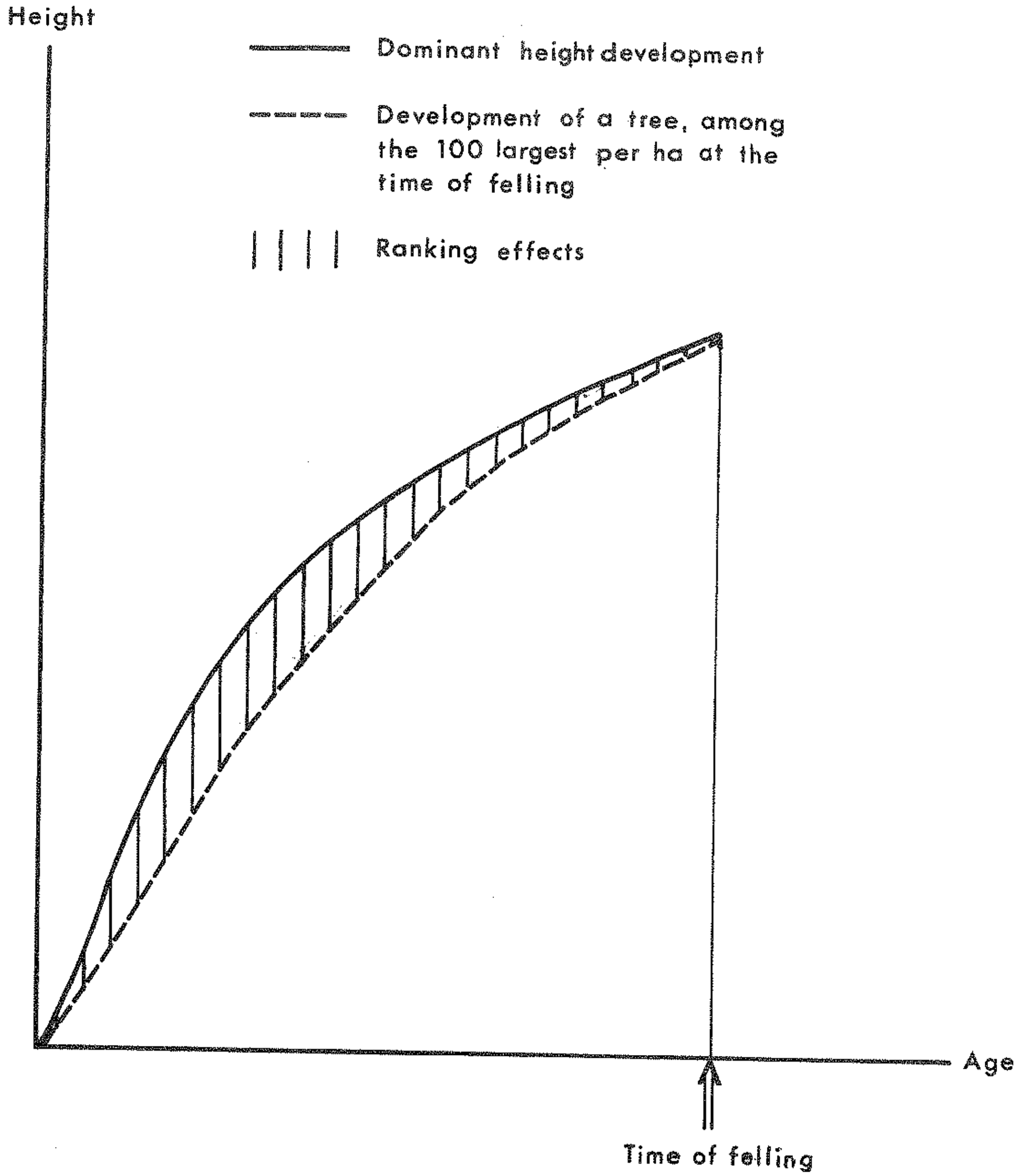


Figure 3. Schematic representation of the ranking effects.

The model is (in logarithmic form) fitted to a data set according to the principles of least squares. This fairly complicated operation is performed in three steps.

Step 1. Locate, for every height development (plot) the point where height and age are most safely determined. As we work with the model in logarithmic form, these points are defined by the plotwise arithmetic means of logarithmic height and the corresponding values of age. The points occur of course at different ages for different developments. We call the points $(t_m(i), h_m(i))$

Step 2. Fit the model (2) simultaneously to data. This is done in the following substeps

2.1 Give (i e guess, but think first) starting values to the parameters b_{11} - b_{23}

2.2 Force the model to fit the points $(t_m(i), h_m(i))$ by means of, for example, a simple step-halving procedure. The parameters b_{11} - b_{23} are kept constant during this substep, and thus the fitting is done by means of the "plot parameter" A(i) only. In this way every height development (plot) gets a value of A(i)

2.3 Compute new values of b_{11} - b_{23} by means of the Gauss-Newton method of non-linear regression (HARTLEY, 1961). During this substep, the parameters A(i) are kept constant

2.4 Repeat from 2.2 until convergence

Step 3. Perform a linear regression analysis, where the residual from the function obtained in step 2 principally is the dependent variable. In this way it is possible to check the results from step 2 - the outcome from step 3 shall not alter the results from step 2 very much. We can also introduce new variables in the function and, for example, check for influences from site properties on the shape of the site index curves. Step 3 is thus a type of "stage-wise" regression analysis.

Models similar to (2) have been used by many researchers for the construction of site index curves. In some investigations, the parameters A(i) have been replaced with some simple function of site index. Site index is directly or indirectly measured from data, which, in most cases, requires a data set where all plots have reached the reference age. (BRICKELL, 1966; LUNDGREN & DOLID, 1970; BECK, 1971). This technique might lead to some bias due to regression fallacy (WALLIS & ROBERTS, 1956). Site indices measured might, especially close to the borders of the data set, be affected of what we call random influences. As site index is measured at a fixed age, the shape of the curves might be affected. This risk of regression fallacy is the reason why we linked our site parameter A(i) to safely determined points occuring at different ages for different developments.

Another way of solving the fitting problem was demonstrated by

FRANZ & RAWAT, 1974. They fitted the Chapman-Richards model to each height development separately. In this way they got a number of estimates of the parameters of the model. Using these estimates as data, they constructed regression functions expressing the relationships between the parameters of the model and some measure on site quality. A similar way of fitting the Chapman-Richards model was also investigated by HÄGGLUND, 1972. However, his separate fits gave results quite different from those obtained by simultaneous fitting. The reason for this is probably that simultaneous fitting means a consequent minimizing of the sum of squares around the variable h (dominant height), while separate fitting partially means minimizing the sums of squares around the parameters of the model. In HÄGGLUND, 1972 the conclusion was drawn that separate fitting does not generally lead to site index curves which represent a least squares solution of the actual problem.

The site index curves used in the HUGIN model have been checked in many ways. By means of grouping of data and by introducing site properties in the height development functions we have checked that the form of the curves is not seriously affected by site conditions. The representativity of the data sets used has been checked with new data. To summarize, the checks indicate that

- the curves describe the data sets used to construct them correctly.

- there are no strong effects of site conditions on curve form at fixed site index. This conclusion is of course not general as Sweden is a fairly small country

- checks with new data indicate a good representativity of the curves.

In the HUGIN model, the function (2) is used for estimating site index in computer from measurements of dominant height and age at breast height. As the equation to be solved at this operation is non-linear, an iterative solution must be used. Several methods can be used for this purpose. We have chosen a simple and sufficiently fast step-halving routine.

ESTIMATING SITE INDEX FROM SITE PROPERTIES

As mentioned above, site index curves can be used for site index estimation only in those cases when the stand is acceptable as an indicator of site productivity. However, on many of the plots treated by the HUGIN model, the stands are not acceptable. To be able to estimate site index on these plots, we have to design a method for assessing site index from site properties. The study performed for this purpose (HÄGGLUND & LUNDMARK, 1977) is briefly reported below. The aim of our work was to establish functional relationships between site index (for Scots pine and Norway spruce) and site properties. In order to facilitate the practical use of the results of the study, the site properties under consideration were restricted to those which could easily be measured in the field.

Data for the study was obtained from the National Forest Survey, an annual inventory of the Swedish forests. In this inventory, circular plots with a radius of 10 m are laid out on forest land. The two largest (by diameter) trees on each plot are used for measuring dominant height and age at breast height. Further, a number of measurements of site properties are recorded. These measurements refer to the geographical location of the plot, to soil conditions and to ground vegetation. Some important variables are exemplified below.

Geographical location:

Latitude.

Altitude.

Distance to coast. Areas closer to coast than 50 km are distinguished.

Climatic regions. Some regions of maritime or continental climate are distinguished.

Soil properties:

Soil texture. The mechanical composition of the soil is converted to an index of soil texture.

Soil depth. Four classes are distinguished.

Thickness of humus layer.

Soil profile type. The depth of different layers is recorded.

Soil moisture. The main criterion is the vertical position of the watertable. Five classes are distinguished.

Surface/subsurface water flow. Assessed from the inclination of the ground surface and the lenght of the slope above the plot. Five classes are distinguished.

Ground vegetation:

The ground vegetation is used as an indicator mainly of the nutritional status of the soil. The vegetation is recorded as independent descriptions of the bottom and field layers, which are combined to forest types according to the principles proposed by TAMM & HOLMEN, 1961.

In figure 4, a plot from the National Forest Survey is shown together with some variables important for us. Our original data set comprised about 15 000 such plots. However, we could of course only use those plots where site index could be estimated in an unbiased way with site index curves. A number of selection rules were formulated in order to ensure that only such plots were included in the final data set. After this selection procedure, the data set comprised 3 385 plots. This means that about 80% of the original plots were discarded, the most common reason being that the stand was not "pure" Scots pine or Norway spruce. The selected data set was used to estimate the parameters of a model, describing the relationship between site index and site properties. The model is actually an integrated multiplicative growth model, originally

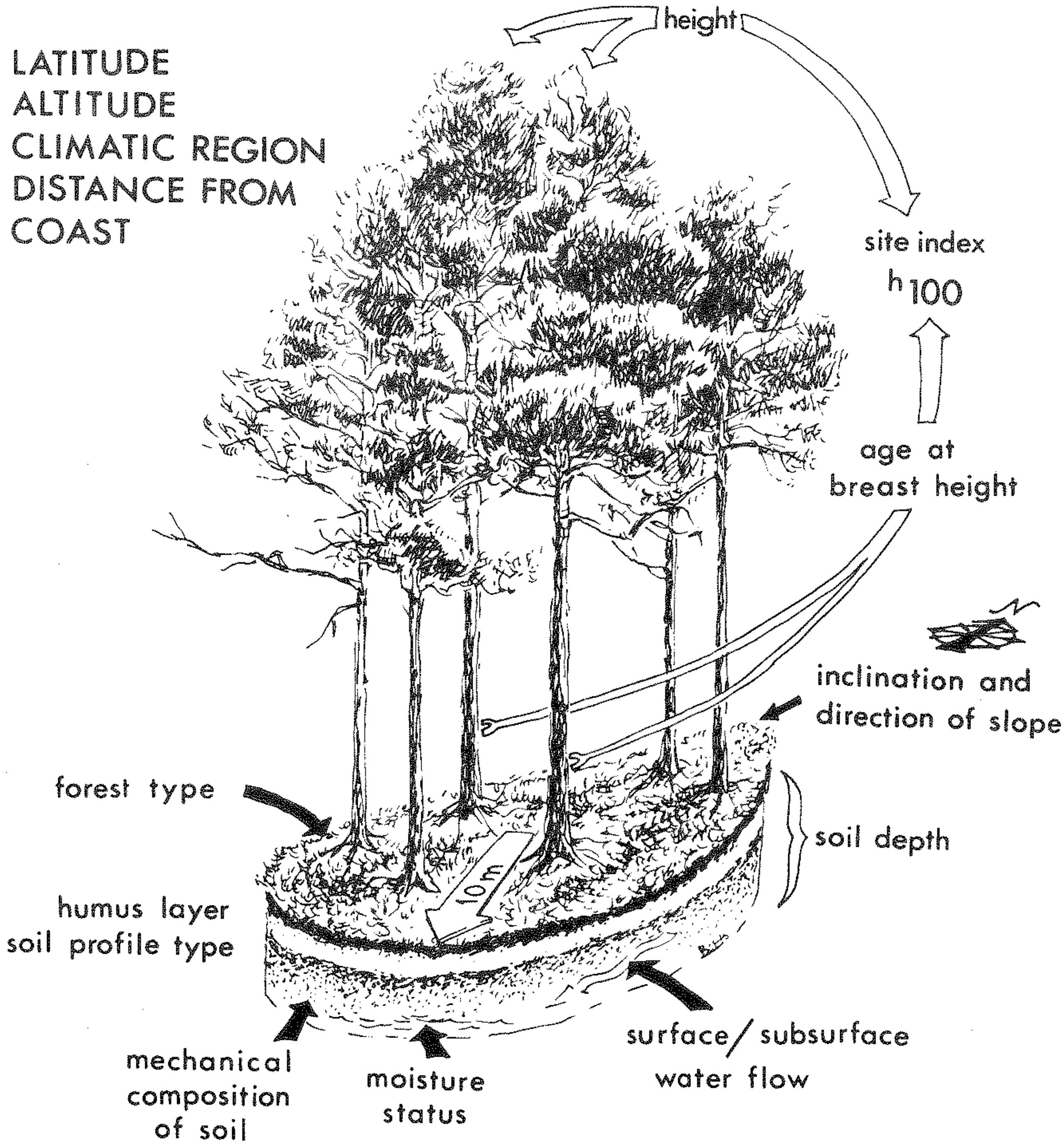


Figure 4. The National Forest Survey plot with some variables important in our investigation.

formulated by JONSSON, 1969. The basic assumption in the model is that the effects of different growth factors work together in a multiplicative way. As it is the effects (and not the factors) which are interacting multiplicatively, the relations between different growth factors and their effects must be modelled. As a proper description of this work must necessarely be fairly voluminous, we here only refer to HÄGGLUND & LUNDMARK, 1977.

The process under study is fairly complex and involves some very complicated interactions. In order to make the model reasonably simple, we roughly solved the problem of complicated interactions by dividing the data set into processing groups and fitting the model separately to the data in each group. In this way we could, for example, take into account the fact that the effect of temperature climate varies for different levels of soil moisture. In total, ten processing groups were formulated. The groups were defined by species (Scots pine/ Norway spruce), soil (mineral soils/peatlands), soil moisture (three classes) and forest type (three classes).

In figure 5, the regression function obtained for the processing group "Norway spruce on mineral soils, moist and slightly water-logged soils" is shown as an example of the results.

The main results of the study can be summarized as follows

- the effects of latitude, altitude and mobile soil water on site index are greater for Norway spruce than for Scots pine
- the more moist the soil, the stronger is the positive influence of mobile soil water
- the negative effect of increasing altitude is greater if there is no regular flow of mobile soil water than if such a flow exists
- within a soil moisture class, the effect of mobile water is greater the less fertile the forest type is
- the climatic regions have, in some cases, strong influences on site index. A continental region in the central part of southern Sweden has a positive effect on site index for Scots pine on very dry and dry soils. A maritime region along the eastern coast of southern Sweden has a negative effect on site index for both Scots pine and Norway spruce on mesic soils when the forest type is herbs, grasses or grounds without field layer
- deep soils have a positive effect on site index for Scots pine on very dry, dry and mesic soils
- textural index influences site index in most Scots pine processing groups. The coarser the soil, the lower is site index
- the influence of forest type is mainly as expected. This means that herb types, grass types and grounds without field layer are better than dwarf-shrub types which are better than lichen types.

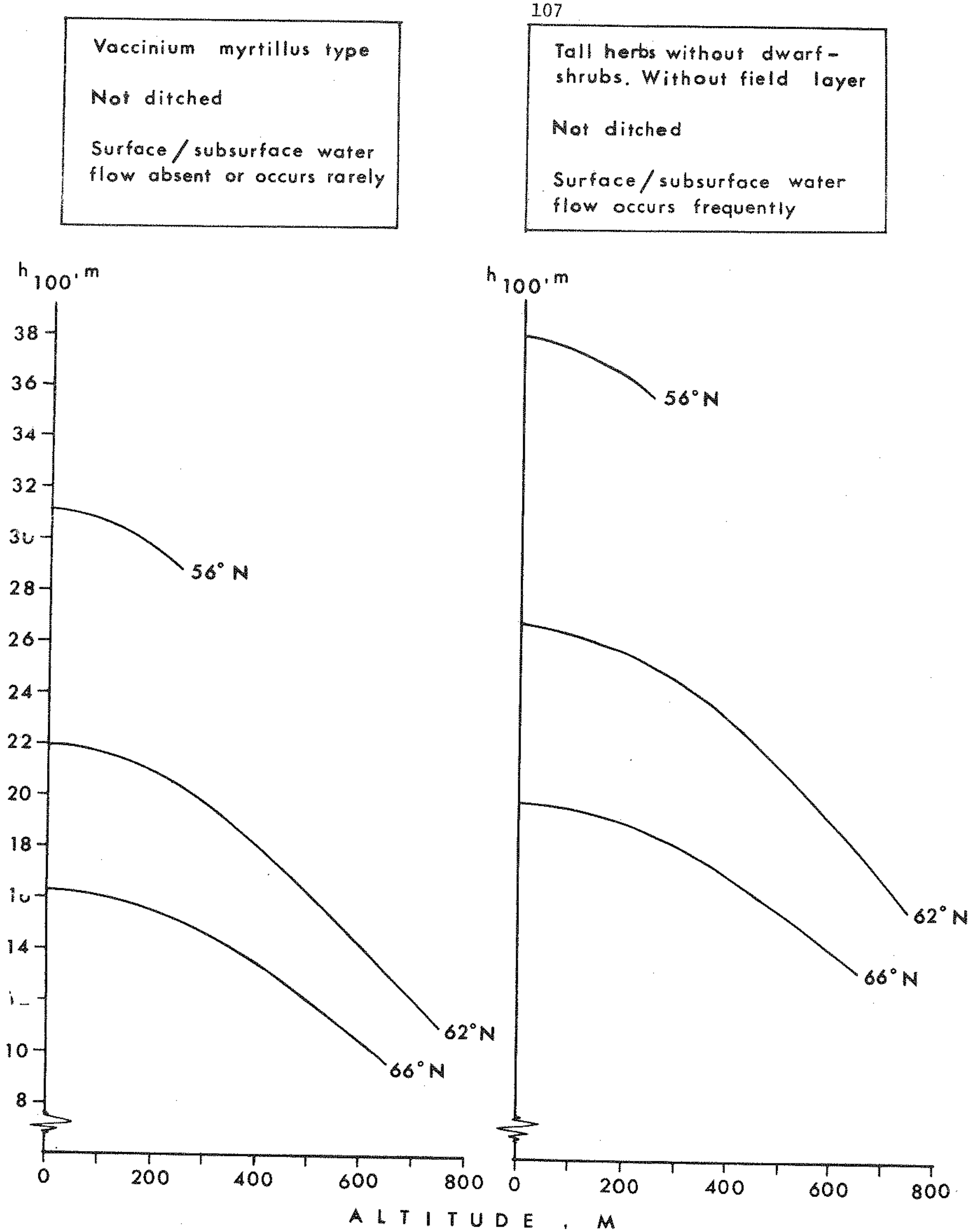


Figure 5. Norway spruce on moist and slightly waterlogged soils. Some partial relationships.

The results of the study were checked against new data - 809 temporary plots, laid out for yield research purposes. The check gave a fairly satisfactory result as the overall difference between estimates of h_{100} performed with site index curves and from site properties was not more than 0.21 m. Hence, there does not seem to be any overall bias in the functions. However, when the checking material is divided in processing groups, some big and statistically significant differences appear. We do not know for sure whether these differences are due to the fact that the functions are biased within the groups in question, or whether the checking material is too small to permit a division into ten groups. Some uncertainty concerning the quality of our results thus remains, but we have started doing some complementary work in order to clarify the situation.

Many topics can be raised in connection with our study. Here, however, we shall only mention four

- does the selection procedure affect the representativity of data? In other words, when we discard about 80% of our data as "non-acceptable stands", do we at the same time ignore some important combinations of site properties?
- we know that many measurements in our data set are subject to considerable error. We also know that, in regression analysis, random errors in independent variables might lead to a "thinning out" of the regression functions (MADANSKY, 1959). Is this of practical importance?
- we can observe that the residuals from our functions have a strong, negative trend over stand age. We explain this as an effect of the fact that the distribution of our data set is skewed in relation to site index/stand age. Is this the true explanation?
- the composition of the ground vegetation layers, and thus the forest type, might change according to variations in stand density, stand age and tree species composition. This might cause some problems when applying the results in practice.

These questions are discussed in HÄGGLUND & LUNDMARK, 1977, but we cannot say that we found completely convincing answers. Nevertheless, we will use our functions in the HUGIN model. We do not think there is any better way to estimate site index in those situations where there is no acceptable stand to rely on. However, as mentioned above, we will continue our work with the functions in order to reduce the number of question-marks which still surround them.

CHANGE-OF-SPECIES FACILITY

When making a forecast of forest growth over a long period of time, it is often realistic to assume change of species after clear-cutting on a fraction of the area under study. To make proper decisions on this question, it is necessary to know the site quality in terms of, for example, site index for different species. However, when using site index curves, the site index estimates obtained are linked primarily to the species of the existing stand. In order to overcome this difficulty, we have to design and use relationships between site indices for different species growing on equal sites. One such relationship between the site indices for Beech and Norway spruce in southern Sweden was presented by CARBONNIER & HÄGGLUND, 1969. A new investigation, in the first stage concerning a comparison between Scots pine and Norway spruce, started in 1975. A first report was published by LEIJON, 1977.

The most proper way to perform this new investigation would, of course, be to establish controlled experiments. This method is also used to some extent, but it will take a long time to get the results. In order to produce results more quickly, we have chosen to base the present study on temporary plots, laid out in the existing forest. Pure stands of Scots pine and Norway spruce, growing adjacent to each other on equal sites are searched for by the staff of the National Forest Survey and by forest owners. Reported stands are first of all thoroughly investigated in order to secure that the sites really are equal. If the stands and sites are accepted, 2 - 3 circular plots are objectively placed in each stand. Site indices are estimated with site index curves and a number of site properties are measured. The output of the investigation will amongst other things be functions where site index for one species is the dependent variable, site index for the other species and measures of site properties independent. These functions will be incorporated in the HUGIN SQ-system.

ACCURACY

The main principle for choosing which method to use for site index estimation in the HUGIN model is to use the <u>unbiased</u> method giving the highest precision. We have already stated that site index curves should be used in "acceptable" stands and thus we have indicated that the curves give higher precision in estimated site index than do the use of site properties. In the following we will discuss the accuracy of the two methods. First we point out that a number of checks indicate that the methods are free from large scale bias. Hence, it seems reasonable to regard all errors as random and limit the discussion to precision only.

Generally, getting measures of precision is an important part of the work with the HUGIN SQ-system. We might need to know the precision of estimated site indices in order to calculate the overall precision of the yield forecasts. Further, as site index is often used as an independent variable in regression analysis, we might also need precision estimates to be able to assess the eventual "thinning out" of regression functions, following from random errors in independent variables (MADANSKY, 1959).

In general the error of a site index estimate results from

- prediction errors. These errors originate from the functions used to predict site index. For example, most stands have a dominant height development which does not exactly follow the site index curves

- measurement errors
- sampling errors. In those cases when site index estimates from sample plots must be generalized, sampling errors occur.

In the following we mainly discuss those errors which are most evidently method-dependent, the prediction errors. We start by briefly summarizing a study of site index curves (HÄGGLUND, 1975).

As stated above, we define the "true" value of site index for a stand as

$$\bar{h}_{100}(i) = 1/(t_2 - t_1) \int_{t_1}^{t_2} h_{100}(i, t)\, dt$$

$h_{100}(i)$: "true" value of site index for stand (plot) no i

t_1, t_2 : ages; t_1 is a low age and t_2 is well beyond the normal rotation

$h_{100}(i, t)$: site index, estimated with site index curves for stand (plot) no i at age t years. $h_{100}(i, t)$ is assumed to be free of measurement and sampling errors.

In practice, the integral is replaced by a sum. The prediction error (S_p) is a function of age and can be written as

$$S_p^2(i, t) = E(h_{100}(i, t) - \bar{h}_{100}(i))^2$$

The symbol E is for expectation. To estimate S_p, we have used data from repeated measurements of permanent plots. The data set, which comprises 203 plots in Scots pine, has not been used for the construction of the site index curves. Figure 6 shows the result of repeated estimation of site index in the data set. As can be seen (it can also be shown numerically), there is no trend in site index over age. Hence, the curves used to estimate site index are unbiased, and all deviations from "true" site index are regarded as random.

The evident problem in estimating S_p from the data set described is to estimate "true site index" for each plot. The developments observed do by no means stretch over a whole rotation. To overcome this difficulty, we constructed a simulation model, which described the data set and could be realized over long periods of time. The model is on the form

$$h_{100}(i, t_j) = \beta_1(t_j)h_{100}(i, t_{j-1}) + \beta_2(t_j)h_{100}(i, t_{j-2}) + \epsilon(i, t_j)$$

i : plot no

t_j : age at measurement no j. The ages t_1, t_2 ... are equidistant in time. The interval between them is 5 years. At each age t_j, $h_{100}(i, t_j)$ is determined with linear interpolation

β_1, β_2 : coefficients to be estimated

ϵ : stochastic component

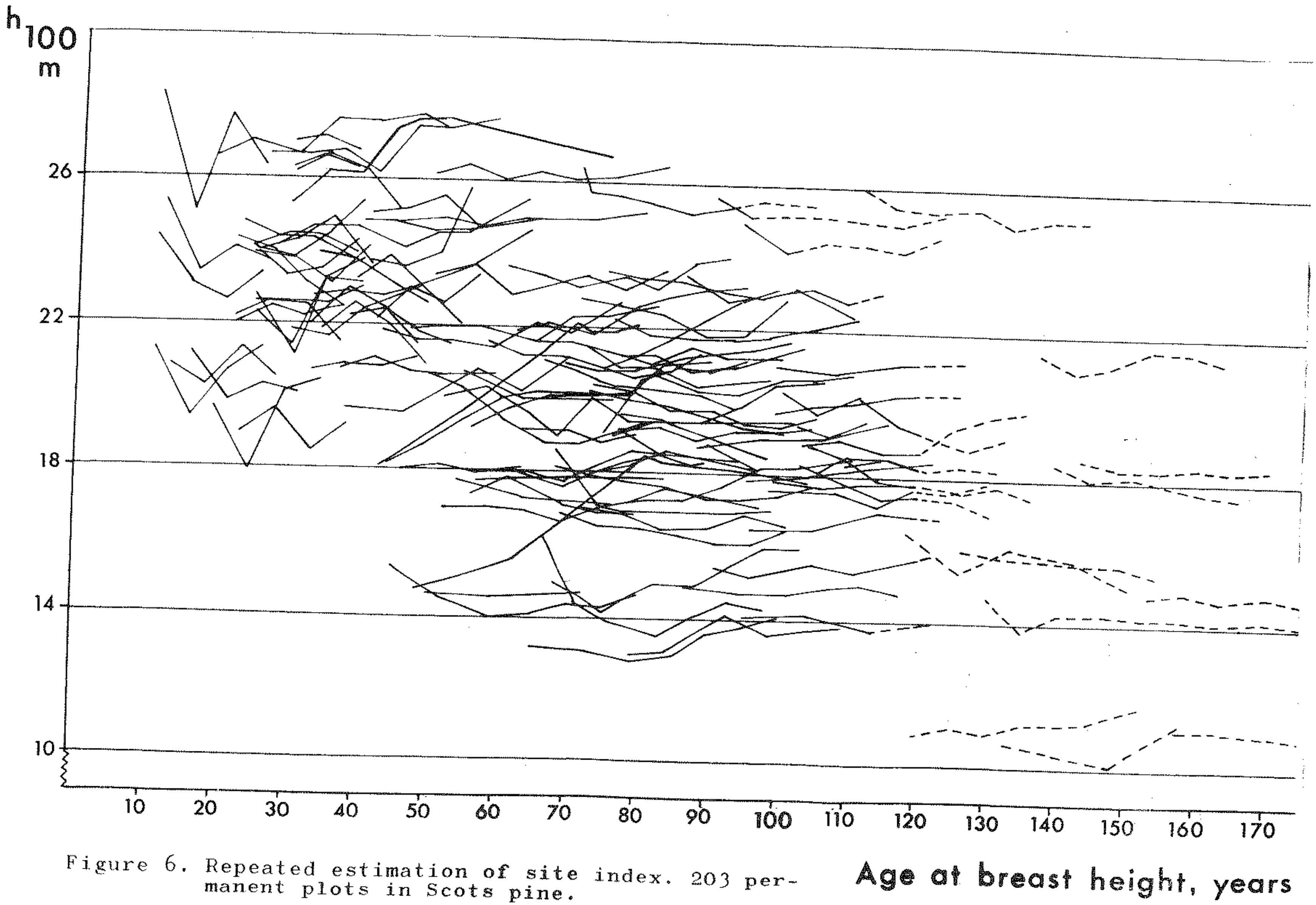


Figure 6. Repeated estimation of site index. 203 permanent plots in Scots pine.

The model is formally a second-order, autoregressive, stochastic one. The coefficients β_1 and β_2 were, as well as ϵ, estimated for different ages t_j with regression analysis. Investigations showed that ϵ is approximately normally distributed.

The model was used to simulate repeated estimates of site index over long periods - figure 7 is an example. By making a large number of simulations (8192), we got stable estimates of S_p, which are shown in figure 8. As can be seen, the prediction error of site index (standard deviation per plot) is about 2 m at an age at breast height of 10 - 15 years. At increasing age, the error decreases quickly to a minimum of 0.6 - 0.7 dm at 80 - 90 years. After that, the error increases slowly with increasing age. These figures are valid for Scots pine on homogeneous sites.

Turning to the method of using site properties, the estimation of precision is fairly simple. In this case we can use either the sum of squares obtained from regression analysis, or the outcome of the check with yield research data reported earlier. The former way of estimating precision means using data from the National Forest Survey ("inventory data"), and thus probably getting estimates of precision which refer to average practical conditions. The other method ("research data") gives result which are non-representative but comparable to those figures for site index curves shown in figure 8. Table 1 below gives the estimates of precision obtained with the two methods. It should be observed that the figures include both prediction and measurement errors (we cannot separate them). However, the prediction errors are by far the larger.

Table 1. Standard deviation, m, of h_{100} per plot when using site properties. The figures include both prediction and measurement errors.

	Inventory data	Research data
Scots pine	3.0	2.1
Norway spruce	3.6	3.0

We see that there is a considerable difference between the estimates of precision obtained from inventory and research data.

Figure 9 shows a comparison between the precision of the two methods - using site index curves or relationships with site properties - for estimating site index. The standard deviations illustrated refer to Scots pine and include both prediction and measurement errors. Comparisons should be made between those curves marked "research data". We see that site index curves from a stand age of about 10 years give the highest precision. For much of the age scale, the superiority of site index curves is very apparent. This is the reason why we state that site index curves should always be used as soon as the stand is in acceptable condition.

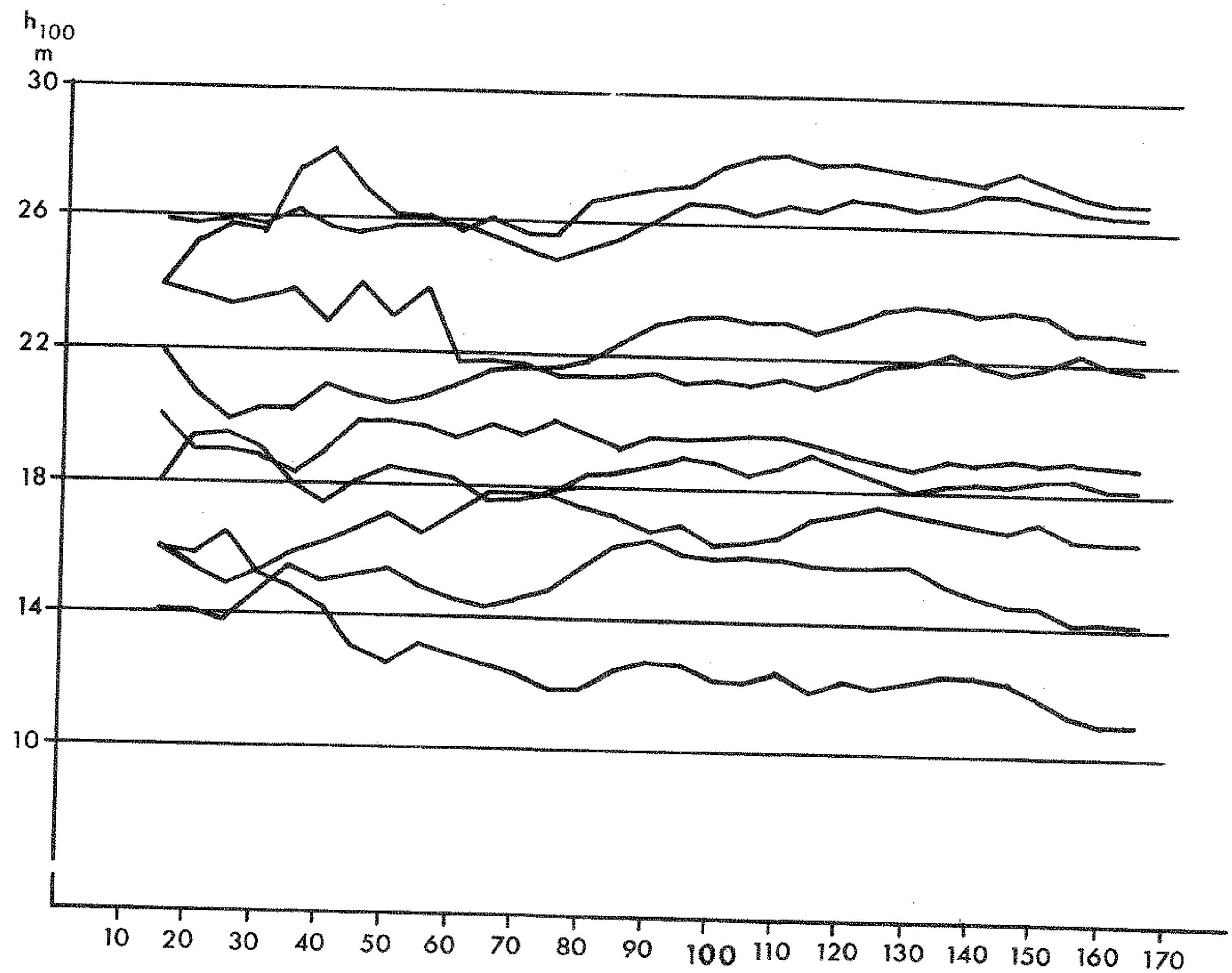


Figure 7. Some examples of simulated series of h_{100}

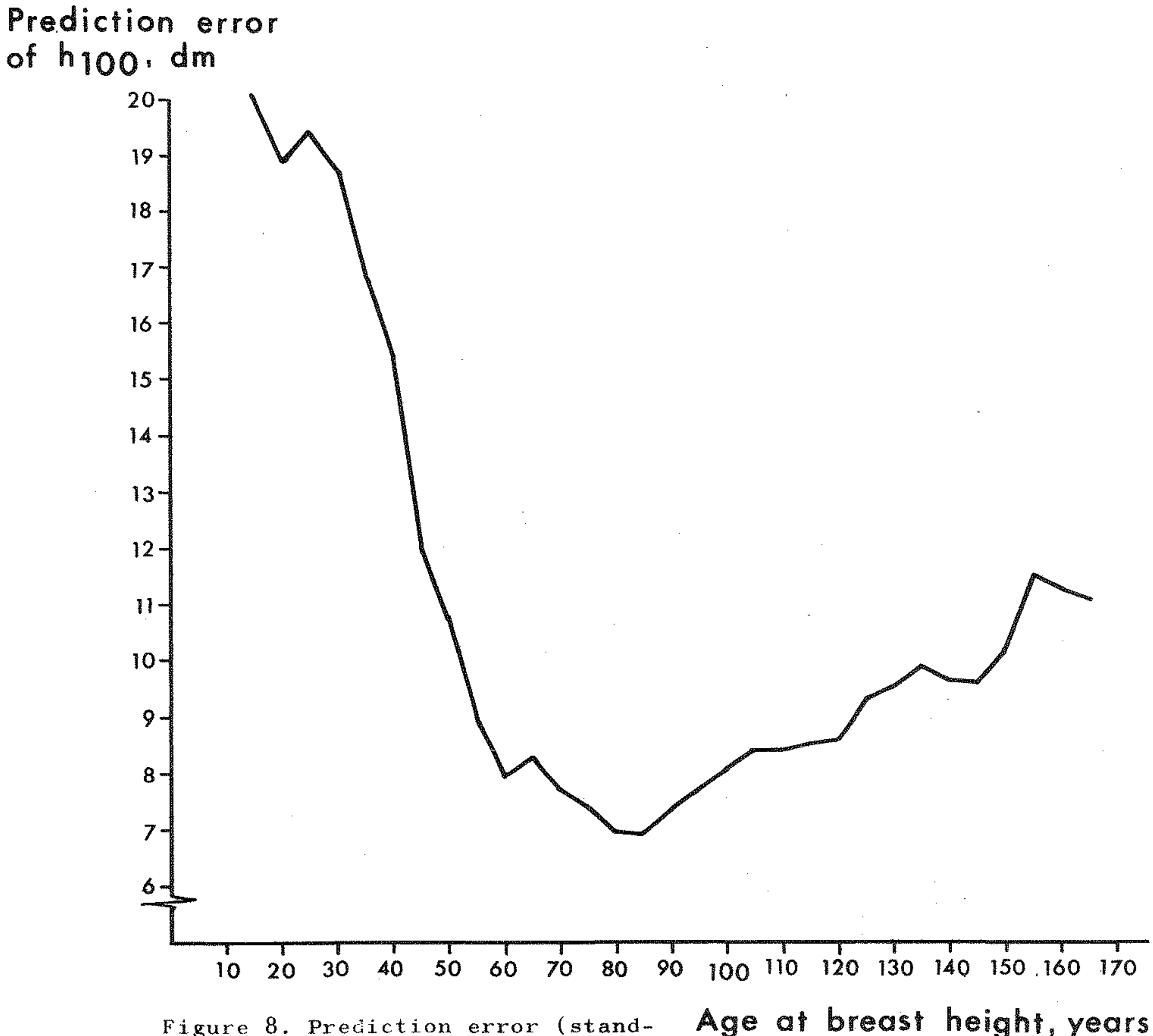


Figure 8. Prediction error (standard deviation per plot) over age at breast height.

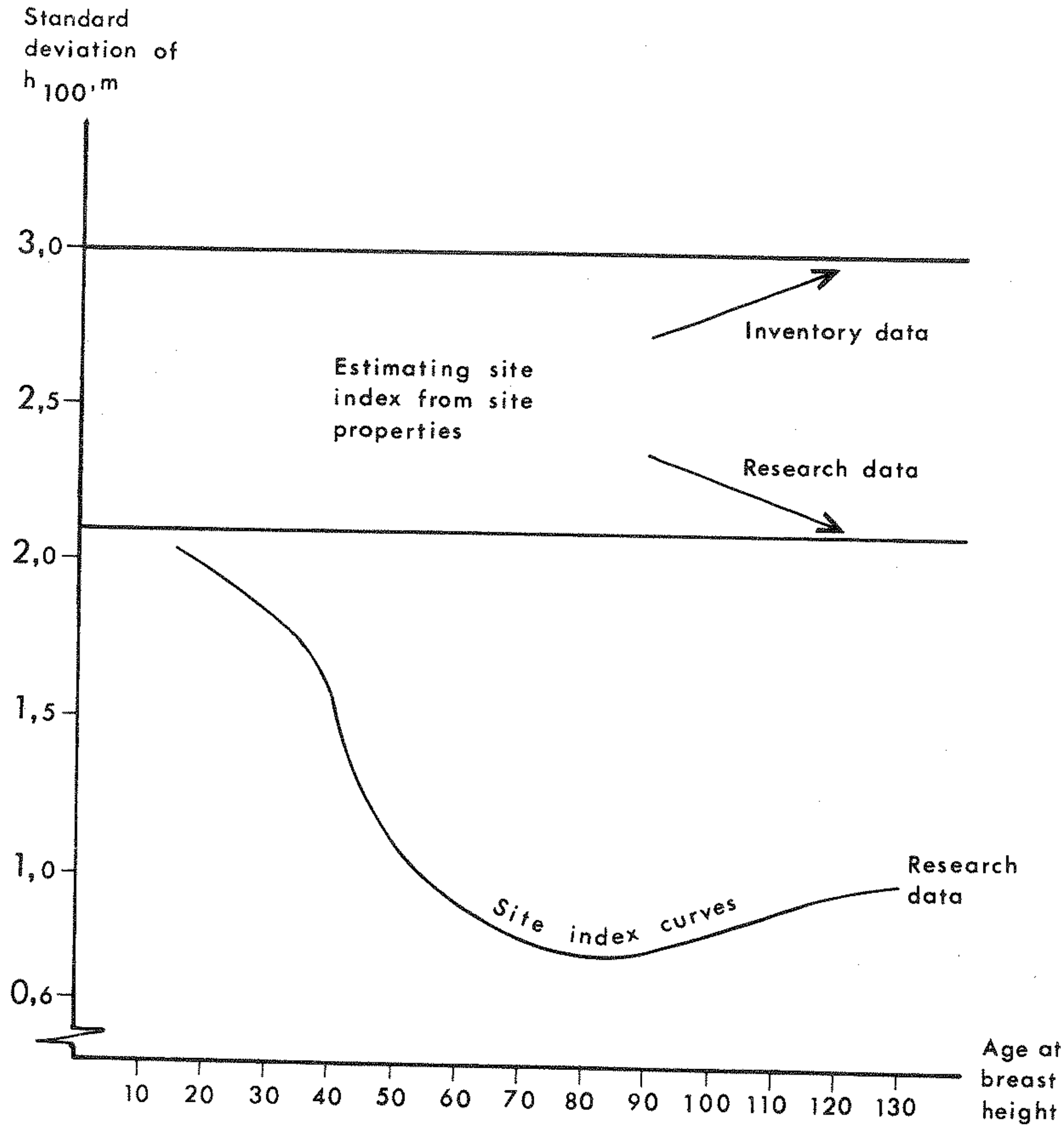


Figure 9. Comparison between the precision obtained when using site index curves and when estimating site index from site properties. The standard deviation includes both prediction and measurement errors. Scots pine.

DESIGN OF THE SQ-SYSTEM IN THE HUGIN MODEL

The SQ-system described has been implemented as a part of the HUGIN model. More precisely, the system is used in a primary processing part of the model, foregoing those parts which are directly used for the forecasting of yield. Figure 10 shows schematically how data flows through the SQ-system. It should be noticed that this design is the one used today - work is presently being done to make the system more complete and the outcome of this work might change the design to some extent.

By use of the HUGIN SQ-system, every plot gets one site index for Scots pine and one for Norway spruce. Further, if there is an acceptable stand of any other species on the plot, the site index for that species will also be estimated. In this way we believe we get a SQ-system, which makes it possible for us to study many interesting alternative site/species strategies for the future timber production of Sweden.

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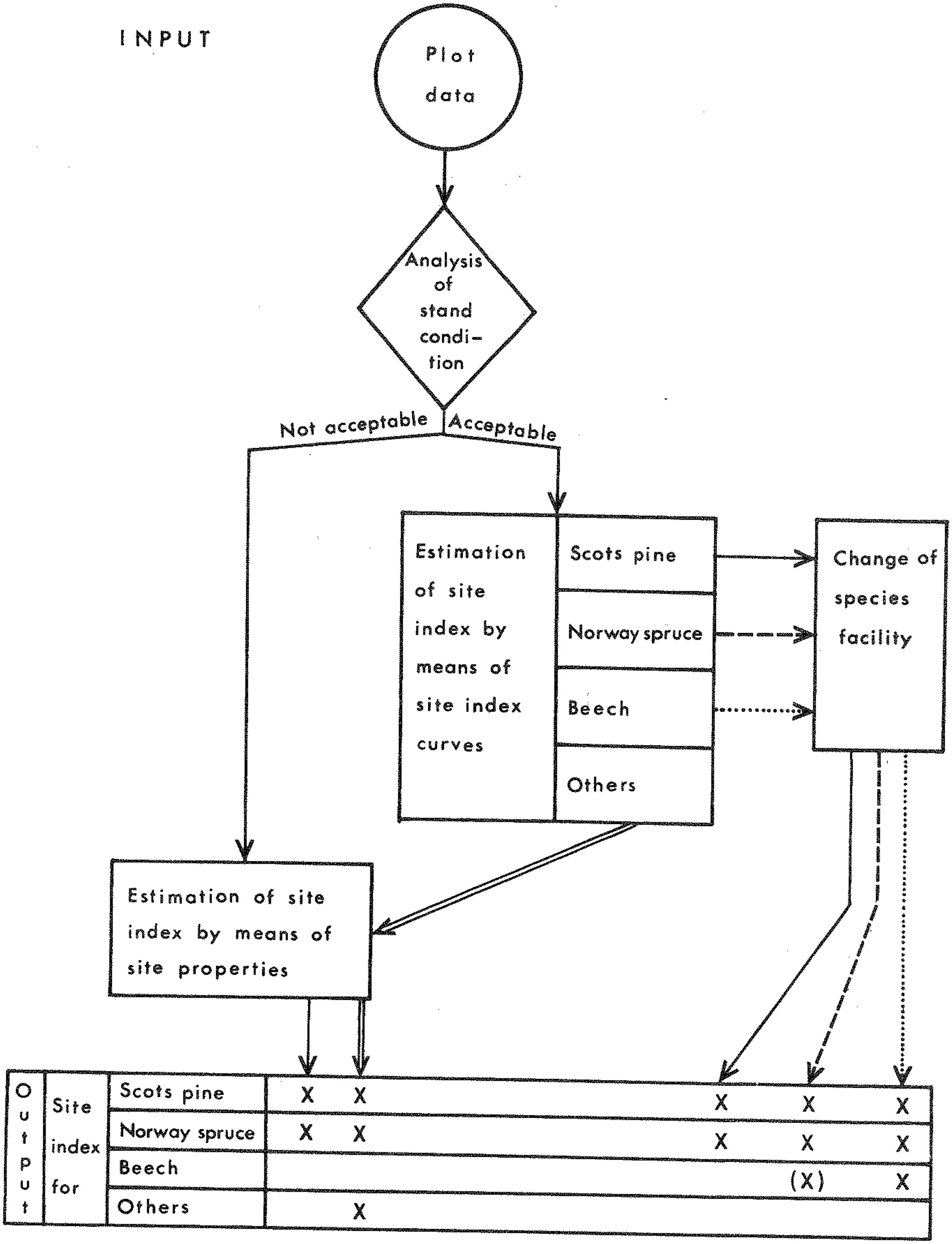

Figure 10. Data flow in the HUGIN SQ-system.

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A GROWTH MODEL AS A BASIS FOR LONG-TERM FORECASTING OF TIMBER YIELDS

(A preliminary version)

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1. General

In a previous study (Jonsson 1974), the author has described a method for the long-term forecasting of timber yields and application of the method in regional estimates of potential cut.

This study deals with growth functions for individual trees based on data from the northern half of Sweden collected by the Department of Forest Survey. Combined with other functions (see below), these functions can now be used in forecasts of forest development, according to simple logging alternatives, in this part of Sweden for periods of up to 50 years hence. The logging alternatives can be varied in respect to their point in time, the nature of the operations and their intensity.

This paper deals with the model of growth functions and also with the accuracy of the functions.

<u>List of functions used</u>

	Number		
	pine	spruce	birch
Diameter growth functions	3	3	2
Height curve functions	6	6	6
Bark functions	2	2	2
Volume functions	2	2	2
Thinning response functions	3		
Natural thinning functions	1		
Total: per tree species	13	13	12
together	4		
Total number of functions used	42		

2. Growth functions

2.1 General

The growth of a tree is the result of a complicated process embracing a multitude of factors. Generally speaking, it is relatively simple employing experimental investigations to determine the influence of one or more factors on a given phenomenon. However, this assumes that regular trials can be arranged and that the results can be awaited.

Other studies are based on phenomena which have occurred under natural conditions before the start of the investigation. They are therefore of a non-experimental nature - with all the statistical problems that this implies (see Wold 1956). In such cases, it is not possible to vary one or more factors and keep the other factors influencing growth under statistical control. Instead, all relevant factors are active without control, and it may be difficult to distinguish the influence of one or more factors from the influence of others.

In such cases, the analysis employs schematic models based on the knowledge available on the growth process. Attempts are then made using empirical materials to quantify the role of essential factors in this process or, in other words, to estimate some of the parameters in the model. The models should be realistic, but, at the same time, usable in practice.

2.2 Growth model

2.2.1 Objective and conditions

Every growth model is designed according to its intended purpose and to the conditions existing as regards observation data, etc. The objective of this study is to quantify the influence on annual diameter growth of both the tree and the environment, with consideration given to the influence of the weather. With access to material including both growth particulars and other relevant data, it should be possible to estimate the separate and the combined influence of essential factors on growth.

2.2.2 Construction of the growth model

2.2.2.1 General

A growth model is therefore constructed for the above case. The approach and its motivation are described in Jonsson (1969).

Among the factors affecting tree growth are the properties of the tree. The extent to which these properties influence growth is in turn affected by the site factors, i.e. forces and matter surrounding the tree, which release, limit, favour, retard or hinder the constitutional growth process.

For the period of time between t_1 and t_2, the growth may be expressed as

$$i(t_1,t_2) = \int_{t_1}^{t_2} i(t)\,dt = \int_{t_1}^{t_2} \varphi\,\{I(t),\ J(t),\ K(t)\}\,dt \qquad (2.1)$$

where i = growth

I, J and K are vectors with a large number of components,

where I = the internal, growth-determining factors

J = the external, non-climatic growth-determining factors and

K = the external, climatic growth-determining factors.

$K(t)$ is a multivariate, stochastic process with two periodic components (year and day) and variations around these.
$K(t)$ also influences the development of $I(t)$ and $J(t)$.

Baule´s law (1917) states that the interaction between the various growth factors is multiplicative, i.e. the effects of the various growth factors multiply each other (cf. Jonsson 1969, page 34).

Let us assume that the vector J includes components (J_2) which the above law applies to, but that the way in which the other components (J_1) and I influence the growth is unknown. Let us assume further that the effect of K is multiplicative. From this and from (2.1) the growth between t and t+dt may be written as $i(t)\,dt$, where

$$i(t) = f\{I(t), J_1(t)\} \cdot g\{J_2(t)\} \cdot h\{K(t)\} \quad (2.2)$$

Since functions f, g and h must be positive, both sides of the equation may be expressed in logarithmic form, thus

$$\ln i(t) = \ln f\{I(t), J_1(t)\} + \ln g\{J_2(t)\} + \ln h\{K(t)\} \quad (2.3)$$

It is assumed that t_1 and t_2 limit calendar year j. During this time, the non-climatic factors undergo successive changes. The condition of the trees and the soil vary according to the season as has been demonstrated by several researchers (cf. Jonsson 1969, page 35). Among other things, the conditions are affected by the weather. As a result of this weather, the effect of non-climatic factors will vary during the year.

It is further assumed that f_j and g_j are the logarithm of the mean values during year j of $f\{I(t), J_1(t)\}$ and $g\{J_2(t)\}$, with normal weather effect during year j. Thus, these expressions represent the average influence that would prevail if all conceivable weather conditions were distributed during year j. This implies that f_j and g_j are conditional expected values for year j.

Let us now assume that

$$\varepsilon_1(t) = \ln f\{I(t), J_1(t)\} + \ln g\{J_2(t)\} - f_j - g_j.$$

then

$$\ln i_j = f_j + g_j + \ln \int_{t_1}^{t_2} e^{\varepsilon_1(t)} \cdot h\{K(t)\}\,dt. \quad (2.4)$$

2.2.2.2 The influence of climatic factors

If the logarithm of the integral expression in (2.4) is designated H_j and the mean value for year j of all possible values of H is $\bar{H}_j$, then

$$\ln i_j = f_j + g_j + \bar{H}_j + (H_j - \bar{H}_j). \tag{2.5}$$

In this expression, $H_j - \bar{H}_j$ is the logarithm of the annual ring index adjusted for autocorrelation and divided by 100[1)] (cf. Eklund 1954).

If $\bar{f}_j$ and $\bar{g}_j$ are expected values under normal weather effects during the whole life of the tree up to and including year j, and if $\varepsilon_{2j} = f_j + g_j - \bar{f}_j - \bar{g}_j$, then

$$\ln i_j = \bar{f}_j + \bar{g}_j + \bar{H}_j + \varepsilon_{2j} + (H_j - \bar{H}_j). \tag{2.6}$$

In this expression, $\varepsilon_{2j} + H_j - \bar{H}_j$ corresponds to the logarithm of the total annual ring index[1)] divided by 100 (cf. Jonsson 1972). This logarithm is designated $\ln u_j$.

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1) Definition of annual ring index

Let b_j^* be expected value of i_j on the condition that K(t) = the real weather (say F(t)) up to the point t_1, when the year j begins.

Let similarly, the same be valid for b_j up to the point t_0 = the "birthday" of the tree or a fixed point of time.

We now can write

$b_j^* = E(i_j \mid \{K(t) = F(t);\ t \leq t_1\})$

and

$b_j = E(i_j \mid \{K(t) = F(t);\ t \leq t_0\})$

Definition: "adjusted" annual ring index $= 100 \dfrac{i_j}{b_j^*}$

total annual ring index $= 100 \dfrac{i_j}{b_j}$

Therefore

$$\ln i_j - \ln u_j = \bar{f}_j + \bar{g}_j + \bar{H}_j, \tag{3.7}$$

which may also be written

$$\ln \frac{i_j}{u_j} = \bar{f}_j + \bar{g}_j + \bar{H}_j. \tag{2.8}$$

However, the annual ring index occurring in practice is the mean value of a given population of trees (P). The annual ring index for individual trees naturally deviates from that for the population. In the case of an individual tree (s) we write the following:

$$\ln u_{sj} = \ln u_{Pj} + \varepsilon_{3sj}, \tag{2.9}$$

where ε_{3sj} represents the deviation of the individual tree in this respect. It seems reasonable that this component has a normal distribution with a mean value of zero and with a stable variance for the trees in the population.

Thus, for an individual tree we get the expression

$$\ln \frac{i_{sj}}{u_{Pj}} = \bar{f}_{sj} + \bar{g}_{sj} + \bar{H}_{sj} + \varepsilon_{3sj}, \tag{3.10}$$

which, if $\frac{i_{sj}}{u_{Pj}}$ is designated $\bar{i}_{sj}$, may also be written

$$\ln \bar{i}_{sj} = \bar{f}_{sj} + \bar{g}_{sj} + \bar{H}_{sj} + \varepsilon_{3sj}. \tag{3.11}$$

Accordingly, $\bar{i}_{sj}$ in the latter expression is the growth under normal weather conditions throughout the life of the tree (up to and including year j).

2.2.2.3 The influence of external, non-climatic factors

In expression (2.11), $\bar{g}_{sj}$ designates the logarithm of the effect of the components in J_{2sj} under normal weather conditions throughout the life of the tree.

This we write in the following way (cf. Jonsson 1969, pages 21 - 22).

$$\bar{g}_{sj} = \ln \{C_{sj} \cdot \prod_{r=1}^{m} (1 + \bar{q}_{rsj})\}, \tag{2.12}$$

where C_{sj} is a constant and $\bar{q}_{rsj}$ is a function of component No. r in J_2 under normal weather conditions throughout the life of the tree.

The significance of $\bar{q}_{rsj}$ is evident from the following identity

$$\bar{g}_{rsj} = \ln \{C_{rsj} \cdot (1 + \frac{e^{\bar{g}_{rsj}} - C_{rsj}}{C_{rsj}})\}, \tag{2.13}$$

and thus

$$\bar{q}_{rsj} = \frac{e^{\bar{g}_{rsj}} - C_{rsj}}{C_{rsj}}. \tag{2.14}$$

From this it can be seen that $\bar{q}_{rsj}$ denotes a percentage change in the influence on growth.

From (2.12) we obtain

$$\bar{g}_{sj} = \ln C_{sj} + \ln \prod_{r=1}^{m} (1 + \bar{q}_{rsj}), \tag{2.15}$$

which, using Taylor's theorem, may be developed and written

$$\bar{g}_{sj} = \ln C_{sj} + \sum_{r=1}^{m} (\bar{q}_{rsj} - \frac{(\bar{q}_{rsj})^2}{2} + \ldots). \tag{2.16}$$

2.2.2.4 Total, theoretical model

If $\bar{f}_j$ is written $E\{\ln f(I_j, J_{1j})\}$, then our theoretical model will take the following form

$$\ln \bar{i}_{sj} = \bar{H}_{sj} + \ln C_{sj} + E\{\ln f(I_{sj}, J_{1sj})\} + \\ + \sum_{r=1}^{m} (\bar{q}_{rsj} - \frac{(\bar{q}_{rsj})^2}{2} + \ldots) + \varepsilon_{3sj}. \qquad (2.17)$$

However, the construction of our model is not yet sufficient for practical application and must therefore be further developed.

In the model,

$$\bar{q}_{rsj} - \frac{(\bar{q}_{rsj})^2}{2} + \ldots = E\{Q_r(J_{2rsj})\}, \qquad (2.18)$$

where Q designates a function in J_{2rsj} (which is thus a value of the J_2-component No. r for tree s and year j).

Using the same designations, we obtain

$$\ln \bar{i}_j = \bar{H}_{sj} + \ln C_{sj} + E\{\ln f(I_{sj}, J_{1sj})\} + \\ + \sum_{r=1}^{m} E\{Q_r(J_{2rsj})\} + \varepsilon_{3sj}. \qquad (2.19)$$

However, we wish to employ the conditions of the different growth-determining components in I and J at the start of year j, such as these conditions would have been under normal weather conditions throughout the life of the tree. Let designations with the prime symbol refer to these conditions. Let us also assume that the conditions develop during year j according to a "normal" pattern.

Thus,

$$\ln \bar{i}_{sj} = \bar{H}_{sj} + \ln C_{sj} + \ln f^{\sim}(I^{\sim}_{sj}, J^{\sim}_{1sj}) + \sum_{r=1}^{m} Q^{\sim}_{r}(J^{\sim}_{2rsj}) + \varepsilon_{3sj}. \quad (3.2$$

However, these "normal" conditions are not always known; the known conditions are sometimes only those that have arisen during the weather conditions prevailing during the life of the tree. Nonetheless, in many cases these real conditions will probably be almost identical to the "normal" ones.

2.2.3 Specification of the model

2.2.3.1 General

The task we are now faced with involves finding suitable variables and analytical expressions for the model functions. Using empirical material, we then estimate the parameters in these expressions, obtaining growth functions.

As precisely as possible, the variables should describe, or be a measure of, the properties of the growth-determining factors. The selection of these variables is limited to the observations contained in the available material and is also dependent on limitations in connection with the use of the functions.

The analytical expressions to be selected should either have a built-in control which corresponds to known conditions or be flexible, governed either totally or partially by the material in cases where our knowledge is inadequate. Systematic errors can be introduced by means of strict control and, through excessive flexibility, features in the material may be included that are not general but caused by a random component.

The selection of analytical expressions constitutes a vital element which forces us to make more or less subjective appraisals.

2.2.3.2 Material

The model has been designed for use in a special study. We chose to construct the model from data of sample trees and sample plots contained in the National Forest Survey. However, this material is not primarily intended for yield studies. Consequently, we cannot expect the material to contain all the requisite data. For instance, we know nothing about previous fellings in the sample-plot stand.

The sample plots are laid out systematically in the forests in Sweden and comprise circular plots with a radius of 6.64 m.

2.2.2.3 Interaction between internal and some external factors

The interaction between the internal factors and the external, non-climatic J_1-factors has not been specified above (cf. 2.2.2.1). We assume that the vector J_1 can only assume a limited number of values, and that for each one of the values $\ln f^{*}(I^{*}_{sj}, J^{*}_{1sj})$ can partly be approximated by a second-degree expression in a variable which describes the properties of the internal factors of the tree. The remaining part is approximated by another tree variable, common to all J_1 values. This is assumed to apply to all trees of a given tree species. If the J_1 vector comprises classes in the Jonson classification system for site quality, and if the former variable describing the properties of the internal factors consists of the diameter of the tree at breast height, and the other variable is the number of annual rings at breast height, the following general model may then be obtained:

$$\ln f^{-}(I^{-}, J_1^{-}) = \alpha^{(k)} + \beta_1^{(k)} \cdot d + \beta_2^{(k)} \cdot d^2 + \beta_3 \cdot a + \beta_4 \cdot a^2$$

where k = Jonson site quality class
d = dbh
a = number of annual rings at breast height, and
$\alpha, \beta_1, \beta_2, \beta_3, \beta_4$ = parameters.

Consideration is also given to the influence of age in that the above parameters are estimated separately for three different age classes.

2.2.3.4 The influence of other external, non-climatic factors

The position of the tree in the stand relative to other trees is expressed as the ratio between the dbh of the tree and the dbh of the largest tree in the sample plot. This diameter ratio is roughly equivalent to the tree class which designates the position of the tree in the stand in respect of the height of the tree. The effect of this relative position is expressed in the model in the form of a second-degree function in the diameter ratio.

The basal area at breast height in the sample plot is used to express the density of the stand around the tree. A change in the density (e.g. as a result of thinning) implies an effect on growth, which effect will gradually increase during a period of several years before it subsequently decreases. Unfortunately, as mentioned earlier, informations of such changes are not included in the National Forest Survey material. However, whether or not felling has taken place in the stand can generally be established by field studies, in which the stands are assigned to one of the cutting periods, a or b. This classification is based on a subjective assessment of the cutting requirement. If cutting within ten years from the time of the inventory is deemed necessary, then the sample plot is assigned to cutting period a. If cutting is desirable in 10 - 20 years time, the sample plot is assigned to cutting period b.

Recent thinning has probably been carried out in stands in the b-class sample plots and, consequently, the favourable influence of the thinning on the growth of individual trees will be greater than in stands of trees with the same basal area in a-class sample plots. To make this evident, the model has been constructed so that the influence of density on the growth is written in the form of a second-degree function in the basal area for each of the two cutting-requirement classes separately.

The tree itself is included in this basal area, the real purpose of which is to denote the density of the stand around the tree. This is done with an eye to the practical application of the growth functions and is considered permissible, since the diameter square (d^2) of the tree also constitutes a variable. This assumes the value of its coefficient, which eliminates the effect of the basal area of the tree on the growth.

The influence of the geographical location of the sample plot is written in the same manner as that employed previously (Langlet 1936), i.e. with a linear function in each of the three variables, viz. latitude, altitude and the product of these variables.

The model is an example of how one or more growth-determining properties can be divided into classes (e.g. site quality class, cutting-requirement class, etc) and how the influence of one or more variables in each class can be described. Special expressions for these variables are necessary for each property.

2.2.4 Total specified model and example of diameter growth function

The final model is as follows

$$\ln i_d = \alpha^{(k)} + \beta_1^{(k)} \cdot d + \beta_2^{(k)} \cdot d^2 + \beta_3 \cdot a + \beta_4 \cdot a^2 +$$
$$+ \beta_5 \cdot \frac{d}{d_g} + \beta_6 \cdot \left(\frac{d}{d_g}\right)^2 + \beta_7^{(p)} \cdot G + \beta_8^{(p)} \cdot G^2 +$$
$$+ \beta_9 \cdot B + \beta_{10} \cdot H + \beta_{11} \cdot BH . \quad (2.21)$$

where i_d = five-year "normal" diameter growth (Note: not one-year as in the model)
k = Jonson site quality class
d = dbh
a = number of annual rings at breast height
d_g = diameter of the largest tree in the circular plot
p = cutting period
G = basal area of the sample plot
B = latitude
H = altitude

(See Figures 1 and 2).

2.2.5 The accuracy of the diameter growth functions

A number of diameter growth functions have been produced. We shall now deal with an elucidation of the accuracy of the functions. The standard deviation of individual trees between the real and estimated growth at breast height is approx. 50 - 60 % for pine and spruce aged between 5 and 125 years, and approx. 65 % for trees aged between 101 and 200 years. These figures may seem somewhat high. However, we must remember that the functions provide a forecast of the growth of individual trees. For practical purposes it is sufficient to know the total growth for a larger number of trees within a given area. For the purpose of elucidating the accuracy in the calculation of growth for a number of trees, the following processing was performed.

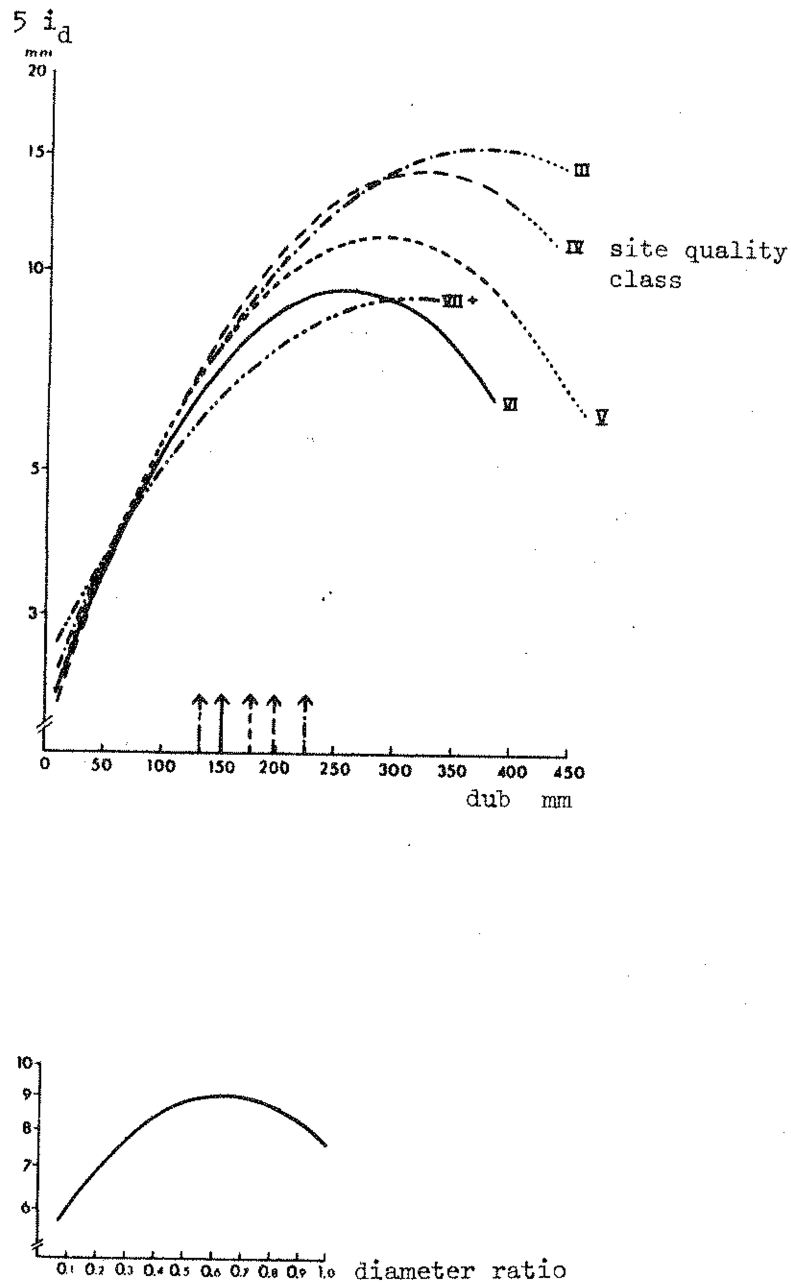

Figure 1. Partial relation between five-year diameter growth and the diameter of tree on sites of different classes and the diameter ratio.
The arrows indicate the average diameters in the material.

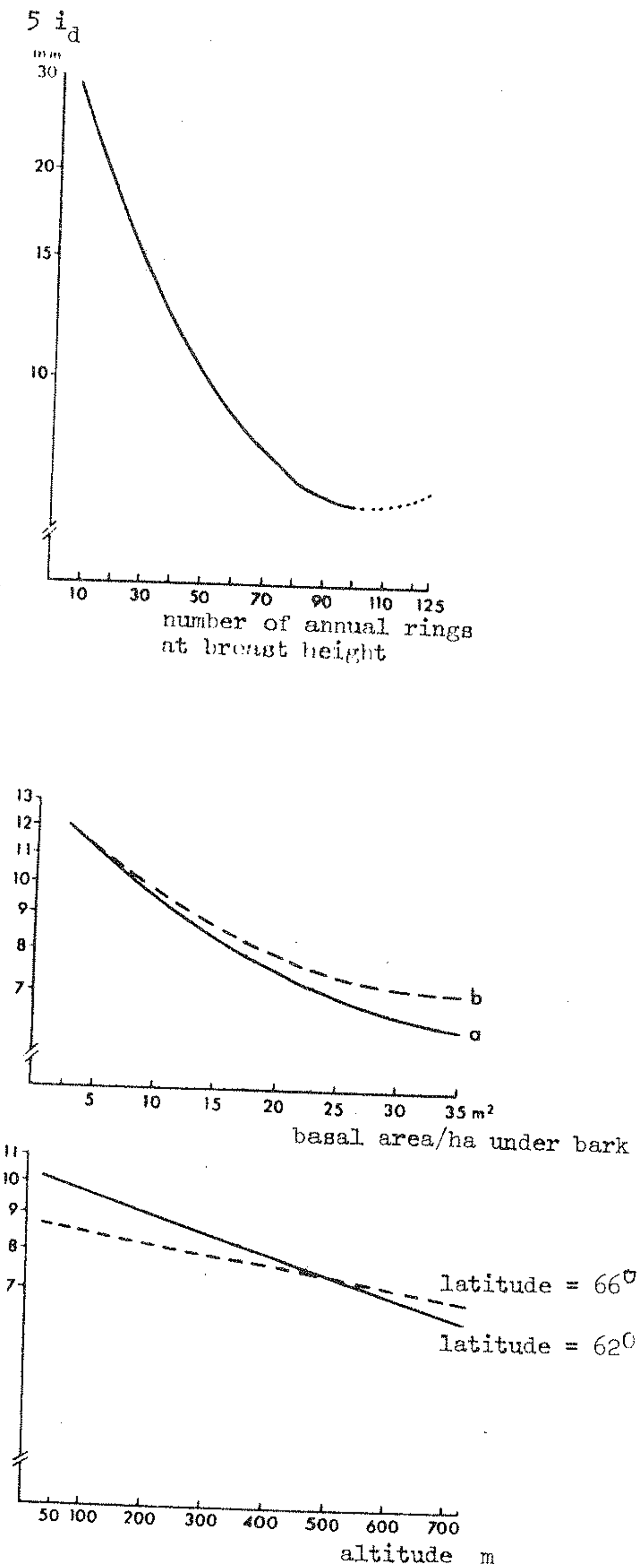

Figure 2. Partial relation between five-year diameter growth and the age of the tree (measured at breast height), the basal area of the sample plot, and the latitude and altitude.

The area studied (i.e. the northern half of Sweden) was divided into four latitude groups, each group then being subdivided into three altitude areas. Accordingly, twelve geographical areas were obtained. The trees included in the material were then divided into four age classes within each of the geographical areas. The mean observed growth of the trees in each age class was then compared with the estimated growth. The difference, or the residual, is an expression of the magnitude of the error in the given class and for the number of trees concerned.

The question then arises as to whether these errors are purely random or whether they also contain a systematic component; for instance, as a result of the influence of age and geographical location being stated incorrectly in the functions. In the case of pine and spruce, an attempt was made to illustrate this important aspect as follows.

Twelve geographical areas and four age classes within each area gives a total of 48 classes and, consequently, 48 residuals as well. The age classes embrace the intervals, 5-25, 26-50, 51-75 and 76-125 years, measured at breast height. If we plot these residuals on a graph against the number of trees in each class, the result obtained will be as shown in Fig. 3. This gives us some information about the scatter or, in other words, the error in the functions used.

If, ideally, this error were independent, having the same distribution around zero, then nearly all (95 %) of the points would lie within the unbroken lines, which describe a curve in the illustration. These lines are derived from $\pm 2\sigma/\sqrt{n}$, where σ denotes the standard deviation of the function and n the number of trees. In reality, the points are not arranged in this way, although not far from it. We may therefore conclude that the error inherent in the determination of the mean growth is favourable and has only a slightly disturbing effect on the final result in the case of many trees from a given age class within a limited geographical area.

It may also be claimed that the effect of both age and geographical location is expressed apparently without any major systematic errors in the function.

The vital question now is whether the functions can be used to forecast the development of a stand without appreciable errors being incurred. This was studied by means of a comparison of relatively well-documented, real growth with those calculated. We are of the opinion that the cases compared agree well (cf. Jonsson 1974).

3. Applications for the functions

As mentioned earlier in the introduction, the 42 functions have been used to make forecasts of the development of the forests within the parts of Sweden covered by the study for periods of up to 50 years hence and according to simple logging alternatives. The logging operations can be varied in respect of their point in time, their form and their intensity.

The functions are intended for use with the material from the National Forest Survey, with individual treatment and development of stands, or parts of stands, in the sample plots (radius of sample plot = 6.64 m).

Thus, the development of individual trees is calculated separately.

The stand development is based on the observed condition of the stand at the time of inventory and is calculated for consecutive five-year periods. The results are noted 0, 5, 15, 25, 35, 45 and 50 years after the time of inventory.

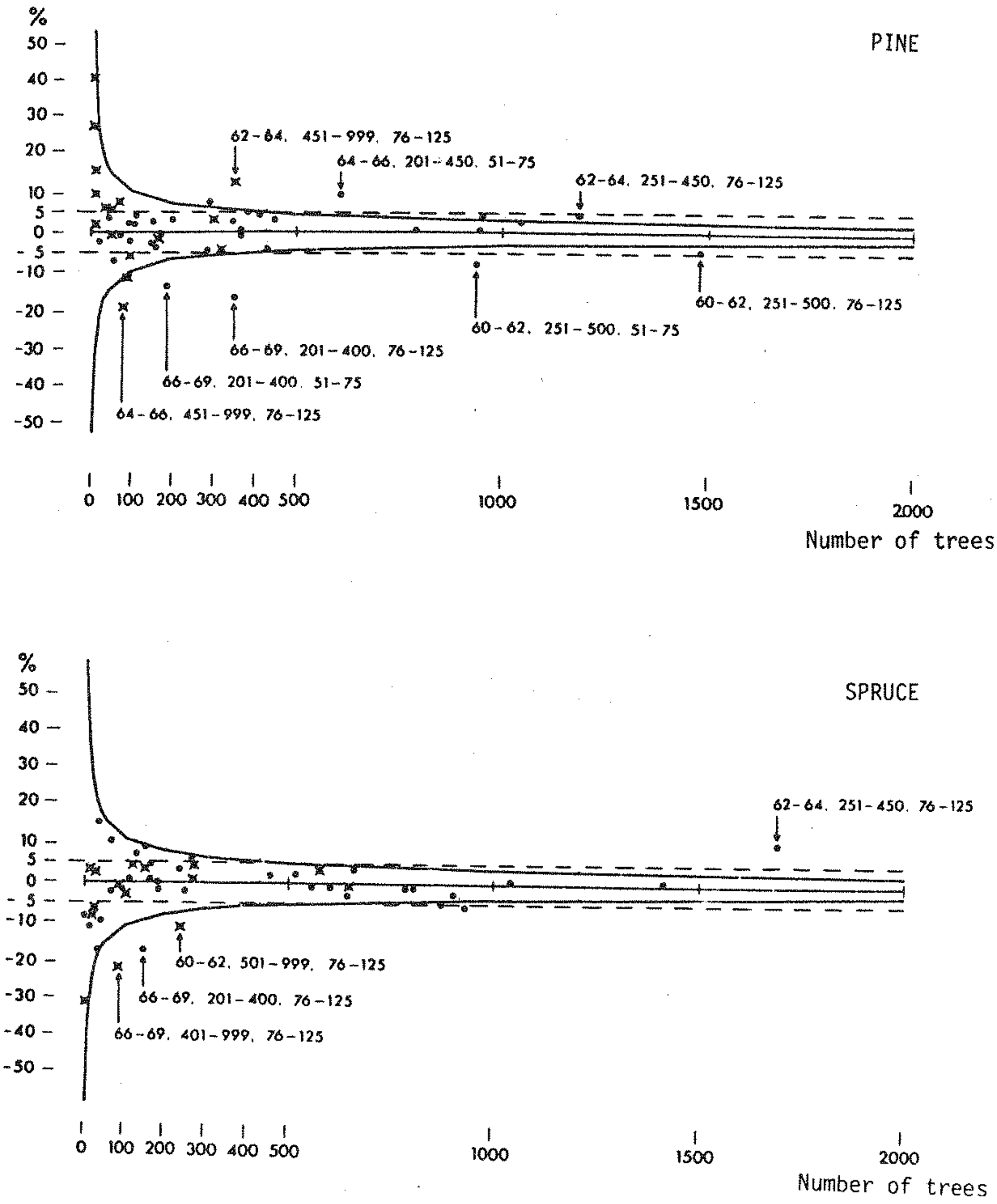

Fig. 3 Residuals plotted against the number of trees in each class. The unbroken lines, which describe cones on the graphs, are derived from $\pm 2\sigma/\sqrt{n}$, where σ denotes the standard deviation of the function and n the number of trees.

The crossed points refer to data on high-altitude areas. The figures referring to the points that are most noticeably outside the "cones" represent latitude group, altitude and age class.

Computer programs for the calculations are available.

A detailed description of the work is presented in Jonsson (1974, only in Swedish).

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ON THE COMPUTATION OF ANNUAL RING INDICES

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The method described in this paper has been applied at the Swedish College of Forestry since the 1960's (see Jonsson 1969, 1972).

A stochastic model

We consider a group of n trees, which are conceived of as a sample from a large population. The annual rings of the n trees have been recorded for a certain period. We denote by a_{ij} the width of the ring formed by the ith tree in the jth year of the period. By h_{ij} is denoted a corresponding hypothetical value: the ring width that would have occurred if the weather influences had been normal during the whole life of the tree.

We now make a number of assumptions on the relation between the actual (a_{ij}) and the hypothetical (h_{ij}) ring width. We first write

$$\ln\left(\frac{a_{ij}}{h_{ij}}\right) = m_j + \varepsilon_{ij} \tag{1}$$

Here m_j denotes the population mean - in the year j - of $\ln(a_{ij}/h_{ij})$. It represents the influence of the weather on the annual diameter growth in the population. The terms $\{\varepsilon_{ij}\}$ are deviations of the individual trees from the population mean m_j. They are supposed to be random variables with expectation 0 and a common variance, σ_j^2 (say). We have here a multiplicative model for the way in which various factors influence the diameter growth. As in other cases, a model of this type is best treated by a logarithmic transformation: From the assumed constancy of the variance of the ε's it is seen that the best linear estimate of m_j is

$$\frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{a_{ij}}{h_{ij}}\right) = \frac{1}{n}\sum_{i=1}^{n} \ln a_{ij} - \frac{1}{n}\sum_{i=1}^{n} \ln h_{ij}$$

To shorten the subsequent formulas, we write

$$x_j = \frac{1}{n} \sum_{i=1}^{n} \ln a_{ij}$$

$$f_j = \frac{1}{n} \sum_{i=1}^{n} \ln h_{ij}$$

for the logarithmic means of the actual and hypothetical ring widths, respectively. Since only the values $\{a_{ij}\}$ - and hence $\{x_j\}$ - are known, we need some additional assumptions to get further. These assumptions will essentially be that the "weather component", m_j, is <u>stationary</u> (in j) and that the component representing all other influences, f_j, is <u>a smooth</u> (<u>evolutive</u>) <u>function</u> of j which can be approximated by a simple mathematical expression such as a polynomial in j of low degree. This latter assumption seems reasonable if the number (n) of trees is not too small.

Estimation of the index

Our estimates of m_j now take the familiar form of residuals from a function graduating the series $\{x_j\}$. In the following, we take this function to be a polynomial of degree g.

If the series $\{m_j\}$ can be considered as purely random (thus a series of identically distributed independent elements), the obvious way to estimate the coefficients of the polynomial is to minimize

$$\sum_j (x_j - b_o - b_1 j - \ldots - b_g j^g)^2 \qquad (2)$$

However, there exist good reasons to assume that the nature of the series $\{m_j\}$ is not purely random. A simple model that appears to suit the situation much better is the autoregressive scheme of first order (see e.g. Jonsson 1969). We then assume

$$m_j = \gamma\, m_{j-1} + z_j \qquad (3)$$

where z_j is "purely random". The term γm_{j-1} in (3) represents a carry-over effect from one year to the next.

Using this approach, the estimates b_o, b_1, ..., b_g, and an estimate c (say) of the autoregression coefficient γ are obtained by minimizing

$$\sum_j \{x_j-b_o-b_1j-\ldots-b_gj^g-c[x_{j-1}-b_o-b_1(j-1)-\ldots-b_g(j-1)^g]\}^2 \tag{4}$$

In comparing results obtained by basing the estimation on (4) instead of (2), it is seen that the values of the coefficients $\{b_k\}$ do not differ very much.

Cf. fig. 1, which shows graduations with polynomials of degree 1 and 4 according to the two principles. The x_j-values are means of logarithms of widths of annual rings of 15 pines from the Muddus area in northernmost Sweden. However, if we apply ordinary regression methods to estimate the standard errors of the b's, the two approaches give divergent results. One example may suffice to indicate this divergence. If we base the estimation of a linear trend b_o+b_1j upon (2) and also base the error calculations on the assumption that $\{m_j\}$ is a purely random series, the value obtained for $\varepsilon(b_j)$ will be only half of the value that would have been appropriate if the true model were (3) with $\gamma = 0.6$.

A remark may be made concerning the "retransformation" of the logarithmic values. Let $\hat{m}_j$ be the estimates obtained from the residuals. The directly corresponding index is

$$I_j = 100 \exp(\hat{m}_j)$$

However, we may wish to adjust the index in such a way that the sum of the widths of all annual rings (a_{ij}) adjusted by the index of the respective years equals the total sum $\sum_i\sum_j a_{ij}$. Using the properties of the lognormal distribution (see e.g. Parzen 1960 p. 348) we find that we should then make a slight adjustment by computing the index from the formula

$$I_j = 100 \exp\left(\hat{m}_j-\frac{s^2}{2}\right)$$

where s^2 is the variance among the residuals $\{\hat{m}_j\}$.

It should finally be pointed out that if the index of annual rings is determined by fitting a smooth trend function to the empirical data (in logarithmic or other form), then the index can only reflect the short-term variation in the influence of weather factors.

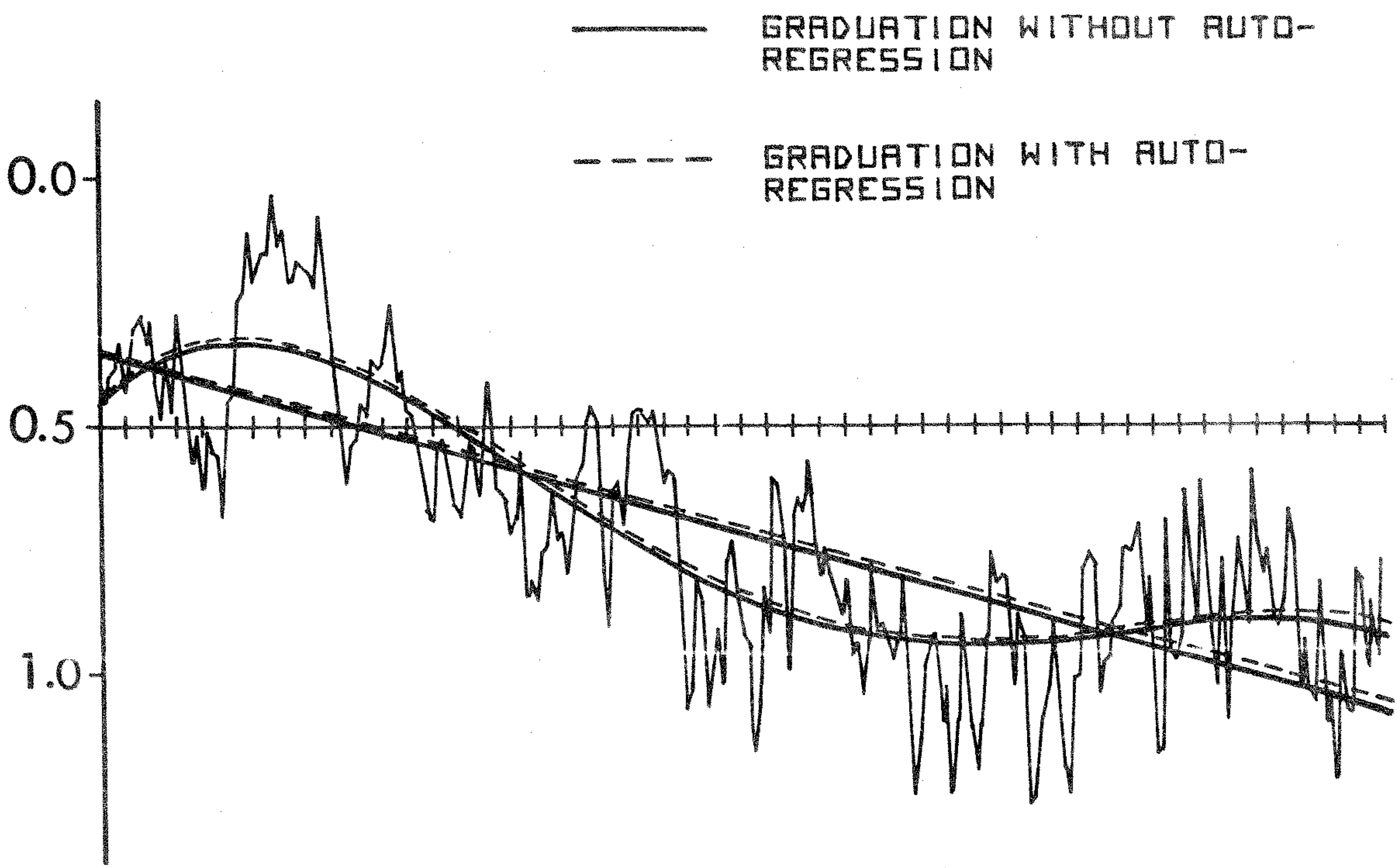

FIGURE 1. GRADUATION OF AVERAGE LOGARITHMS OF WIDTHS OF ANNUAL RINGS IN 15 SCOTS PINES FROM MUDDUS NATIONAL PARK (1723-1972)

If the weather is undergoing a long-term movement, the effect of this change will be included in the graduating function. However, if we include in this function also meteorological factors (Jonsson 1969, p. 244), we at least have a chance to obtain an index that reflects the influence of the weather also over longer periods. If we obtain a good fit by such a function, we can also hope to get a high precision in the determination of the index.

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IMPACT OF PLANTING DENSITY AND JUVENILE SPACING ON THE YIELD OF DOUGLAS-FIR

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SUMMARY

The impact of planting density (2,722, 1,742, 889, 436 and 222 trees/ac) and juvenile spacing on the yield of Douglas-fir, Pseudotsuga menziesii (Mirb.) Franco, was investigated with the Tree and Stand Simulator (Mitchell, 1975b) using height instead of age and site as the independent variable.

High mortality in closely spaced plantations upgrades the vigor of the population through natural selection and increases the average height of the stand as it reduces the number of trees until little evidence of the initial density remains. Close spacing yields the greatest gross volume but accumulated mortality accounts for much of the difference. The standing volume is greater in dense plantations because of the larger number of trees. Heavy mortality removes the advantage leaving little difference until later in the rotation when the benefits of natural selection appear. Basal area and volume growth are similar although the former is accelerated when the trees are growing freely. The average diameter at breast height of widely spaced plantations is large because the trees have bigger crowns for a longer time allowing them to utilize more resources. This advantage is lost when the high mortality in dense stands allows the crowns of survivors to increase in size and produce more diameter growth because the diameters are smaller and the trees are more vigorous. The age and height of the stand when the mean annual volume increment (MAI) culminates on a given site is not affected by spacing, but the increment and therefore the sustainable yield declines at the wider spacings.

Young stands spaced from 1,742 to 436 or 222 trees per acre surpassed the volume of the unthinned controls. The MAI culminated at a higher level but later.

(Keywords: Model, Spacing, Douglas-fir, Yield)

INTRODUCTION

This report[1] examines the simulated growth and yield of stands of Douglas-fir, Pseudotsuga menziesii (Mirb.) Franco, in relation to the number of trees established per acre. The level of stocking is controlled by planting density and juvenile spacing. The results apply to even-aged stands of second-growth Douglas-fir growing on moist sites of medium to high productivity on the coast of British Columbia. Only planted stands are considered but the conclusions should apply to stands of natural origin where the age of the trees and the density of the stand is fairly uniform.

The Tree and Stand Simulator (Mitchell, 1975a, 1975b) used in this investigation allows the crowns of individual trees to expand and contract asymmetrically in a three-dimensional growing space in response to internal growth processes, physical restrictions imposed by the crowns of competitors, and various environmental factors (e.g. site quality, defoliation, animal damage) and cultural practices (e.g. thinning, pruning, fertilization). The crowns add a shell of foliage each year that benefits the tree in diminishing amounts for five years. The volume increment produced by the foliage is distributed over the bole annually and accumulated to provide tree and stand statistics.

METHODS

The model was calibrated to conform with a large industrial data base of 460 permanent sample plots before undertaking the spacing experiments. Most plots ranged in age from 15 to 110 years and were measured 2 to 4 times at an interval of about 5 years for a total of nearly 1200 measurements. The height of site trees was used as the independent variable in place of age and site index to simplify the calibration procedure and generalize the results of the spacing experiments. The yield tables constructed by McArdle, Meyer and Bruce (1949) for Douglas-fir in the Pacific Northwest show negligible difference among sites 80 to 140 when total cubic volume per acre is related to the average height of dominants and codominants. Higher sites produce less volume relative to the height of site trees but this departure may be caused by non-site factors such as stand density. The industrial data base was analyzed by separating plots with full stocking ($0.08 <$ average distance between trees/height of site trees < 0.12; Wilson, 1951) into site classes (< 66, 66-75, 76-85, 86-95, 96-105, 106-115, 116 +; King, 1966). There was no apparent difference between the volume/height relationships of the sites above 75 feet. The lowest site class was obviously different. Consequently, the height

[1]The Productivity Committee of the British Columbia Forest Service and the College of Forestry of the University of Idaho provided the support for this project.

of site trees was used in place of age and site to calibrate the model for number of trees, volume and basal area per acre, and average diameter. The results can be applied to all but the lowest sites.

The experiments are designed to isolate the action of one or more variables of interest. Other variables are removed or held constant to prevent them from confounding the results. It is desirable, for example, to have the same trees involved in each experiment so that the populations are identical genetically. Spacing or number of trees per acre is varied by altering the size of the plot and the distance between the trees rather than the number of simulated trees. However, limitation of the computer make some designs unrealistic. The design of the experiments described in this report is shown in Table 1. All trials are performed with the same population of 400 trees except for the extreme densities. Four hundred trees spaced 14 feet apart occupy 1.8 acres making the run prohibitively expensive because the cost if primarily determined by the size of the plot. Consequently, 49 trees were established in Experiment 8. Four hundred trees in a 0.15 acre plot are insufficient for Experiment 1 with 4-foot spacing because the high mortality associated with this density would leave too few trees per acre to adequately represent the distribution of tree sizes at the end of the rotation. The number of simulated trees was increased to 625 which left about 50 trees when the dominants reached a height of 160 feet.

Table 1. Design of the experiment.

Experiment	Initial spacing (feet)	Number of trees per acre	Number of simulated trees	Juvenile spacing (feet)
1	4x4	2722	625	none
2	5x5	1742	400	none
3	5x5	1742	400	7x7
4	5x5	1742	400	10x10
5	5x5	1742	400	14x14
6	7x7	889	400	none
7	10x10	436	400	none
8	14x14	222	49	none

Trees were planted according to a square spacing regime with some variation to mimic field conditions. A forward and lateral standard deviation of 0.5 feet was requested about the intended location. The resulting spatial distribution was identical in all but the 4x4 and 14x14 regimes where it was extended or curtailed to accommodate the different number of trees. Juvenile spacing in Experiments 3, 4 and 5 was accomplished by removing the shortest trees in the plot when the site trees reached a height of 10 feet. The tallest 25 simulated trees (6 percent) comprise the site trees. Each run was terminated when the site trees reached a height of 180 feet which is 30 to 40 feet beyond the range of the calibration data. Consequently, statistics beyond a height of 160 feet should be interpreted with caution.

RESULTS AND DISCUSSION

The height of site trees is the independent variable in the following discussion. It can be related to age and site by means of the site index curves developed by King (1966).

Planting Density

Planting density refers to the number of trees planted per acre. Competition is the only source of mortality other than juvenile spacing which is considered later.

Number of Trees

Mortality of overtopped trees begins about the time of crown closure or later if the trees are widely spaced (Figure 1) and continues at a rate of about 5 trees per acre per foot of height growth of site trees until the ratio of the average distance between trees to the average height of site trees decreases to about 0.10. This ratio remains fairly constant throughout the remaining life of the stand.

The high mortality associated with close spacing offers an opportunity to upgrade the vigor of the population through natural selection. The best 200 trees selected from a population of 1742 trees (5x5 spacing), for example, will obviously be more productive than the same number selected from a population of 436 trees (10x10 spacing). High mortality also lowers the tree-to-tree variation among the survivors and increases the number of stems per acre towards the end of the rotation. The small differences in the number of trees in the three densest plantations were removed during the summary of the data to prevent them from confounding the comparisons presented in the following sections.

Average Height

The average tree increases in height at a rate which is 20 percent below the site trees prior to suppression and mortality (Figure 2). Competition early in the life of the stand suppresses the height growth of enough trees in the two densest levels to depress the average height. However, the suppressed trees die and the increase in the relative vigor of the populations becomes quite evident. The average height of the trees remaining when the site trees reach 180 feet is given below along with the increase relative to a hypothetical stand with no mortality:

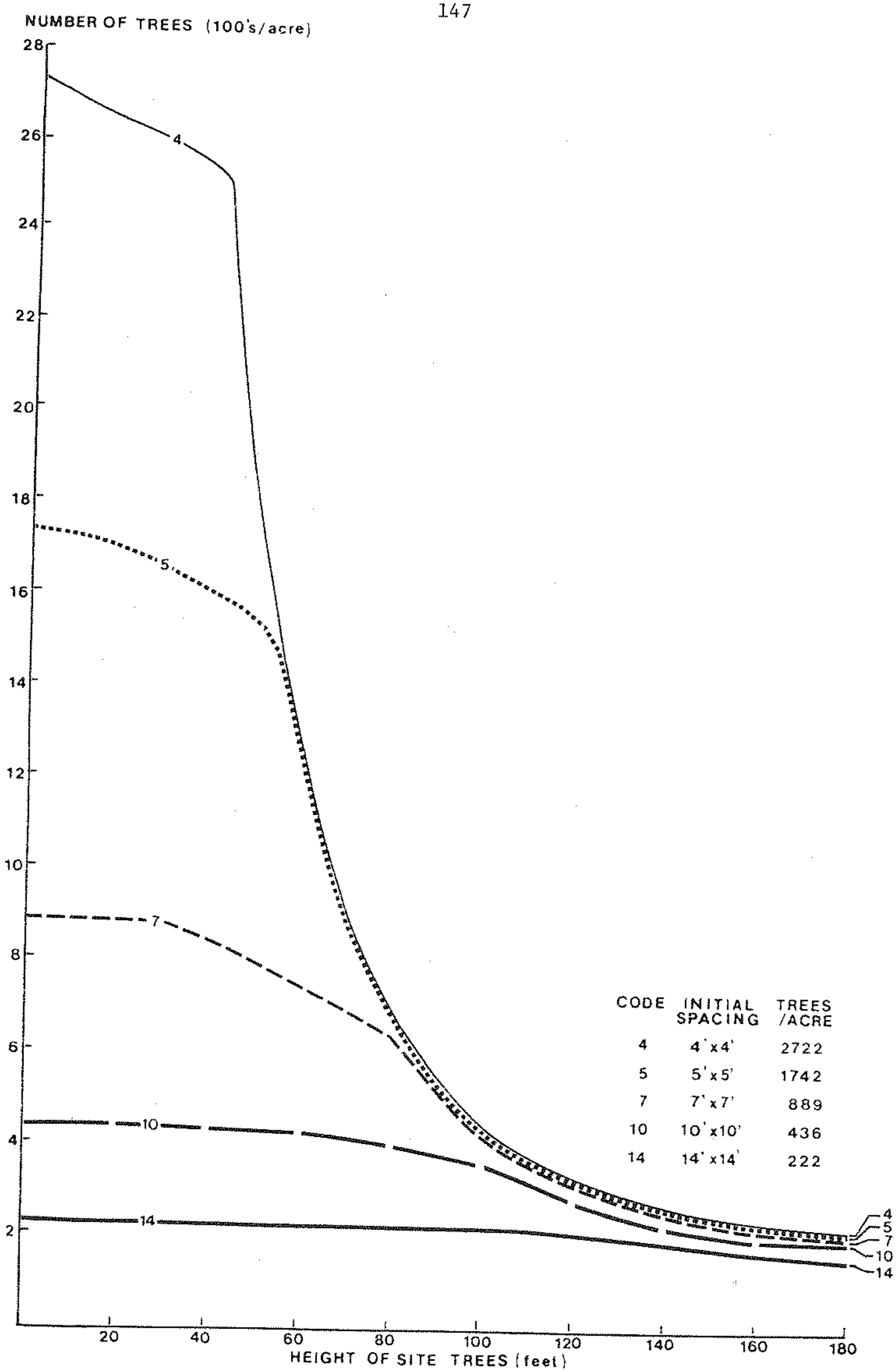

Figure 1. Relationship of spacing and site height to number of trees.

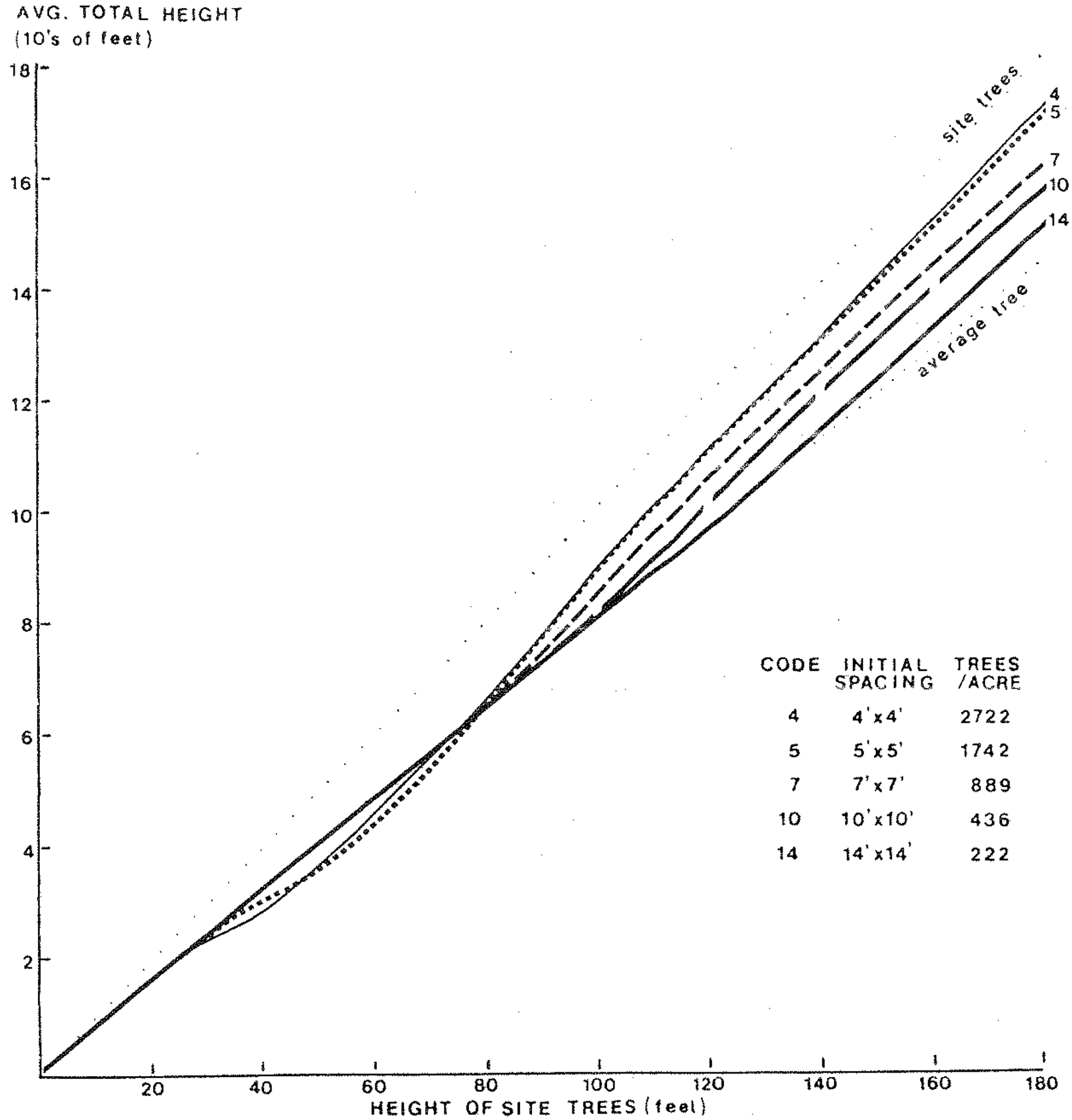

Figure 2. Relationship of spacing and site height to average height.

Spacing	Trees planted per acre	Living trees @ 180'	Average height (feet)	Increase in height (feet)	Relative increase (%)
4	2722	220	174	30	21
5	1742	220	172	28	19
7	889	220	163	19	13
10	436	196	158	14	10
14	222	159	152	8	6
(21)	(100)	(100)	(144)	(0)	(0)

The height growth of a few trees in each stand is retarded slightly by competition at 180 feet but this will not affect the results appreciably. Natural selection has effectively increased the productivity of the site. However, the apparent site index corresponding to a particular initial density may be high or low depending upon the selection intensity employed in the derivation of the site index curves.

Gross Volume

Gross volume includes the standing volume and all previous mortality in an effort to measure the total productivity of the stand.

Close spacing provides the greatest production (Figure 3) because the crowns can occupy and utilize the growing space relatively quickly. Furthermore, the high mortality improves the vigor of the population as discussed previously. The relationship between gross volume and initial spacing is approximately linear. The insert in the upper lefthand part of Figure 3 shows that the production at 160 feet of height increases 700 cubic feet for every foot the spacing is decreased. This implies that a limit of about 30,000 cubic feet could be produced at this height with very close initial spacing.

Total Volume

Total volume represents the standing volume left after the mortality is subtracted from the gross volume. The high mortality associated with close spacing removes a disproportionately high volume from the closely spaced stands leaving very little difference in the standing volumes, with the exception of the widest spacing, when the site trees reach a height of 80 to 100 feet (Figure 4). Consequently, little can be gained from close spacing unless the mortality is salvaged. This may not be feasible considering that the average diameter of trees that die in the 5x5 stand is 5.1 ± 0.4 inches during the period in which site trees grow from 90 to 100 feet. The advantage of close spacing is not realized until later when the stand capitalizes on the beneficial effects of the high mortality and natural selection.

The curves in Figure 4 exhibit a rather obscure pattern which is exaggerated for illustrative purposes in the insert in the upper lefthand corner. Each stand grows freely

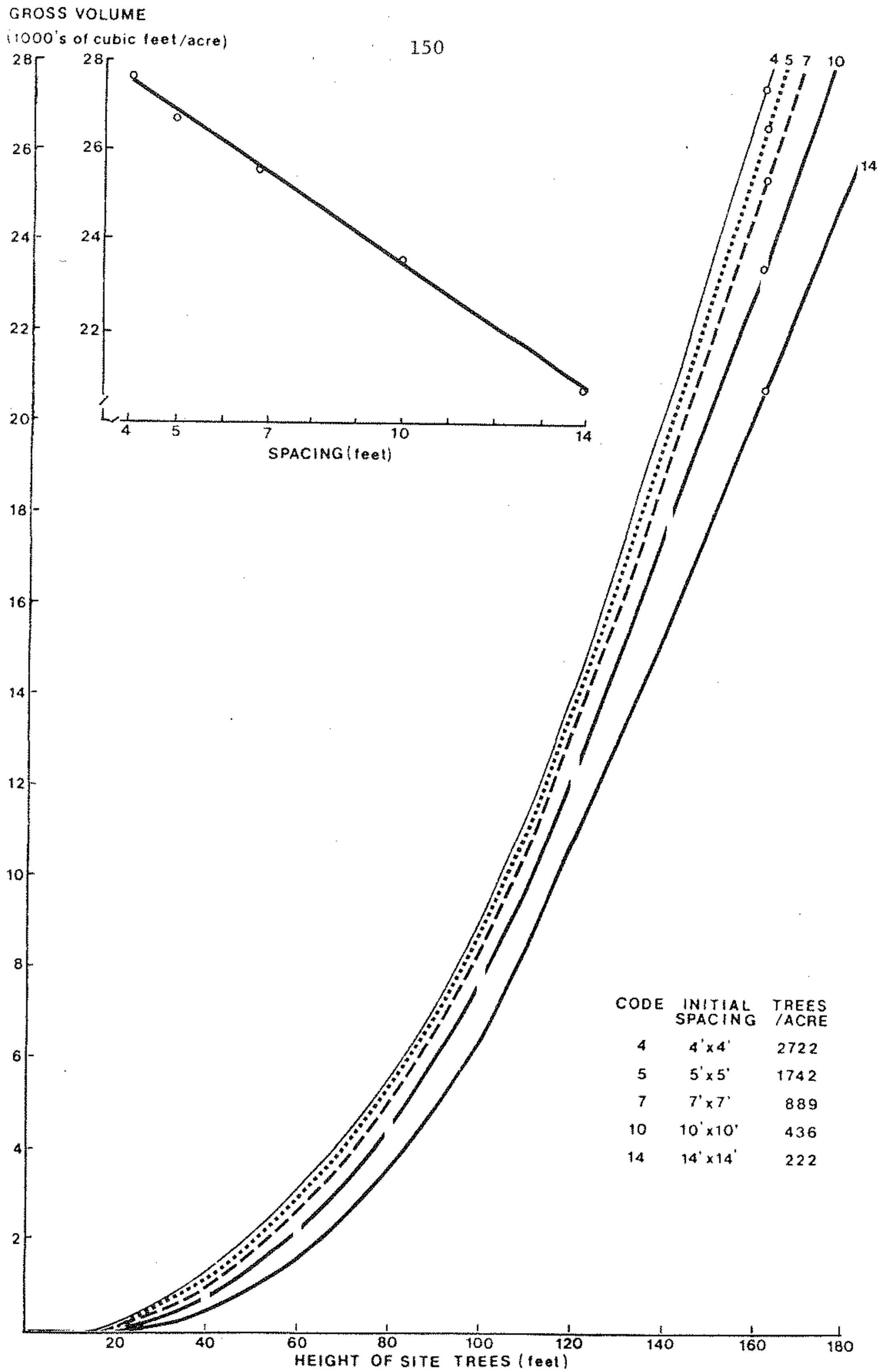

Figure 3. Relationship of spacing and site height to gross volume.

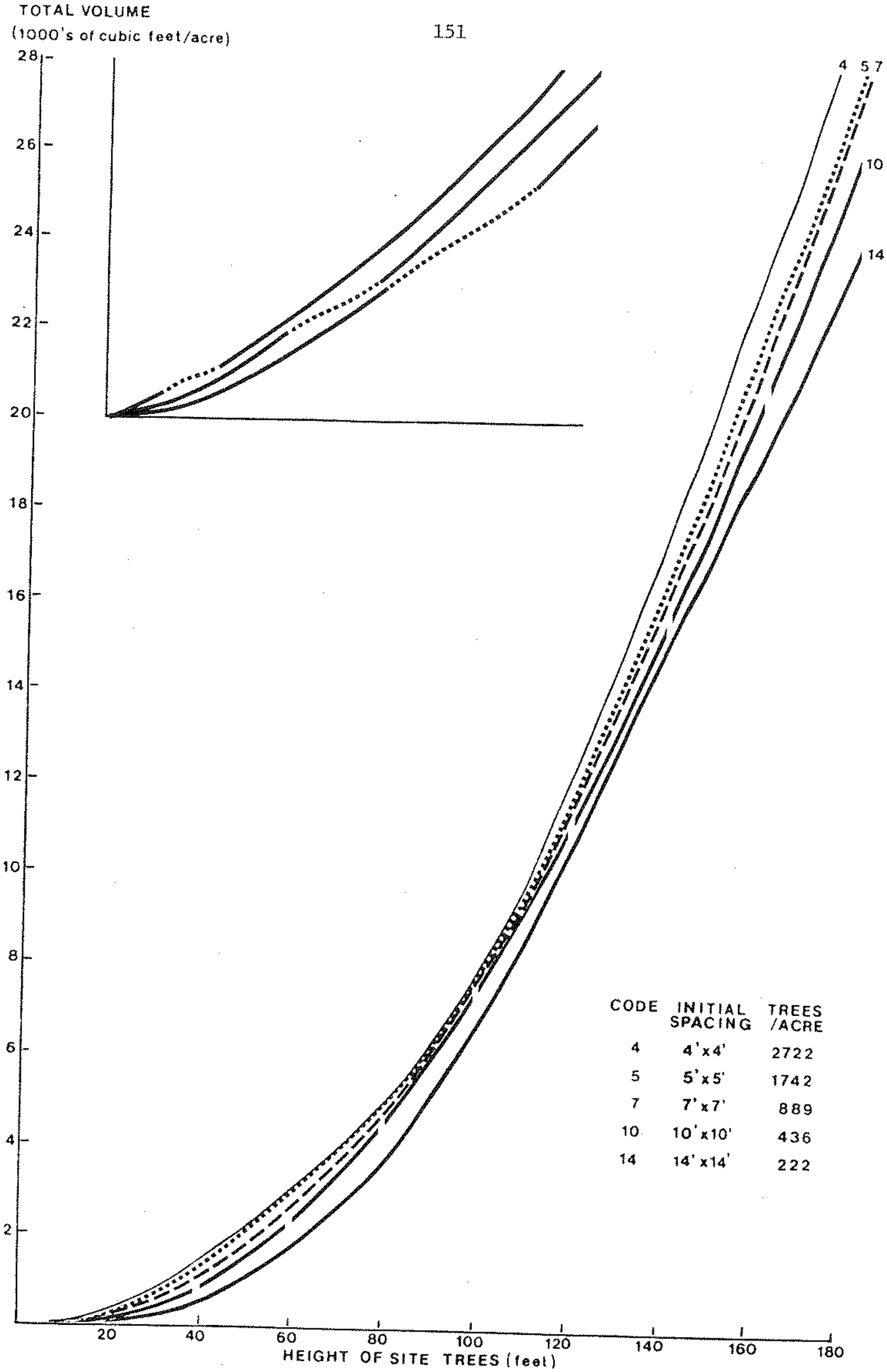

Figure 4. Relationship of spacing and site height to total volume.

initially followed by a period of high mortality relative to the final phase when mortality is minor (Figure 1). The duration of these periods is affected by the initial spacing which ensures that they are out of phase.

Basal Area

Standing volume and basal area bear a similar relationship to height although the "waves" are somewhat amplified in the latter case (Figure 5) and overlap because more growth is distributed towards the base of the bole when trees are widely spaced and have long crowns and a high live crown ratio. All stands concentrate increment at breast height until they reach a volume of 4000 cubic feet (Figure 6) and a height of about 70 feet at which point basal area increment in the 5x5 stand declines because of competition and receding crowns. The 10x10 stand is not affected until the site trees reach 110 feet.

Diameter

The diameter of the tree of average basal area is largely a result of the duration of the period before the crowns close and competition begins (Figure 7). It ranges from 3.5 to 7.0 inches at 50 feet of site height. The separation of the curves remains fairly constant after mortality reduces the number of trees to a common level (Figure 1). The distribution of heights and crown dimensions is similar in all stands during this period giving rise to about the same increment in terms of volume and basal area. All trees are then growing at the same rate and the difference in the bole dimensions are due to past growing conditions. That is, trees originating in widely spaced stands have larger crowns for a longer period allowing them to utilize and incorporate more resources. However, the closely spaced stands have a lasting advantage because the trees are superior genetically. The smaller diameters will also tend to increase faster because a constant amount of area increment is distributed over a shorter circumference. These factors will reduce the differences in diameter towards the end of the rotation. Note that the average diameter of the three densest stands is almost identical at 120 feet, and the trees in the 10x10 stand are only an inch larger.

Mean Annual Increment

The age and height of the stand when mean annual volume increment (MAI) culminates on a given site is not related to the initial spacing but the increment and therefore the sustainable yield does decline at the wider spacings (Table 2). The MAI culminates first on good sites when the site trees are 170 feet tall and 95 years old (Table 2) followed by the medium (140-150 feet at 100 years) and poor sites (120 feet at 108 years). This is considerably later than indicated by McArdle, Meyer and Bruce (1949) because the volume increment of the permanent sample plots used to calibrate the model does not decline noticeably.

Juvenile Spacing

The three uppermost curves in Figure 8 show the gains that can be achieved by precommercially thinning a stand with 1742 stems per acre (5x5 spacing) to two different

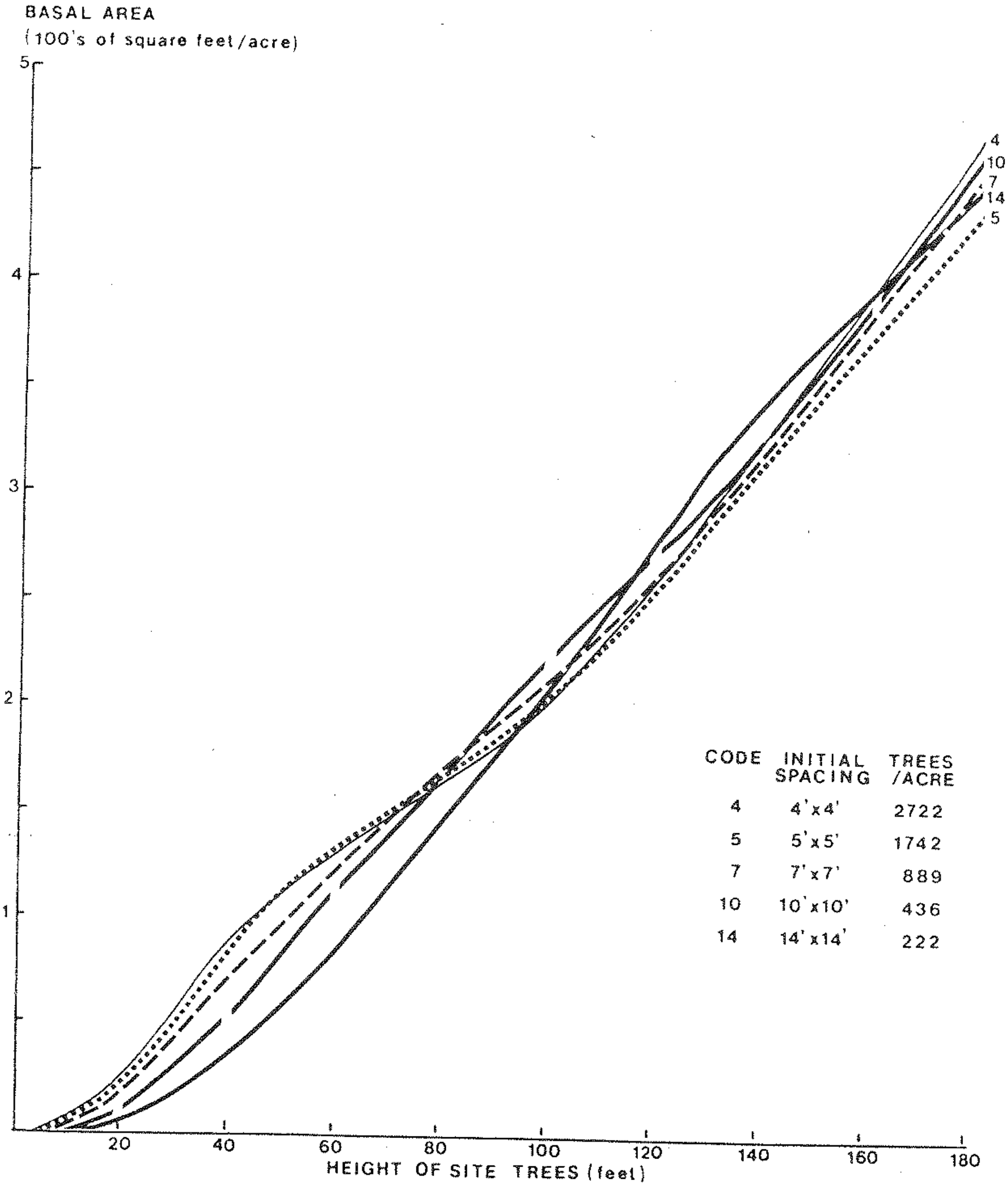

Figure 5. Relationship of spacing and site height to basal area.

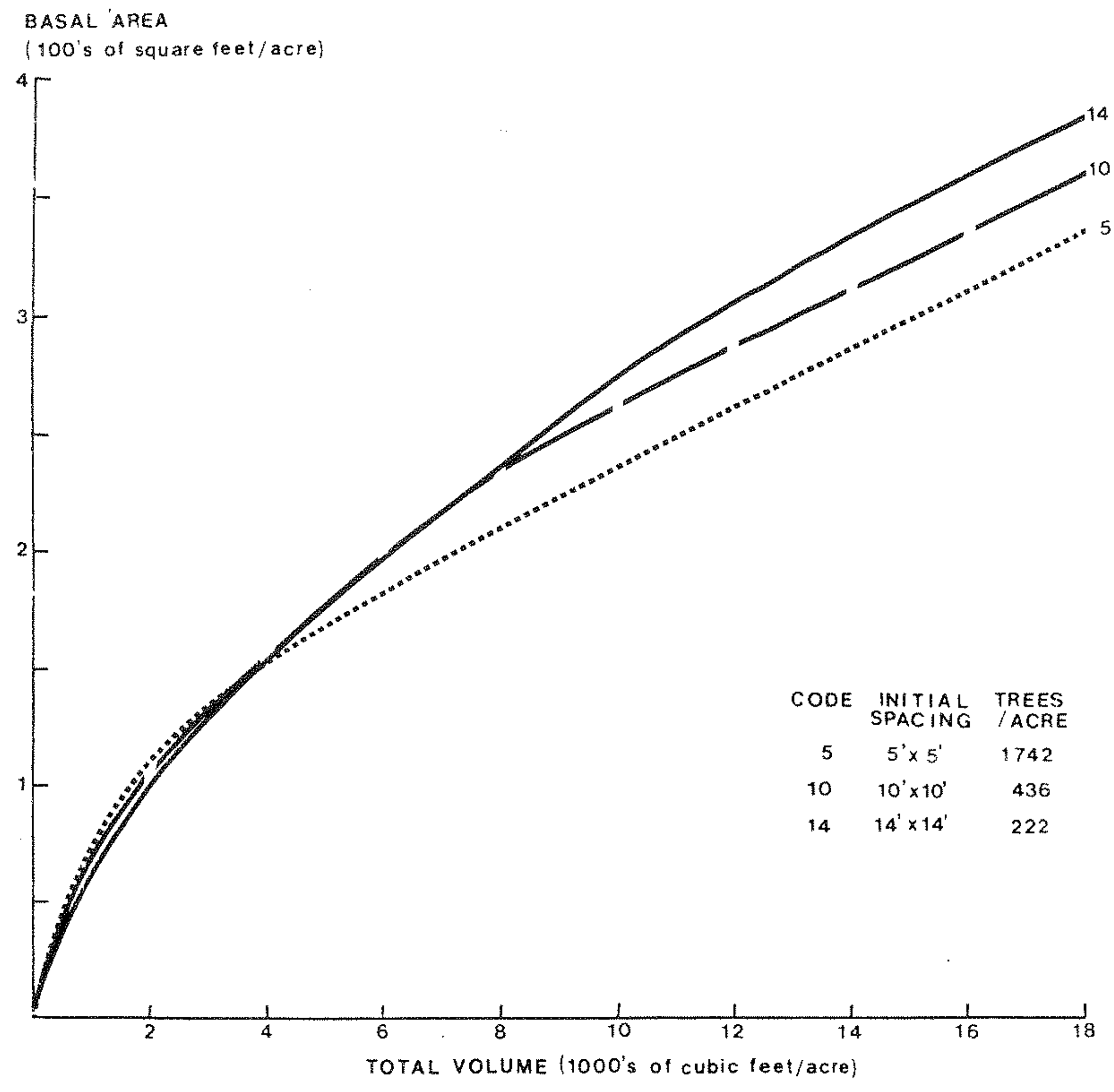

Figure 6. Relationship of spacing and volume to basal area.

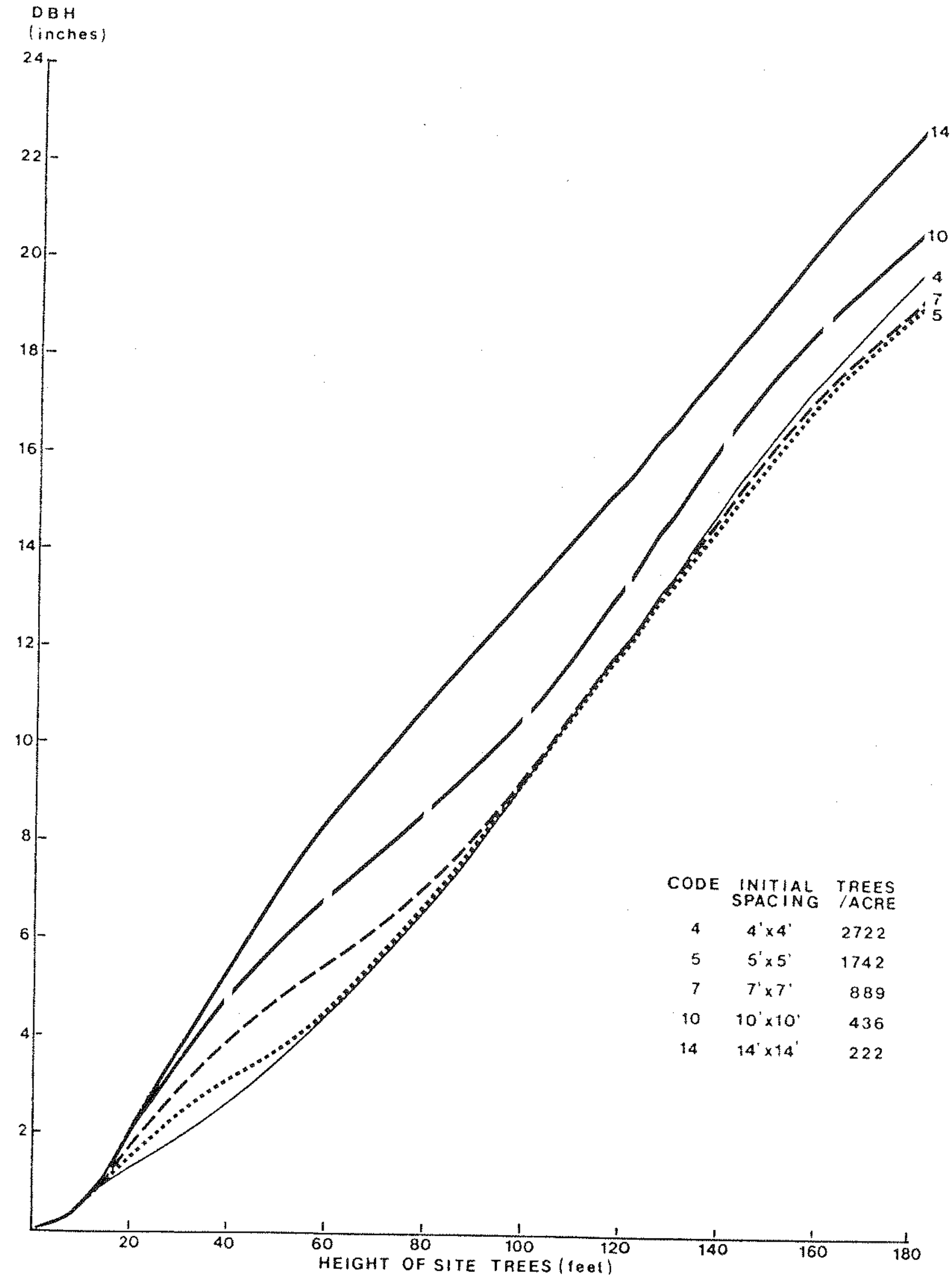

Figure 7. Relationship of spacing and site height to diameter.

Table 2. Culmination of mean annual volume increment (MAI) on good (G) medium (M) and poor (P) sites in relation to spacing.

Spacing (feet)	MAI (cubic feet)			Height (feet)			Age (years)		
	G	M	P	G	M	P	G	M	P
4	280	188	111	170	150	120	95	109	108
5	262	177	107	170	140	120	95	94	108
7	257	173	105	170	140	120	95	94	108
10	247	166	103	170	150	120	95	109	108
14	230	161	96	160	140	120	85	94	108
5 -> 10	274	185	113	170	150	120	95	109	108
5 -> 14	283	192	109	180	150	130	107	109	134

levels. In both cases, the shortest trees were removed when the site trees attained a height of 10 feet. Spacing initially reduced the number of stems, volume and basal area, and increased the average height and diameter. However, the standing volume and basal area recovered and eventually surpassed the unthinned stand. The length of the recovery period was governed by the intensity of thinning. Stands reduced from 1742 (5x5) to 889 (7x7) stems (not shown in Figure 8) recovered before the site trees reached 80 feet but only gained about 30 cubic feet of standing volume thereafter.

Columns 1 to 3 of Table 3 show how the yield at 160 feet is affected by spacing a stand from 5 to 10 feet. Standing volume, basal area and diameter increase 5 to 10 percent and it is up to the manager to decide if 1140 cubic feet of volume will offset the cost of juvenile spacing. Almost any rate of interest is likely to discourage this practice unless the larger diameters increase the value considerably.

The MAI of stands spaced from 5 to 10 or 14 feet culminates higher but later than the control (Table 2).

Table 3. Spaced stand statistics at 160 feet of site height.

	5x5 (1)	5 -> 10 (2)	difference (2)-(1)=(3)	10x10 (4)	difference (2)-(4)=(5)	difference (4)-(1)=(6)
Number of stems	231	222	-9	207	15	-24
Average height	152	153	1	140	13	-12
Gross volume	26725	25867	-858	23630	2237	-3095
Standing volume	21986	23126	1140	20700	2426	-1286
Basal area	376	412	36	396	16	20
dbh	17.3	18.4	1.1	18.8	-0.4	1.5

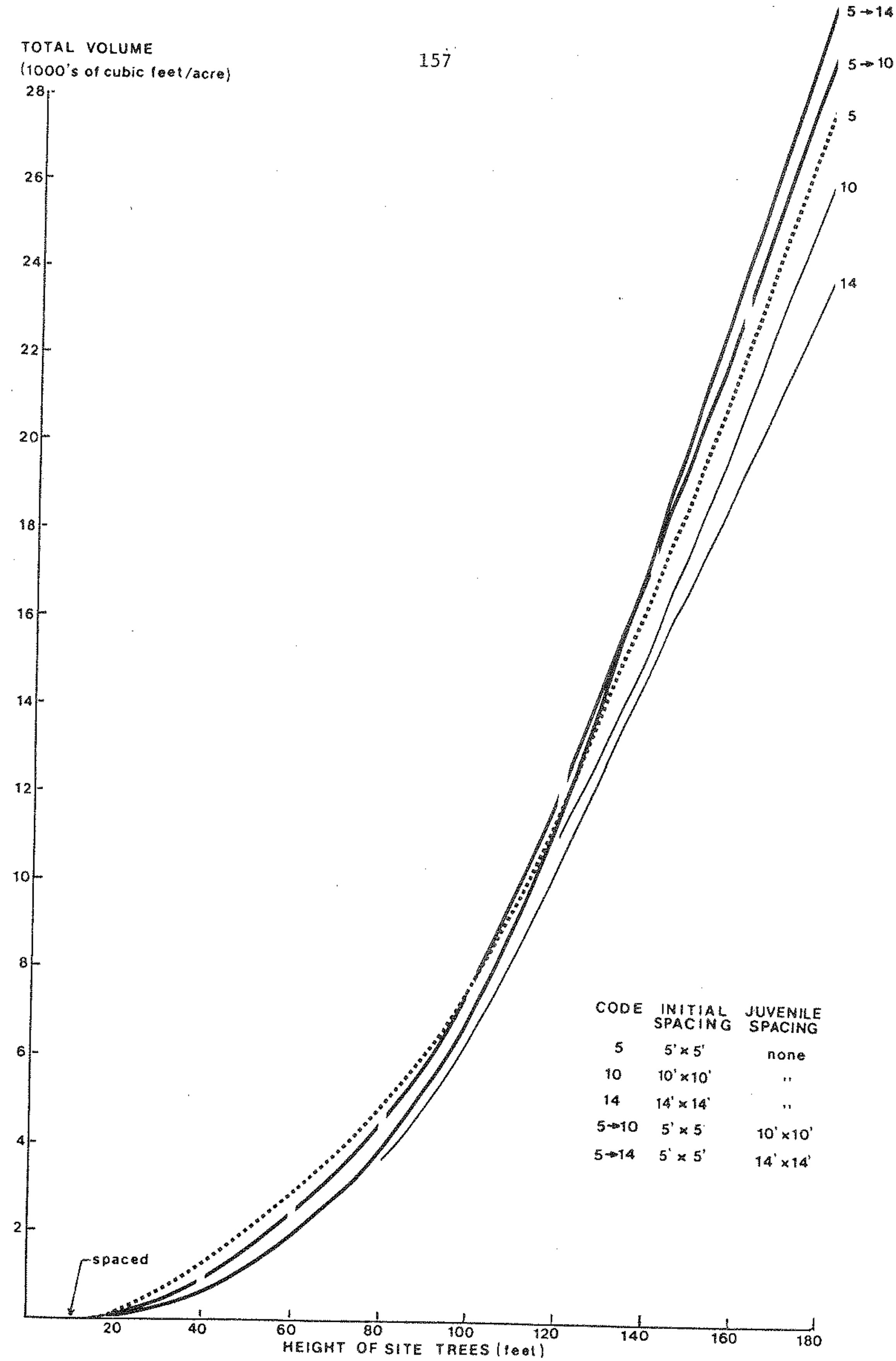

Figure 8. Relationship of juvenile spacing and site height to total volume.

Another management alternative may also be of interest. Should dense plantations be established and later thinned to favor the best trees, or should the plantations be widely spaced initially? A 5x5 plantation spaced to 10x10 is compared with a 10x10 plantation in Figure 8 and Columns 2, 4 and 5 of Table 3. Juvenile spacing increases the standing volume by 2426 cubic feet or 11 percent which again must be related to the extra cost of planting and spacing. The volume is increased by 4,500 cubic feet if 14-foot spacing is desired (Figure 8).

Wide Spacing vs Close Spacing

Widely spaced stands having 436 trees planted 10 feet apart are compared with dense stands spaced 5 feet apart (1742 trees) in Figure 8 to assess the advantages and disadvantages of each regime when trees reach a height of 160 feet. The information summarized in columns 1, 4 and 6 of Table 3 assumes that the manager has the option of planting at either density. Wide spacing reduces the number of stems, average height, gross volume and standing volume while increasing the basal area and diameter. The differences vary between 5 and 12 percent. Close spacing produces an extra 1286 cubic feet of standing volume. Again the manager must decide if this return is sufficient to justify the extra planting costs. Wide spacing would most likely be favored if a realistic rate of interest is applied.

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LONG RANGE FOREST DEVELOPMENT MODELS

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1. Introduction

Forestry is a sector where long term planning models have been applied since very long ago. The main reason for this is the long production period. The development period for a forest stand is mostly between 50 och 150 years, to be compared with one or more crops per year within agriculture. Another important characteristics of forestry is that the forest crop during a considerable part of the growth period either can be harvested partly (through thinnings) or totally (through clear cutting). The forest crop thus both is a product and a production means. In Sweden, like in many other countries, there is a forest law which prohibits the clear cutting of too young stands. The national forest policies mostly aim at securing a sustained or increased annual yield. One dilemma is that the rotation period for maximum volume production is considerably longer than the rotation period for highest profitability when discounting expected costs and revenues at a reasonable rate of return.

The forest development model, which is presented in this paper, is based on data from the Swedish National Forest Surveys. The first applications of the model were in connection with the development of an evaluation system for forest and forest land which was used at the countrywide 1975 taxation of real estates. The present version of the model was used by the 1973 Forestry Commission in order to analyse and describe the present wood balance situation in Sweden. In its first report the Commission stated that the wood consumption in Sweden in 1973 and 1974 corresponded against a gross cut of 84 million m^3 (trunk volume with bark). According to the model calculations and assuming unchanged silvicultural intensity the long term production capacity of the forests was no more than 66-70 million m^3. The continued work of the Commission will concentrate on measures for increased production such as intensified silviculture, fertilization and ditching. Our present efforts to improve the calculation model must therefore take into account that the model should allow the identification and evaluation of a range of future possible management regimes, which means that improved knowledge of the effects of silvicultural measures is essential.

This paper will present the existing version of the model and to some extent outline features which are being introduced during the summer 1976.

2. Objectives of the model

The model aims at describing the probable development of the Swedish forest resources under specified assumptions with regard to future management regimes and levels of cutting.

The objective would not be to identify the optimal management regime, but rather to identify possible and probable management alternatives taking into account the existing ownership structure and ranges of state forestry policies.

The forestry system to be studied (figure 1) contains the following main elements or problem areas.

The Swedish forest resources comprising approximately 24 million hectares of forest land which are described by data from the national forest survey.
Growth processes which are studied through yield research and by increment measurements which are part of the forest survey.
Management regimes which must be identified and discussed together with representatives of forest owners and managers but which partly can be identified by observations made by the field survey teams.
Timber harvesting policies, which can be studied by stump inventories and consumption statistics as far as historical trends are concerned. When it comes to the future the level and intensity of cutting is one of the open variables which can be varied from one alternative to the other in order to indicate possible ranges of future yield.

The calculation model should merge these elements into a dynamic system and allow that the system can be steered and controlled by a number of steering parameters.

One very important qualification is that the model must be simple enough from pedagogical point of view, to allow detailed discussions with forest managers and forest policy-makers on the choice of development alternatives and steering functions and parameters. From pedagogical point of view it is also necessary to break down the calculations into phases or sequences which can be presented and discussed separately. Figure 2 contains a comprehensive description of the model structure.

The development of the model has taken place "in the open" and has been influenced by a number of reference groups with high subject matter competence.

3. The Swedish National Forest Survey

Purpose

The Swedish National Forest Survey continuously provides data for the planning and control of the utilization of the forest resources on a regional and national level. The survey provides data on the current state of the forest and on measures which have been taken. The survey estimates the size and composition, the location and technical properties of the forest resources and the increment. Regeneration measures and the extent of the annual fellings are also observed.

Background

The National Forest Survey has been working since 1923. Four complete surveys have been implemented since then. The first was during 1923-1929, the second 1938-1952, the third 1953-1962 and the fourth 1963-1972.

Originally the survey was carried out county by county in the form of strip surveys in which both area and volume assessment were performed within a 10 m wide survey belt.

During the second National Forest Survey the assessment of the volume was based on circular sample plots (of approximately 140 m^2). The volume assessment and most of the area descriptions have been based on sample plots of this type during the third and fourth surveys. A reorganization was carried out in 1953 which resulted in an annual, low-percentage survey of the entire country.

The reorganization also made it possible to record the annual fellings by means of a stump inventory which refers to the last concluded logging season. In addition, the reorganization has made it possible to estimate annual climatic variations in increment. In 1973 the survey was re-arranged and the field work was increased considerably.

Methods

The National Forest Survey is an annual systematic cluster sampling of the forest resources of the country. This inventory is carried out by the Department of Forest Survey at the Royal College of Forestry in Stockholm.

The present method of inventory means that the entire country is covered each year by a grid net of circular sample plots. These sample plots are located along the sides of squares which are called "tracts". One tract of this type (a cluster) is a work unit intended to constitute a day´s work for the survey crew. The sampling density varies from one part of the country to another. The density is greatest in the south and gradually becomes lower towards the north.

Each year some 1500 tracts are inventoried. This work is carried out by some 20 survey crews with 5-7 members in each crew.

The accuracy of the survey is so adapted that the results - primarily the volume - can be presented county by county with a satisfactory degree of reliability after a five-year interval.

Results are presented in various ways. Some of the main results are presented annually in the Swedish Statistical Yearbook of Forestry. Other more detailed accounts are presented in the form of reports from special studies of the kind which is described in this paper.

The sample plot and sample tree data produced by the National Forest Survey is probably unique as far as biologic inventory data is concerned. It covers practically the entire country below the alpine tree limit, it has been collected by objective sampling methods and the methods used for the recording and documentation of data are uniform throughout the country. The entire material is available on magnetic tapes at one place, i.e. the Royal College of Forestry. The inventory has been carried out continuously since 1953 using much the same methods.

Main elements of the survey

Area inventory. A careful description of the various site and stand properties is made for all sample plots on forest land. This data then provides a basis for various area calculations and for the estimation of total volume, growth and fellings.

Volume assessment. An assessment of the growing stock is carried out on those of the sample plots which are selected as volume plots. All trees with a diameter at breast height of at least 10 cm within a circle with a radius of 10 m are recorded. Trees with smaller diameters than 10 cm are recorded only on parts of the large plot. This data is used to determine the volume per hectare. Combined with data from the area inventory, the total volume can be determined. A certain quota of the trees on the sample plots are selected as sample trees. On those trees factors determining the volume and increment are measured and certain technical properties are described. A bore core taken from each sample tree is processed in the laboratory and provides data on the diameter increment.

Stump inventory. The stump inventory is intended to provide an assessment, by recording the stumps on special "stump plots", of the quantities felled each year and the areas which are affected by various types of cutting. The data recorded for those sample plots includes data on the ownership category and the type of cutting method employed.

4. Treatment classes

The state variables describing the forest resources must refer to some classification system. One possibility would be to keep the sample plots as units, but we have in our model chosen to aggregate these plots into treatment classes.

The aggregation procedure must take the following demands into consideration:

1/ The treatment classes must be as homogeneous as possible with regard to biological growth.

2/ The classes must be relevant for the decision rules used by forestry management.

3/ The total system of treatment classes must be pedagogical and facilitate the discussion with decision makers and representatives from practical forestry.

4/ The number of treatment classes must be reasonable taking the data processing costs into account.

The first point mentioned above tends to increase the number of classes and the last two points to reduce it. We have tried to arrive at a reasonable compromise by aggregating the sample plots into about 100 treatment classes in each of about 10 geographical areas. In addition a breakdown into 2 or 3 ownership classes is being applied.

The treatment classes are defined by means of the following state variables:

- stand age/cutting class
- site index/vegetation type
- density
- cutting priorities based on field observations
- in some areas a further breakdown on forest types and accessibility is applied

The treatment class is regarded as a "stand" in the calculations. The class is described by a number of "state variables", which are sums or averages of the corresponding sample plot values.

The calipered trees are aggregated according to tree species (four classes) and diameters (ten 5-cm classes). Within a subclass the following variables are recorded:

- o number of stems
- o average age (at breast height)
- o average diameter and volume
- o average diameter and volume increment during the last five years.

Age, volume and increments are originally known only for the sample trees. Through an initial "pairing" operation each calipered tree is matched to a sample tree and given the values of that sample tree.

In addition to these state variables there are also variables characterizing the whole stand, such as average site index, density, altitude, vegetation type etc.

From the state variables described above several other variables can be calculated, such as volume per hectare, average diameter, basal area, increment percentage etc.

5. Growth models

Established stands

The dynamic part of the model should project the state of the forest resources period by period.

The most important of the state variables is the volume (cubic metre trunk volume over bark) of each treatment class. Within the treatment class the volume is recorded separately for four different tree species (species groups) and ten diameter classes. Each such subclass is projected individually.

The information about the initial state comes from the survey sample trees and includes data about the present volume and the volume increments during the last two five year periods.

This type of data can be used for projections in several ways. We have adopted as a theoretical framework the growth model proposed by Chapman-Richards, according to which the development of the size W of an organism follows the differential equation:

$$dW/dt = nW^m - K \cdot W \qquad (1)$$

where n, m and K are parameters depending on genetical and environmental factors.

Derivating the basic function above gives, after some simplifications:

$$d^2W/dt^2 = m \cdot (dW/dt)^2/W + K \cdot (m - 1) \cdot dW/dt \qquad (2)$$

Equation (2) expresses the acceleration of growth as a function of size and its rate of change. The equation is remarkably simple. The unknown exponent m in the basic function has disappeared as exponent, which means that the remaining two coefficients (m and K · (m - 1) can be estimated with ordinary least square techniques.

In practice the development is not followed continuously but in periods. The derivatives above are thus replaced by the corresponding differences:

W_n = size at the end of period n

$\Delta W_n = W_n - W_{n-1}$ = increment during period n

$\Delta^2 W_n = \Delta W_n - \Delta W_{n-1}$ = difference in increments between period n and n-1

Equation (2) is accordingly replaced by

$$\Delta^2 W = a \cdot (\Delta W)^2/W + b \cdot \Delta W \qquad (2')$$

As mentioned above the coefficients are dependent on environmental factors. There are no corresponding theory to guide the selection of factors and the functional form of this dependance. By trial-and-error, using residual analysis, we have found the site index, the basal area and a simplified vegetation classification to be appropriate as modifying variables in equation (2').

Summarizing, our projection technique involves the following two steps:

A. Initial calculation using least square technique of "increment difference function" based on the model equation (2') and data from the survey sample trees.

   Different functions are calculated for different tree species and geographical areas.

   The coefficients in (2') are functions of site index, basal area and vegetation type.

B. A one-period-ahead forecast of volume development is calculated according to the formula:

   $$W_{n+1} = W_n + \Delta W_n + \Delta^2 W_{n+1}$$

   where W_n and ΔW_n are known, either initially based on sample tree data, or, subsequently, as a result of the corresponding calculations for the last period, and $\Delta^2 W_{n+1}$ is calculated using the least square equation from step A.

Young stands

The method discussed above is not applicable to young stands. One reason for this is that the calculations presumes knowledge of the increment during the last five years. Thus, all trees must be at least five years old at breast height (where the increment core is taken).

The growth model, however, is unreliable even for some time after this age, due to the difficulties to incorporate the effects of cleaning and natural losses during the early years of a stand.

Instead we define initially the state of some treatment classes on different sites at the age of about 20-60 years (just before the first thinning). At an appropriate time interval after clearcutting we assume that the clear cut areas have developed stands equal to our defined treatment classes, which are called "planting classes".

The planting classes can be taken from among the initial treatment classes based on the survey data. In that case the classes will reflect previous intensity of forestry management. Alternatively the classes may be defined more or less synthetically with the aim of reflecting anticipated future forestry intensity, for instance that clear cut areas are planted with pine or spruce of a defined quality.

Thinning and fertilization effects

The growth model incorporates methods for handling thinning and fertilization effects based on recent studies of data from permanent sample plots.

6. Management regimes

The system of forest management in Sweden is very homogeneous which facilitates our modeling work. One reason for this is that there are not more than 3-4 tree species which are of economical relevance. The dominant objective of forestry is the combined production of raw material for the pulp and board industry and the sawmilling industry. We therefore apply a management system which aims at a high volume production, the most valuable end-product being timber for the sawmilling industry. For ecological reasons the regeneration of forest stands is by clear cutting and planting or by means of natural regeneration by seed trees, which are being left to grow 10-15 years after the final cutting. Out of the total yield from a forest generation about 30-50 per cent is taken out as thinnings during the growth period. The growth period varies with site and location from about 50 years in south Sweden to 150 years in north Sweden. As was mentioned in the introduction it is imperative that such a long production period would necessitate long range management models.

Figure 3 will illustrate the principal management cycle. The classification into treatment classes apparently also is a classification into development classes or development phases.

In the field survey a classification into both "cutting classes" and age classes is made. In the present model work we apply a combination of cutting classes and age classes for our basic classification. Plots which in the field are classified as mature for clear cutting are aggregated to a member of "clear cut" classes and the rest of the plots are aggregated into age classes.

The clear cutting policy in the model can be steered by priority functions or by simple algoritms which aim at simulating the clear cutting policy (or lack of policy) applied by more than 100,000 private forest owners.

After clear cutting there is a period of stand establishment for which it is not possible to apply growth functions. The waiting time for the birth of a new stand and the early development of the new stand is mainly due to human influences and is in the model represented by the choice of "planting class" at a somewhat later stage.

The next stage of development is the thinning phase. The thinning policy is steered by functions or algoritms similar to those which are used to steer the clear cutting. A complicating factor is that thinning stimulates the growth of the remaining trees, which makes it necessary to apply a "thinning effect model" which influences the basic growth functions. Similar effects are caused by fertilization and ditching, measures which are now being included amongst activities within the model.

The following example indicate in which way the model can simulate alternative management regimes:

(a) The total volume cut per year is determined. The initial values are based on present cutting statistics. The values referring to later time periods are preliminary estimates, which may be changed in later iterations, due to their effects on the standing volume.

(b) The volume cut per year is split between clear cutting and thinning, according to present praxis and estimates regarding future thinning policies.

(c) The treatment classes are selected for cutting according to defined priority rules (until the quota is filled). We have applied two types of rules to mirror the differences of optimality among the many forest owners. As a first step we use a broadly formulated rule such as "clear cut x per cent of all forest in site class y which is over z years of age". In the second step we use a more specific and "optimal" rule based on state variables such as age, expected increment (with regard to site class), actual increment, diameter etc.

7. Output

The basic output from the system is a "balance statement" for each time period showing:

Initial volume
+ increment
- natural losses
- thinning
- clear cutting
= final volume

Fig 4 is a typical example showing the development over ten 10-year periods of the three most important components in the balance statement, i.e.

F = initial volume
T = increment
A = cutting (thinning and clear cutting)

This particular diagram shows that the total volume (F) will decrease rapidly, if the cutting (A) is kept on the present level (84 mill m^3) during the first 10-year period. The diagram further shows that the cutting must be reduced to 66 mill m^3 during the remaining periods, in order to restore the total volume to its present level of 2 200 mill m^3.

8. Further development

A cross-disciplinary project has recently been started within the Swedish College of Forestry in order to bring about a further development of long range models of the type now presented. This project will run for 5 years and engage 50 man-years or more. The project will be working within four problem areas: growth processes, establishment of young stands, management regimes, and model construction. The last mentioned problem area is a minor part of the project since we believe that the limiting factors rather relate to subject matter knowledge than to the mathematical-statistical frame-work.

The constructors of the model described in this paper have also made a very limited effort to build up a computer model of land-use development in a tropical country. This model aimed at demonstrating competing claims between agriculture and forestry and the need for a long range land use programme in order to retain the productivity of land and to secure the supply of food and wood.

The type of models discussed in this paper deals with future land use strategy or land use policy. One can foresee a rapid development of model work on the enterprise level, both of strategical-policy kind and short-term or middle-term models of tactical-operative kind. Models of the latter kind exist, but they are mostly not sufficiently realistic with respect to biological foundation.

Nils-Erik Nilsson

Karl-Georg Bergstrand

Figure 1. The forestry system

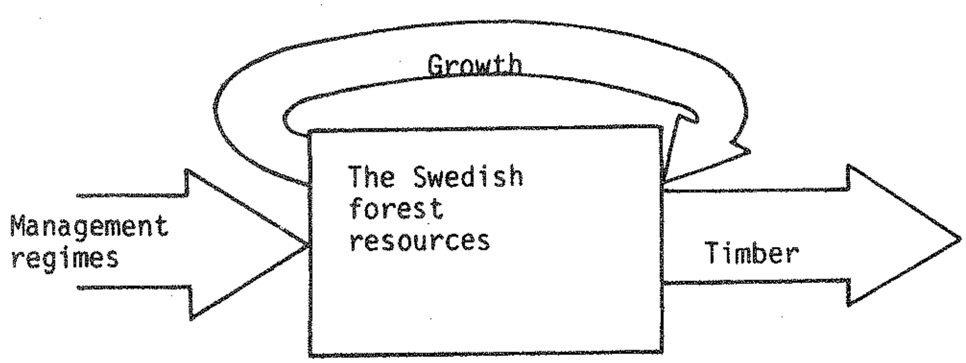


Figure 2 Model structure

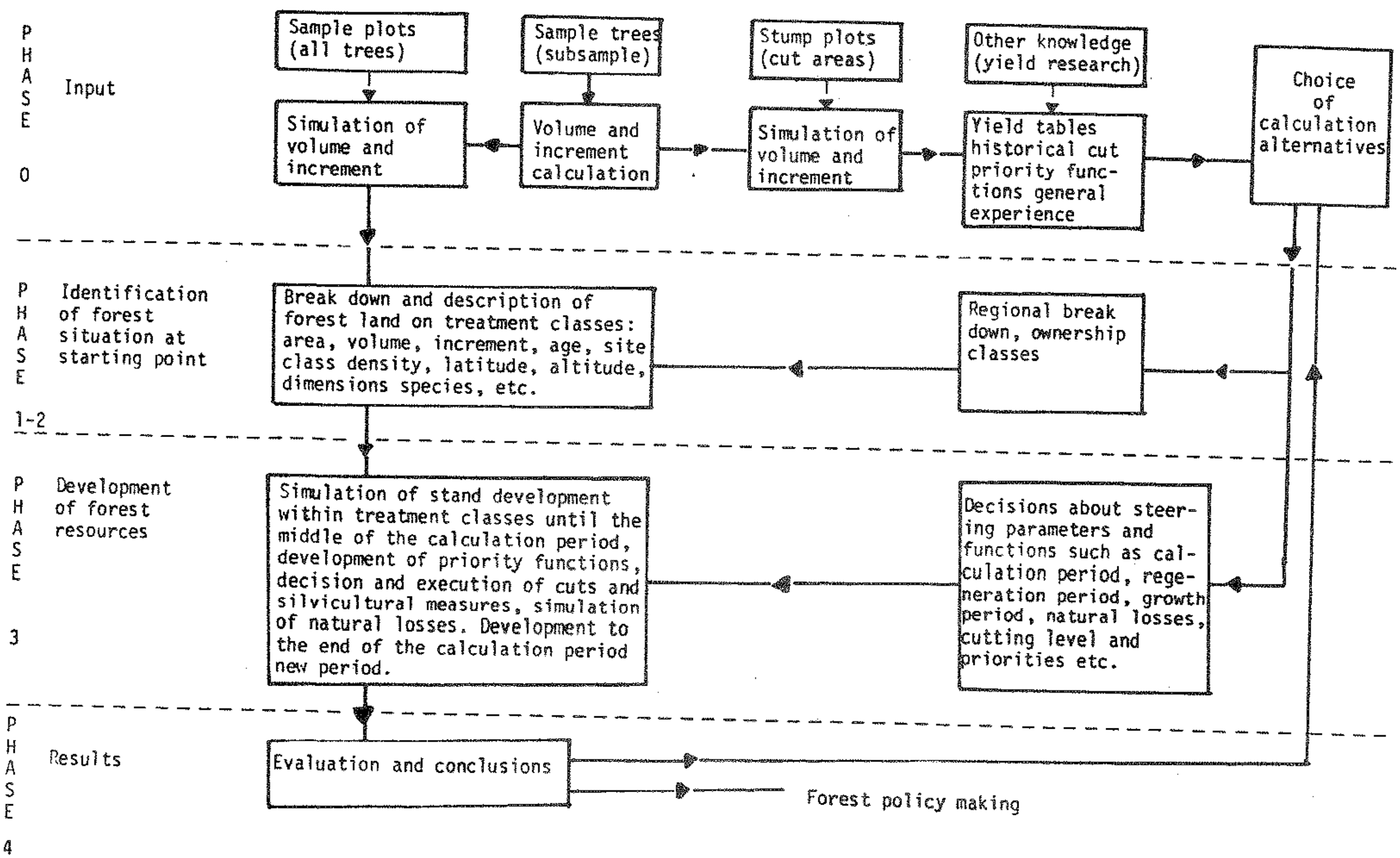


Figure 3. Management cycle

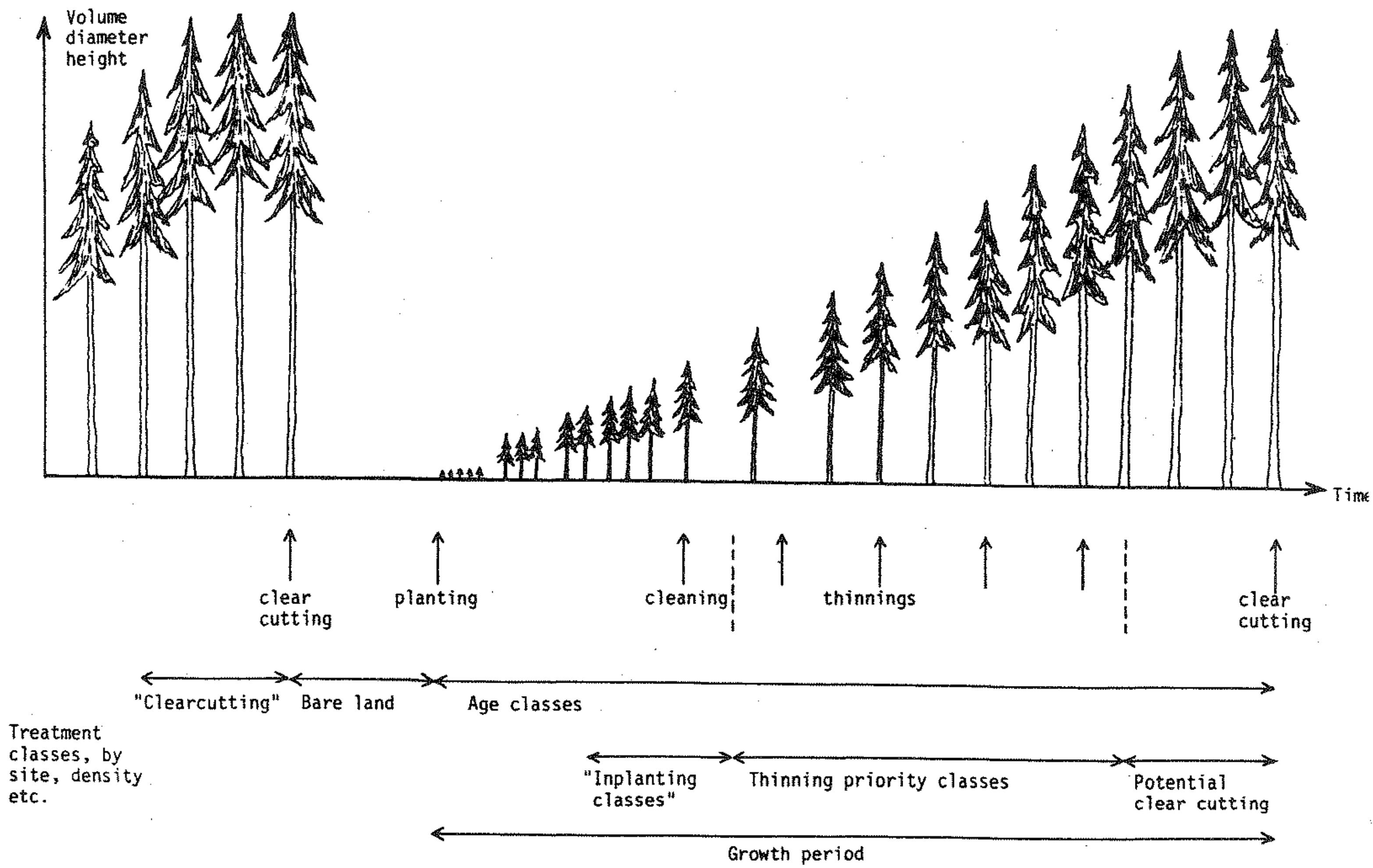
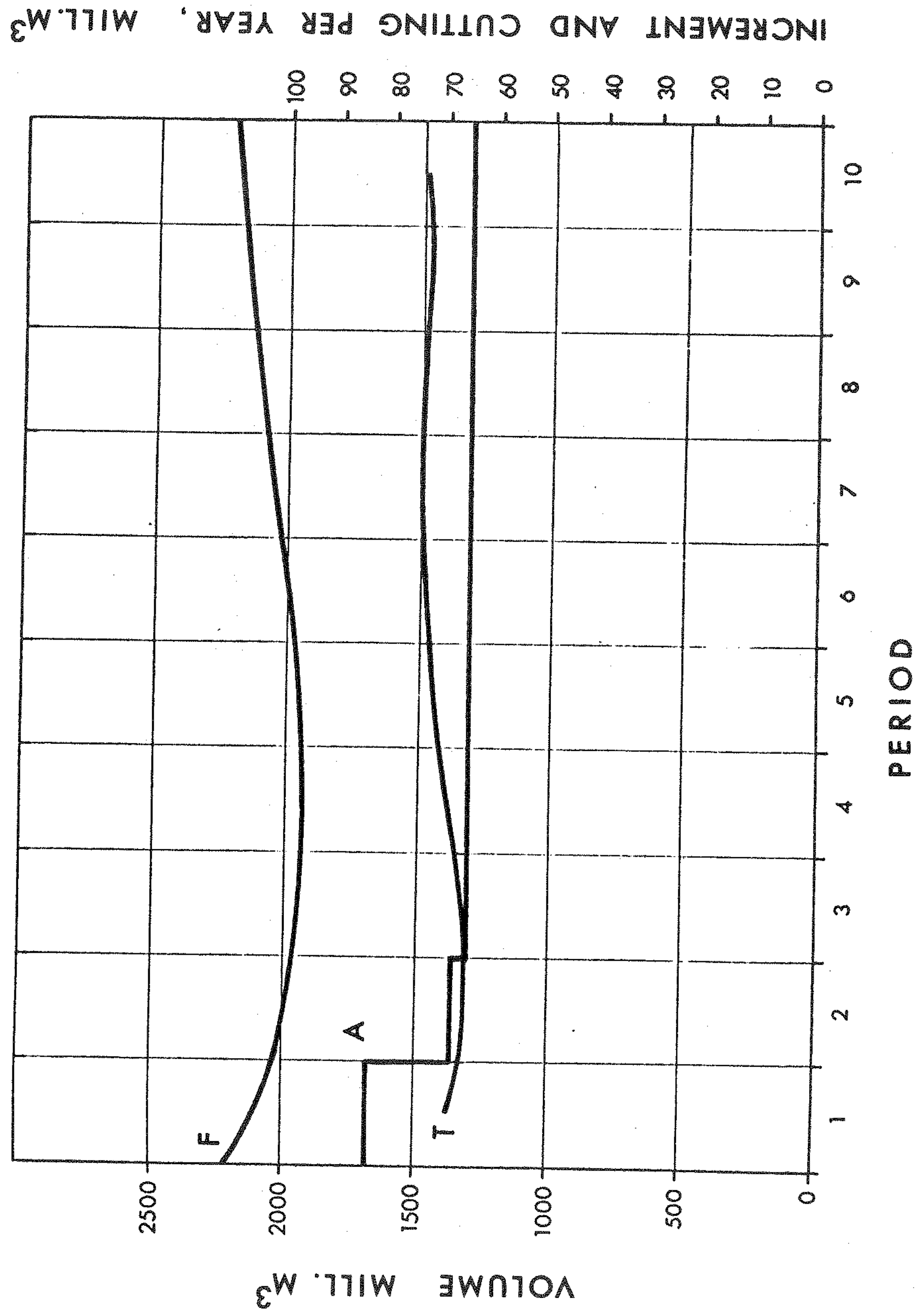


FIG 4


INCREMENT AND CUTTING PER YEAR, MILL.M3
100
90
80
70
60
50
40
30
20
10
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VOLUME MILL. M3
2500
2000
1500
1000
500
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PERIOD
1
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ESTIMATING INDIVIDUAL TREE GROWTH WITH TREE POLYGONS

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INTRODUCTION

Individual tree growth models have received considerable attention in recent years. Such models can be used for individual tree growth and stand simulations for growth studies and fertilization experiments.

In the past different approaches have been taken to evaluate the effects of competition on individual tree growth. In many cases average density, expressed as number of trees or basal area per area unit, proved to be unsatisfactory as it represented stand averages which often were not well correlated with individual tree characteristics. Spurr (1962) described a method to determine stand density at a particular point in terms of basal area per area unit but did not specifically develop a inter-tree competition index. Bella (1971) developed a competition index, the Competitive Influence-Zone Overlap (CIO) which takes into consideration the influence zone of each tree and the amount and nature of interaction.

Brown (1965) suggested a different approach, the area potentially available or APA. Other modifications of this method were described by Stöhr (1963) and Moore et al. (1971).

This paper reports on the application of the APA or tree polygon method and several alternatives to the estimation of tree growth in a stand of tuliptree (*Liriodendron tulipifera*).

TREE POLYGON METHODS

The tree polygon method developed by Brown (1965) is based on the assumption that each tree in a forest stand has potentially available to it half the distance to a neighboring tree. The stand area of the tree then is determined by erecting perpendicular bisectors to the lines from the center tree to its neighbors. The intersections of these perpendicular bisectors represent the corner points of the tree polygon or APA (area potentially available) (Figure 1).

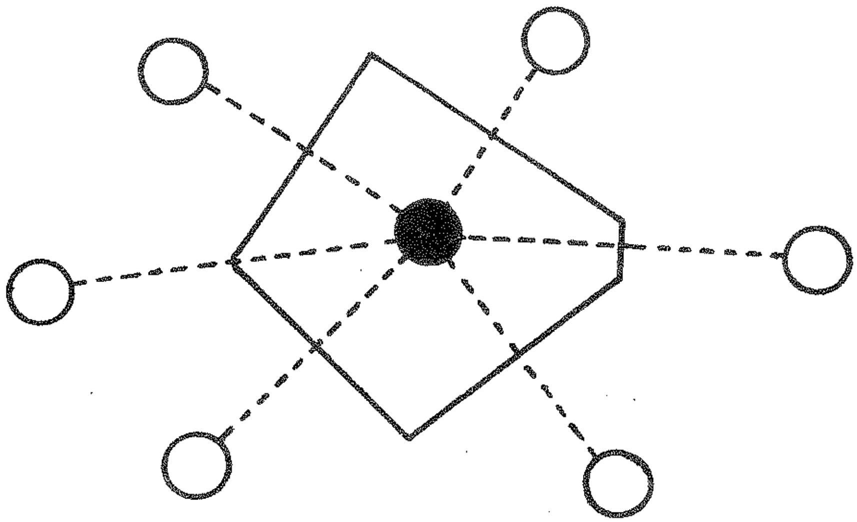

Figure 1. Tree Polygon

Trees that are too distant from the center tree will be excluded from the APA automatically, effective neighbor trees do not have to be identified explicitely. The tree polygons are mutually exclusive, i.e. there is no overlap between two adjacent polygons.

The distance allocated to the center tree is calculated as

$$a_i = \tfrac{1}{2}A_i$$

where

A_i = distance from center tree to ith neighbor

a_i = distance from center tree to perpendicular line

In Brown's definition of a tree polygon the relative size of the trees was not considered. An alternative definition was suggested by Stöhr (1963). In this method the lines connecting the center tree with its neighbors are divided proportional to the diameters of the trees so that larger trees are allocated more growing space than smaller trees. The portion allocated to the center tree (tree 0) is calculated as

$$a_i = \frac{d_o}{d_o + d_i} A_i$$

where

d_o = diameter of center tree

d_i = diameter of ith neighbor

A_i = distance from center tree to the ith neighbor

a_i = distance from center tree to perpendicular line

Moore et al (1971) define tree polygons derived by division proportional to basal area or d^2.

$$a_i = \frac{d_o^2}{d_o^2 + d_i^2} A_i$$

Both of these methods take into account the spatial distribution of trees and their relative sizes in terms of diameter or basal area, but they do not consider directly the relative competition of trees in space.

Tree competition is to a large extent influenced by the relative height of trees. To test the effect of tree height in defining the tree polygon area three models were developed. The first model divided the connecting lines from center tree to neighbors proportional to tree height.

$$a_i = \frac{h_o}{h_o + h_i} A_i$$

The second model divided the distance proportional to $d \cdot h$.

$$a_i = \frac{d_o \cdot h_o}{d_o \cdot h_o + d_i \cdot h_i} A_i$$

The third model divided the distance proportional to $d^2 \cdot h$.

$$a_i = \frac{d_o^2 \cdot h_o}{d_o^2 \cdot h_o + d_i^2 \cdot h_i} A_i$$

The concept of tree polygons can be extended into the third dimension by defining a tree growing space represented by a geometric solid. Ideally, crown dimensions and relative status would be included in such a model. Jack (1968) used such a concept to devise a competition factor that took into consideration tree polygon area and the relative heights of the neighboring trees.

Measurement of crown dimensions is very time consuming and expensive, and measurement errors are normally high. Therefore, in this study the three dimensional growing space of a tree was approximated by a geometric solid which was derived by multiplying the tree polygon area by the height of the center tree. The tree polygon may be constructed by any of the six methods discussed previously. These models will approximate the actual growing space to different degrees.

Tree characteristics can be expected to be correlated with the growing space, as a tree will grow more rapidly and attain larger dimensions faster if a larger growing space is available, everything else held constant. To test the correlation between alternative definitions of growing space and tree growth, correlation coefficients have been derived.

PROCEDURES

A 23 year old tulip tree (Liriodendron tulipifera) stand (Illini Plantations, Urbana, Illinois) was measured in 1974 and again in 1977, and the X,Y coordinates of all trees and their diameters were recorded. In addition the height of every tree in the stand was measured in 1977. The total stand consisted of 340 trees of which 231 trees were selected as sample trees. A buffer strip around the plot was not included in the analysis to avoid any potential bias in defining the tree polygons of border trees. Stand characteristics are displayed in Table 1.

A computer program was designed to calculate two dimensional growing areas and three dimensional growing space for all six methods.

Table 1 Stand characteristics of Illini Plantation tulip tree stand

	Year	
	1974	1977
Average diameter (cm)	14.72	16.97
Total basal area (m^2)	4.36	5.84
Number of sample trees	231	231

For each tree the X and Y coordinates, the diameter, and height have to be specified. For a center tree the computer program determines the polygon area by evaluating the size and location of the 5 closest neighboring trees in each quadrant, i.e. a total of 20 trees are included in the calculations. Tree polygon areas and three dimensional growing space were calculated for all six methods described. Correlation analyses were performed to test how well tree basal area growth is related with two dimensional growing area and three dimensional growing space.

RESULTS

To examine the relationship between growing space and tree growth, correlation coefficients were derived for tree basal area growth and growing area (two dimensional tree polygon) and for basal area growth and three dimensional growing space.

Table 2 Correlation Coefficients

Method	Tree polygon with distances to neighbors divided	Correlation coefficient between tree basal area growth and	
		two dimensional growing area	three dimensional growing space
1	equal	0.241	0.515
2	proportional to diameter	0.744	0.766
3	proportional to height	0.549	0.635
4	proportional to $d \cdot h$	0.776	0.775
5	proportional to d^2	0.822	0.816
6	proportional to $d^2 \cdot h$	0.825	0.811

The correlation between basal area growth and the tree polygon constructed by equal division of the lines connecting the center trees and its neighbors, is lowest with $r = 0.241$. For tree polygons constructed by a method that divides the distances proportional to diameter, height, basal area, or volume the correlation coefficients increase considerably (Table 2). The highest correlation coefficient was found for method 6 that divides the distances proportional to d^2h. It could have been expected that the correlation coefficient was very high for tree polygons constructed proportionally to d, d^2, or d^2h due to the high correlation of basal area growth and tree dimensions. For example, the correlation coefficient for basal area growth and diameter was 0.873 and for basal area growth and height 0.635.

If now tree polygons are constructed that are highly correlated with diameter, height, or both then we could expect basal area growth to be correlated with tree polygon area regardless of the original relationships involved. The stronger the correlation between tree polygon area and the diameter or basal area, the stronger will be the correlation between tree polygon area and basal area growth. For the tuliptree stand the highest correlation coefficients for tree polygon area and diameter were found with methods 5 and 6; the correlation coefficients for basal area growth and tree polygon were highest with these two methods also.

The correlation between tree basal area growth and three dimensional growing space were found to be considerably higher than the correlations with two dimensional growing area for methods 1, 2, and 3, and somewhat smaller for methods 5 and 6.

To test whether tree growing area or tree growing space can explain a significant amount of variation of tree basal area growth, regression analyses were conducted. Alternative regression models were defined to predict tree basal area growth from a set of independent variables for each of the methods discussed.

The models that explained the largest amount of variation for all methods tested were of the form:

(1) $Y = b_o + b_1D + b_2H + b_3GA$

and (2) $Y = b_o + b_1D + b_2H + b_3GS$

where Y = tree basal area growth
D = diameter at breast height
H = total tree height
GH = growing area
GS = growing space

The R^2 ranged from 0.7835 to 0.7948, the standard error of prediction was equal for all twelve equations with 0.0023.

Table 3 Multiple regression coefficients

Tree polygon method	Regression Model	
	$Y = f(D,H,GA)$	$Y = f(D,H,GS)$
1	0.7835	0.7844
2	0.7850	0.7873
3	0.7851	0.7865
4	0.7880	0.7899
5	0.7910	0.7927
6	0.7944	0.7948

The variables growing area or growing space were for all six methods significant, i.e. these variables explained a significant amount of variation in tree basal area growth. For all six methods, the models with three dimensional growing space as independent variable explained a higher percentage of variation than the models with growing area as independent variable.

SUMMARY AND CONCLUSIONS

Individual tree basal area growth is well correlated with two dimensional growing area and three dimensional growing space. Correlation coefficients are lowest for tree polygons based on inter-tree distances only. They are somewhat higher for tree polygons constructed proportional to height and become highest for polygons constructed proportional to diameters or basal area. The area of tree polygons constructed proportional to d^2 or d^2h has the highest correlation with tree basal area growth.

Tree growing area and growing space did prove to be significant in predicting basal area growth from a set of independent variables. The regression equations predicting basal area growth from diameter, height and growing space explained in all cases a higher percentage of variation than the equations containing two dimensional growing area. The models containing the diameter squared terms (i.e., models 5 and 6) explained more variation than other models and the three dimensional models containing d^2 terms explained the largest percentages with 79.27 percent and 79.48 percent respectively.

Further experiments should be conducted to study the interrelationships of tree basal area growth and tree polygon area for other forest conditions as these individual tree growth models can provide useful information for scientists and managers concerned with tree growth.

ZUSAMMENFASSUNG

Die Korrelation von Standfläche und Zuwachs und Standraum und Zuwachs von Einzelbäumen wurde anhand von 231 Individuen eines Liriodendron tulipifera Bestandes getestet. Standflächen und Standräume die mit Berücksichtigung der Stammdurchmesser oder Stammgrundflächen konstruiert wurden zeigten höhere Korrelationswerte als solche die proportional zu Höhen oder durch Halbierung der Entfernungen vom Zentralstamm zu den Nachbarn konstruiert wurden. Regressionsanalysen mit Stammgrundflächenzuwachs als abhängigen Variablen zeigten, dass sowohl Standfläche wie auch Standraum einen statistisch signifikanten Anteil der Gesamtvarianz erklären. Regressionsgleichungen, die Standflächen oder Standräume enthielten die proportional zu d^2 oder d^2h konstruiert wurden hatten höhere R^2-Werte als andere Gleichungen. Die Variable Standraum führte in den sechs verschiedenen Modellen zu höheren R^2 Werten als die Variable Standfläche.

Key Words: Tree growth, Growing space, Growing area, APA

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GROUPING FOREST GROWTH SIMULATION PROCEDURES INTO A GENERAL USER-ORIENTED SYSTEM

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Abstract--A generalized simulation procedure can project timber growth for 12 United States species or species groups according to 20 published sets of relationships, as well as employ user-provided relationships. The procedure is now part of MULTIPLOY, a computerized timber management evaluation system widely used in the United States Forest Service. MULTIPLOY's enhanced simulation capability permits forest management analysts to use just one computer system to: (1) simulate growth of a variety of species, and to modify growth for local circumstances, (2) combine forest establishment and growth with financial evaluation, and (3) directly compare one growth procedure or species with another.

To meet pressing needs for a user-oriented computer system that could evaluate timber growing opportunities, the Forest Service developed MULTIPLOY, a special simulation system for developing and evaluating alternatives for managing individual timber tracts (Row 1974). MULTIPLOY facilitates both technical and economic analysis of investments making the best use of information available. MULTIPLOY also is extremely user-oriented, without requirements for detailed formats of input, sequence of input records, and special codes or classifications.

Timber simulation can use yields entered as tables, or supplied by built-in or user-developed mathematical procedures. Routines the user can call will estimate major environmental impacts; simulate fires and pests; project prices, costs, and other financial factors; evaluate economic returns in several ways; and provide information for further planning and resource allocation.

The MULTIPLOY system configuration

Figure 1 shows the general scheme of the computer system. In processing a problem, the first phase interprets the natural language memo, codes the information, and edits the codes for completeness and consistency. The second phase processes each trial within the problem. Types of run can either (1) make sensitivity tests on a wide range of decision parameters such as rotation age or stocking density, (2) make stochastic trials, with growth rates, prices, costs, or other factors varying according to selected probability distributions, (3) perform case trials, with each trial a different project, described by information read from a record or (4) simply evaluate one trial. A trial can start with an existing stand, or bare land.

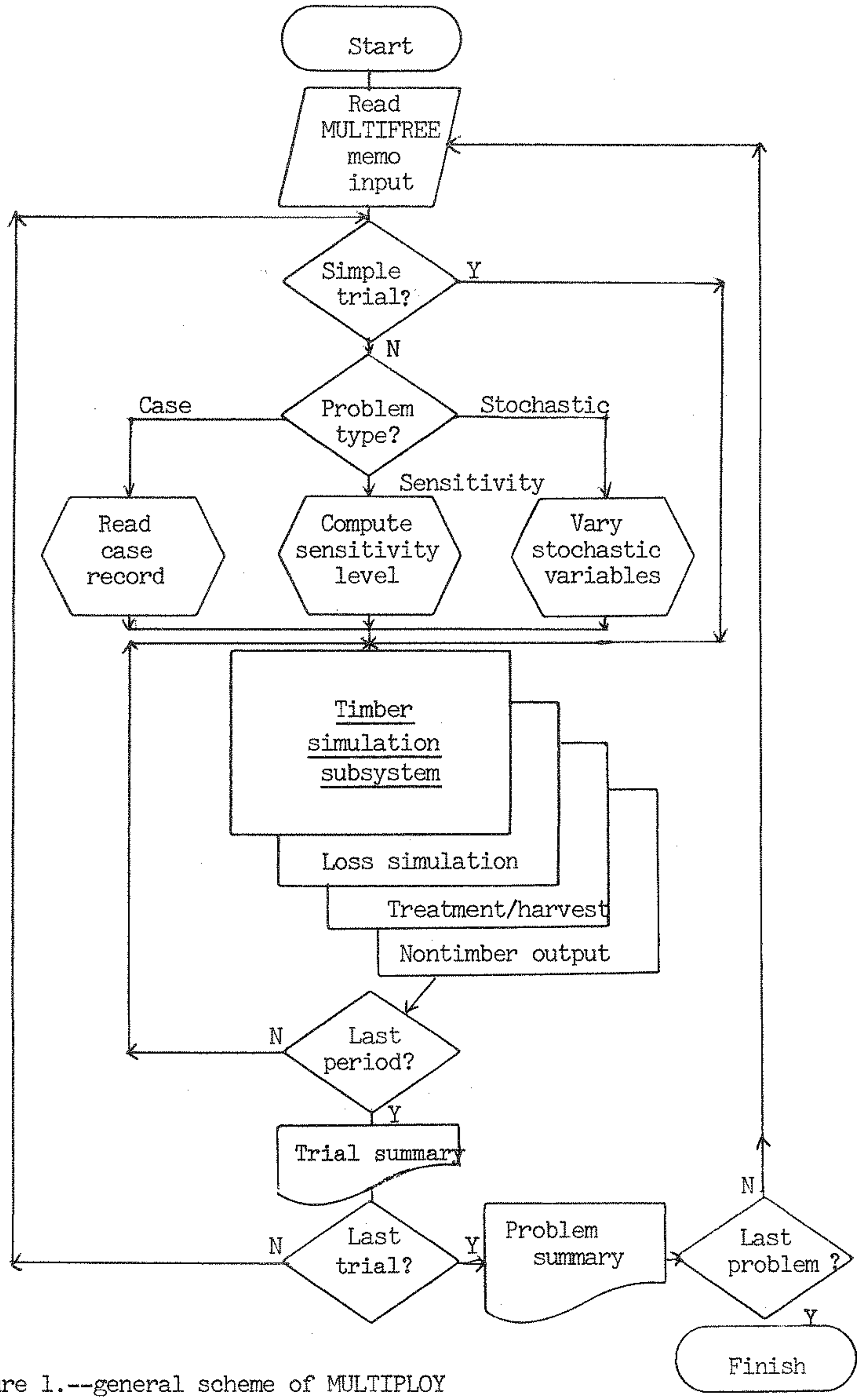

Figure 1.--general scheme of MULTIPLOY

Within the trial processing loop, a second loop governs the advance of time period by period. Period length may vary from one to many years, but must stay the same throughout a problem. A regeneration lag can be inserted between rotations. Within each time period four simulation processes occur--timber growth (or establishment); losses from fire, storm, or pests; treatment or harvest; and nontimber outputs. The management regime can progress with an initial regime for the first rotation (or remainder of present rotation for an existing stand), then change to a subsequent regime for the second rotation. The simulation sequence of periods can cease after one or two rotations, or continue for 50 periods.

The output of MULTIPLOY includes a timber stand and output table, a trial summary with financial results, and a summary of all trials, in which the user can select the information tabulated. The system can process several entire problems consecutively.

Yield tables vs. simulators

Though analysts and planners within the USDA Forest Service have used MULTIPLOY extensively since 1975, almost all users have employed yield tables entered via the input memo. Simulation routines have seen little use, and then only on a research basis.

Yield tables are useful in only limited situations. They are applicable to stands regenerated by techniques that control number of trees established within a narrow range, such as planting. They are usable, though less satisfactory, when uniform timber is thinned to a prescribed density and the size of the trees left varies little. But for other treatments, yield tables, whether "normal" or "managed," are awkward and inflexible.

When projects treat stands in varying conditions, "custom" simulation of yields in each project is more sensitive to conditions of existing stands. Determining the sensitivity of financial returns to varying intensity of treatments, such as thinning to various basal areas, is accomplished more readily by growth simulation.

MULTIPLOY is capable of incorporating adaptations of timber stand generators or simulators developed by biometric research. In the summer of 1977, the timber simulation system was redesigned and fitted with specifications for 20 simulation procedures to enhance its usefullness as a mensurational tool. This paper specifically discusses the details of the timber simulation procedures, including both programmed relationships and provisions for modification by users.

Types of timber stand simulation compatible with MULTIPLOY

In recent decades forest biometricians have developed timber stand simulation procedures in varying degree of detail of stand and process description. Many are modeled after Clutter's (1973) concepts of compatible growth and yield models. Other procedures like that of Stage (1973) model to greater degree biological processes. Types of published simulators, from the most general to the most specific, are:

1. Aggregate stand simulators, using average descriptive measures for stand height, diameter, basal area, number of trees, and volume. Some separate the component smaller than merchantable diameter based upon relationships to average descriptors.

2. Diameter distribution simulations, using distributions described by mathematical functions such as beta, Weibull, binomial, or exponential distributions.

3. Stand table projections, using a table of trees by diameter class. Growth of trees within a size class results in movement of trees to larger classes by specified rule.

4. Individual tree simulations, with sample trees representative of the stand, and not described by spatial locations.

5. Individual tree simulations, where each tree is located spatially in reference.

Because MULTIPLOY was already a large system, the growth procedures selected were limited to the first two types. The system could be adapted to stand table projection (type 3). The information required for individual tree simulations ruled out directly incorporating procedures for types 4 and 5. But such simulators can generate tables that could separately be used in MULTIPLOY.

Furthermore we specified that each selected growth procedure must be adequately described in the literature, with published examples of simulated management regimes against which we could check our adaptations. The species or species group concerned had to represent a significant timber type under management in the United States. A wide variety of other simulation procedures for foreign species would have been equally adaptable.

The growth procedures selected are shown in table 1. For some species or species groups several growth procedures were available for separate geographic areas. Unfortunately, all were for even-aged stands. We have since learned of other simulation procedures that are also adaptable to MULTIPLOY.

Some simulators are only applicable to plantations, and others to natural stands, as shown in column 3, and some have specific projection periods, as shown in column 5. Also shown are the authors of the biometrical analysis (column 6), site index year (column 7), and applicable ranges of sites and ages (column 8). The site productivity range generally covers 90 to 95 percent of land in the forest type. But often the procedure is applicable only to younger ages for which mensurational data is available.

Lastly, the fourth column shows the classification of projection procedure used. This is:

Table 1.--Applicable growth-simulation procedures incorporated

Species	Region	Regeneration type 3/	Projection type 1/	Projection period 4/	Authors	Site index year	Site range	Age range
Loblolly pine	Coastal Plain	N	RNS	V	Schumacher & Coile	50	60-120	20-80
	Piedmont	P	BDF	V	Lenhart & Clutter	25	40-80	10-30
	Highlands	P	WDF	V	Smalley & Bailey Bailey & Dell Smalley & Bower	25	40-70	10-40
Slash pine		N	RNS	V	Schumacher & Coile	50	40-100	20-80
		P	BDF	V	Bennett & Clutter	25	40-80	10-30
Ponderosa pine	Black Hills	N	GED	10	Myers	100 2/	40-70	20-150
	Southwest	N	GED	10	Myers	100 2/	40-70	20-150
	Interior	N	RNS	V	Lynch	100	40-70	70-150
Douglas-fir	Coastal	N	GEV	V	Bruce, DeMars & Reukema	100	80-200	10-100
Shortleaf pine		N	RNS	V	Schumacher & Coile	50	40-100	20-80
		P	WDF	V	Smalley & Bailey & Dell Smalley & Bower	25	30-60	10-40
Longleaf pine		N	RNS	V	Schumacher & Coile	50	50-100	20-80
Oak-hickory		N	GEB	V	Dale	50	55-85	20-110
Yellow poplar		P	GEB GEV	V V	Beck & Della-Bianca	50	90-130	20-70
Aspen	Lake States	N	GEB	V	Schlaegel	50	65-90	20-60
	Lake States		GED	V	Schlaegel	50	65-90	20-60
Red pine		N	GEB	V	Buckman	50	45-60	30-160
Lodgepole pine		N	GED	10	Myers	100 2/	40-70	20-150
Mixed conifers		N	GED	10	Alexander, Shepperd, & Edminster	100 2/	40-110	20-160

1/ Definitions for these classifications are in text.
2/ Site index year for western species assumed to be 100 when unspecified in literature.
3/ Regeneration type: N = natural, P = planted.
4/ Projection period: V = variable, 10 = 10 years.

Procedure	Symbol
RETURN TO NORMAL STOCKING	RNS
RETURN TO NORMAL BASAL AREA	RNB
RETURN TO NORMAL VOLUME	RNV
GROWTH EQUATION FOR BASAL AREA	GEB
GROWTH EQUATION FOR VOLUME	GEV
GROWTH EQUATION FOR DIAMETER	GED
BETA DISTRIBUTION FUNCTION	BDF
WEIBULL DISTRIBUTION FUNCTION	WDF
STAND TABLE PROJECTION	STP

The return-to-normal growth procedure has been used rarely in recent growth and yield research, but numerous timber growth and yield studies that use the approach are still valuable. Most modern growth and yield research employs growth equations for basal area, diameter or volume (either for the entire stand or by diameter classes).

In using mathematical distribution functions for diameter or basal area of stands, growth is projected as the stand becomes older by estimating stand descriptors independently for each period. These include survival of trees, the largest and smallest diameters, and parameters of the distribution function. It has proved successful for unthinned planted timber stands in the Southeast (Lenhart and Clutter 1971, Bennett and Clutter 1968, and Bailey and Dell 1973).

Within a problem the memo instructions may specify a different simulation process be used (1) for "treated" regimes vs. "untreated" regimes, (2) in first rotation as succeeding rotations, and (3) before vs. after thinning, or in any of the eight combinations of these classes.

Functions of timber simulation covered

Essentially two aspects of timber simulation, at a minimum, must be included in a growth procedure. First is stand establishment. For aggregate stand simulation, several descriptors must be assumed or simulated to develop a static description at the time of establishment. These are:

1. Age at which the stand is "established," after which period-to-period growth can be simulated

2. Height of dominant/codominant trees or average trees.

3. Any two of the following four items:

    a. Number of trees established
    b. Diameter of trees established
    c. Basal area of trees established
    d. Stocking index, if one is available for species

    From any two items, the others can be computed.

The second aspect is the estimation of period-to-period growth, taking into account effects of treatments. In predicting growth, several processes must be simulated. These are:

1. Height growth.

2. Size growth, either growth in average diameter, or gross or net growth in basal area, cubic volume, or changes in parameters of diameter distribution functions.

3. Mortality in basal area or volume, if size growth is in gross basal area or volume.

4. Mortality in numbers of trees.

From descriptors at the beginning of a period, the growth processes must project at least a measure of height, size, and number of trees at the end of the period. From these the remaining descriptors may be estimated.

Equation types within processes

The component equation types for each of the projection methods are shown in table 2. No growth projection method uses all types of equations, because several are mutually exclusive. For any of the simulation procedures except the diameter distribution functions, the user can temporarily override any of the component equations by specifying an equation of the form:

(equation name) = (a user-supplied formula)

Users can enter their own formulas for each simulation equation listed with an asterisk in table 2. A use of the user-supplied simulation equations is in modifying yield tables entered directly as input to MULTIPLOY. If a variable, such as total or merchantable volume, is missing, the variable will be automatically computed by MULTIPLOY if the appropriate equation is given. Any combination of entered and computed variables can be used in an equation, so long as the information is present in the system.

However, the user can construct his own entire simulation routine by supplying all of the necessary equations given in table 2. In this case, he can designate his own species name, and needs to provide site index only if he uses site index in one or more of the simulation equations. Figures 2 and 3 show the input into a hypothetical simulator and the resultant yield table.

Organization of the timber simulation system

In entering the timber subsystem, processing control goes to one of four subroutines, depending on the condition of the stand and the simulation process used. Figure 4 shows the flow of control within the timber subsystem. The essential simulation is performed in one of four subroutines.

1. Diameter function simulation, if the procedure uses a diameter distribution function and age is at least establishment age before first cutting.

2. Establishment simulation, if an aggregate simulation process is used and the stand is at establishment age.

3. Natural stand growth simulation, if an aggregate simulation process is being used and the stand is already established.

Table 2.--Functions used in timber growth and development simulation processes

Function	Used In: RNS	RNB	RNV	GEB	GEV	GED	BDF	WDF	STP
HEIGHT = *	x	x	x	x	x	x	x	x	x
NORMAL NUMBER OF TREES = *	x	x	x	-	-	-	-	-	-
NORMAL BASAL AREA = *	x	x	x	-	-	-	-	-	-
RETURN TO NORMAL BASAL AREA = *	-	x	-	-	-	-	-	-	-
RETURN TO NORMAL VOLUME = *	-	-	x	-	-	-	-	-	-
GROWTH IN BASAL AREA = *	-	-	-	x	-	-	-	-	-
GROWTH IN VOLUME = *	-	-	-	-	x	-	-	-	-
GROWTH IN DIAMETER = *	-	-	-	-	-	x	-	-	x
MORTALITY IN NUMBER OF TREES = *	x	x	x	x	x	x	x	-	x
MORTALITY IN BASAL AREA = *	-	?	-	?	-	?	-	-	-
MORTALITY IN VOLUME = *	-	-	?	-	?	-	-	-	-
VOLUME IN TOTAL CUBIC FEET = *	x	x	x	x	x	x	x	x	x
VOLUME RATIO FOR MERCHANTABLE CUBIC FEET = * / VOLUME RATIO FOR MERCHANTABLE BOARD FEET = * (either or both)	x	x	x	x	x	x	x	-	x
MAXIMUM DIAMETER =	-	-	-	-	-	-	x	-	-
MINIMUM DIAMETER =	-	-	-	-	-	-	x	x	-
ALPHA PARAMETER =	-	-	-	-	-	-	x	x	-
BETA PARAMETER =	-	-	-	-	-	-	x	x	

```
TO: MULTIPLOY      'DATE'
FROM: 'ANALYST'S NAME'
SUBJECT: 'SIMPLE HYPOTHETICAL GROWTH PROCEDURE'

* MANAGEMENT REGIME
PREPARE SITE FOR REGENERATION
PRECOMMERCIAL THIN AT AGE 10
SIMULATE TIMBER GROWTH BY GROWTH EQUATION FOR BASAL AREA
ASSUME ESTABLISHMENT AGE IS 20
ASSUME 300 TREES ESTABLISHED
ASSUME 90 SQUARE FEET OF BASAL AREA ESTABLISHED
THIN TO 80 SQUARE FEET BASAL AREA
HARVEST FINAL YIELD AT 60 YEARS

* EQUATIONS FOR COMPLETING STAND TABLE
ASSUME SITE INDEX IS 80
HEIGHT = 10 + SI - 800 / T
VOLUME IN TOTAL CUBIC FEET = .00008 * D ** 2 * H
ASSUME MINIMUM PULPWOOD DIAMETER IS 4.5 INCHES
VOLUME RATIO IN MERCHANTABLE CUBIC FEET = 1 - .8 / D
ASSUME MINIMUM SAWTIMBER DIAMETER IS 8 INCHES
VOLUME RATIO IN MERCHANTABLE BOARD FEET = .07 - .12 / D

* GROWTH EQUATIONS
GROWTH IN BASAL AREA = 15 * SI / T
MORTALITY IN NUMBER OF TREES =  - .05 + 3 / T

* COSTS AND PRICES
COST OF PREPARING SITE IS 40
COST OF PRECOMMERCIAL THINNING IS 15
COST OF PRESCRIBE BURN = 5 - .05 * T
COST OF SALE PREPARATION = 3 * OC
PRICE OF PULPWOOD IS $5
PRICE OF SAWTIMBER IS $60

WRITE USING 80 COLUMN FORMATS
RUN PROBLEM PLEASE
STOP
```

Figure 2.--"Memo" input for MULTIPLOY

'ANALYST'S NAME'
NOV 16, 1977, AT 6:56

'SIMPLE HYPOTHETICAL GROWTH PROCEDURE'

YIELDS USED IN ANALYSIS BEFORE THINNING DEVELOPED FROM --
SIMULATION FROM USER'S EQUATIONS

YIELDS USED IN ANALYSIS AFTER THINNING DEVELOPED FROM --
SIMULATION FROM USER'S EQUATIONS

YEAR		YIELD TABLE BEFORE/AFTER TREATMENT							TIMBER HARVEST					PERIOD	
A	T	HT	BA	NO	DBH	VOLY	VOLO	VAL	DBH	QI	VOLY	VOLO	VAL	REV	COST
0	0	0	0.	0.	.0	.0	.0	0.							
		0	0.	0.	.0	.0	.0	0.	.0	0.	.0	.0	0.	0.	-45.
10	10	0	0.	0.	.0	.0	.0	0.							
		0	0.	0.	.0	.0	.0	0.	.0	0.	.0	.0	0.	0.	-15.
20	20	50	90.	300.	7.4	66.0	3.2	16.							
30	30	63	130.	285.	9.1	120.8	6.3	376.							
		63	80.	146.	10.1	74.5	4.0	239.	8.1	0.	46.2	2.3	138.	138.	-7.
40	40	70	110.	145.	11.8	113.2	6.3	379.							
		70	80.	93.	12.6	82.4	4.7	280.	10.2	0.	30.8	1.7	99.	99.	-5.
50	50	74	104.	93.	14.3	113.1	6.6	395.							
		74	80.	64.	15.1	87.0	5.1	307.	12.5	0.	26.1	1.5	89.	89.	-4.
60	60	76	100.	64.	16.9	112.7	6.7	405.							
		0	0.	0.	.0	.0	.0	0.	16.9	0.	112.7	6.7	405.	405.	-20.

Figure 3.--Yield table output produced by MULTIPLOY from input in Figure 2.

Abbreviations:

E = Establishment age

YT = Yield table

DF = Diameter function

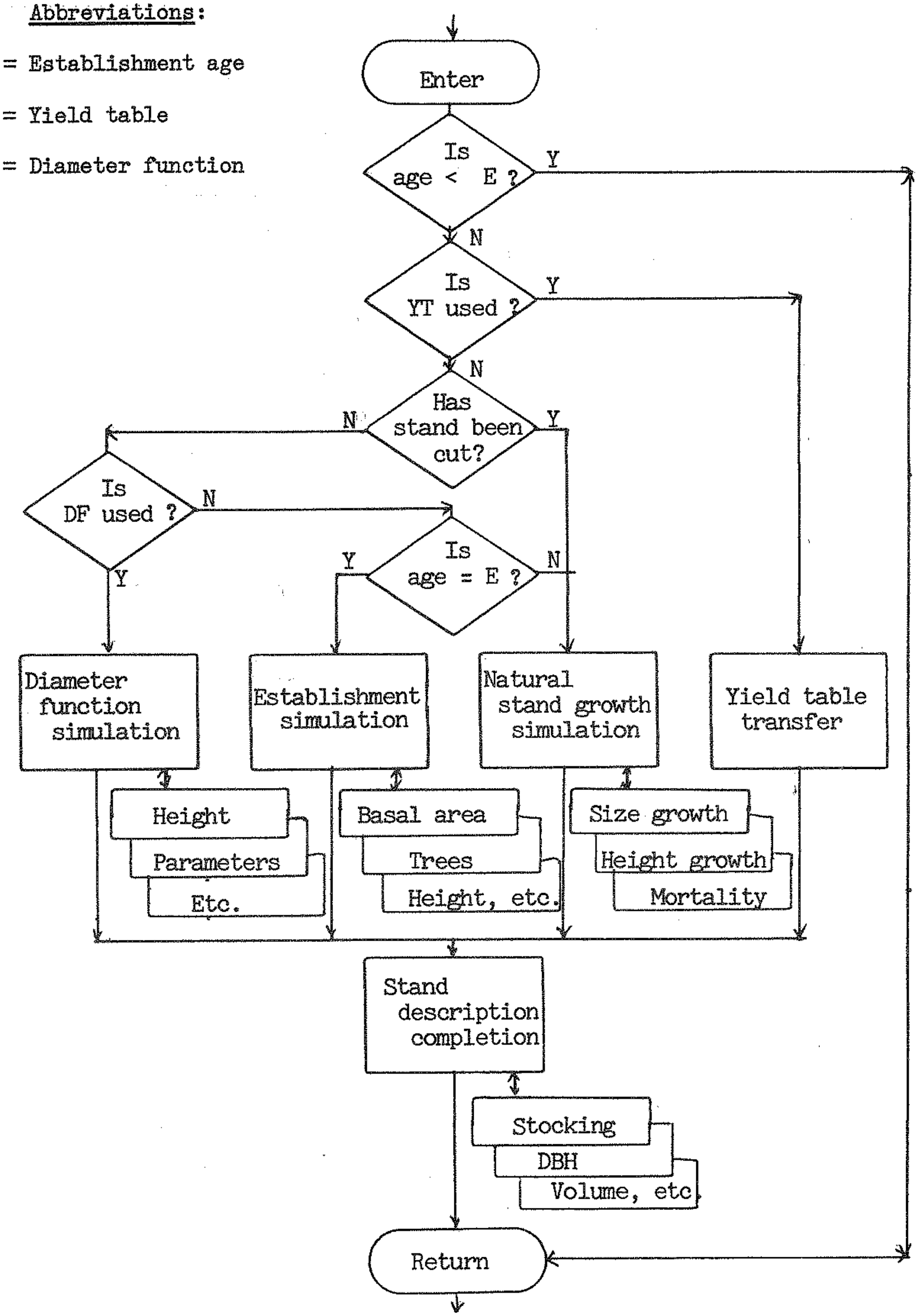

Figure 4.--Scheme of timber subsystem

4. Yield table transfer, if a yield table is being used.

Each of these routines simulates only a limited number of descriptors, such as height, basal area, and number of trees. All of the routines then pass control to a utility subroutine which completes the computation of other descriptors. For example:

1. Height is a monotonically increasing function of site index and age. If any two of these three variables are known, the other can be computed regardless of the form of the function.

2. Basal area of a component is a fixed mathematical function of diameter and number of trees. If any two are known, the third is computed simply.

3. Volume is an increasing function of diameter, height, and number of trees. If any three are known, the utility routine can compute the remaining descriptor regardless of the form of the function.

Each of these simulation subroutines may call several of the function subroutines corresponding to the function types in table 2 as shown in figure 4. In these functions are stored the relationships for each of the species and species groups. Functions for some species comprise several equations.

Since values per unit of timber harvested are influenced by factors such as diameter and volume per acre, timber quality can also be computed by a user-supplied function. This quality can be expressed as an index of the value of the final products manufactured, relative to a standard product grade.

In adapting the growth projection procedures to MULTIPLOY, we took care not to alter relationships in any material way. Nevertheless, the construction of some published simulation processes made the task quite difficult.

Related timber simulation processes

One feature of MULTIPLOY allows the user to specify probabilities of loss from (a) fire and storm, and (b) from insect and disease pests from period-to-period. The probabilities of total or near total loss may be fixed factors or functions of stand age, site index, basal area, or other descriptors known to affect losses. These functions must be entered by the input memo. Related functions that can be entered estimate the proportion of volume likely to be salvaged, and the proportion of value the salvaged cut has to the same volume if cut green.

The system keeps track of the proportion of the original area still intact, and applies the loss probabilities to the proportion that remains each period. The system can evaluate the regime under the assumption that areas lost are regenerated promptly using the same management regime as originally assumed.

The next simulation is thinning, which is in the treatment/harvest subsystem. The subroutine THIN estimates average diameters, numbers of trees, and basal areas of both the cut and remaining components if the stands are cut to a given standard. The most usual standard is either a fixed basal area of the remaining stand or a proportion of the original stand. The most satisfactory system is that of Myers (1968), which is used if no other set of thinning estimation functions has been provided along with the growth estimation procedure.

Precommercial thinning uses the same simulation procedure if the trees are individually selected. If the precommercial thinning is made by chopping strips or cross-hatch patterns, the number of trees and basal area are reduced in proportion to the area chopped.

The final phase of simulation estimates nontimber outputs within each period and is entirely optional. The functions must be supplied by the users and entered via the input memo. Relatively few problems with nontimber output simulation have been processed with MULTIPLOY because specific published relationships relating nontimber outputs with the overstory timber stand are often not available.

Concluding comments

The number of disparate simulation processes that have been developed for various types of timber stands impressed us. Some of these relationships were based on exhaustive statistical analysis of adequate pools of data; others seem dominated by a relatively stereotyped approach or more limited data. We encountered relatively few cases in which alternative approaches and levels of detail were considered, the relationships fitted, and the resulting simulation processes evaluated according to criteria related to user application.

We also noted that seldom have the simulators been validated from growth data collected independently from the data used in the biometric analysis. Validating the simulators and adjusting them to local timber growth patterns may present unexplored problems.

By enhancing the simulation capability of MULTIPLOY we have created a mensuration as well as economic tool for forest planners. It should be a particular assistance for field users and thus enable them to make improved growth and yield estimates.

A generalized system such as MULTIPLOY can encompass a wide variety of the aggregate stand and diameter distribution simulation approaches. For a wide variety of United States species and timber growing conditions, these simulators encompass best available growth and yield information. These approaches represent a realistic compromise between yield tables and detailed tree simulation.

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PREDICTING STAND STRUCTURE

IN

EVENAGED STANDS

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Introduction

The stand structure provides valuable information to the foresters and timber owners. Questions such as: whether it will pay to thin a stand; whether there will be enough veneer size volume to justify a harvest cut; or how much merchantable volume and value may be expected at the time of harvest, cannot be answered satisfactorily without some idea about how the stand is structured. The average stand diameter conveys little information about the spread and distribution of individual tree diameters.

Other than getting stand structure data by cruising, there are two general approaches of getting this information: one, by updating a previously compiled stand table and two, by deriving it from measured or updated stand attributes. The former is data intensive as a stand table must be continuously updated. In the latter approach, on the other hand, the stand structure is quantified by a mathematical function based on stand attributes such as average, minimum and maximum diameter, basal area and number of trees. This is much more flexible and efficient if computer is used to store and update stand information.

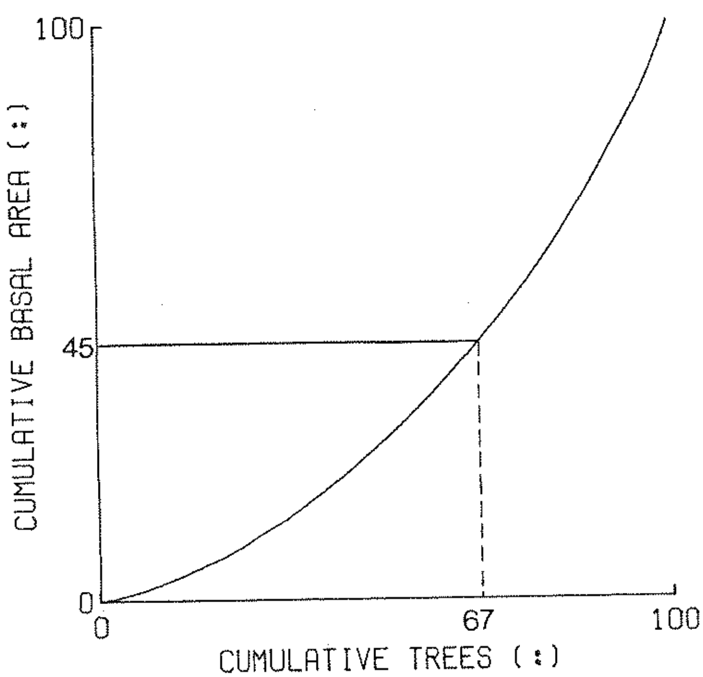
Literature Review

Most of the studies so far have concentrated on explaining known stand structure by a mathematical function. Examples include fitting a normal curve (Gingrich 1967), a log normal curve (Bliss and Reinker 1964), Gram-Charlier Series (Meyer 1930), Pearsonian Curve (Schnur 1934), Pearl Reed growth curve (Osborne and Schumacher 1935, Nelson 1964), Gamma distribution (Nelson 1964), and Weibull function (Bailey and Dell 1973) over a stand table data. Only Clutter and Bennett (1965), McGee and Bella Bianca (1967) and Lenhart and Clutter (1973) attempted to recreate a stand table from known stand attributes using beta distribution.

The procedure for updating of a stand table has been demonstrated by Seth (1974) and Rudra (1968). The main drawback of their approach is the massive amount of data needed. At least two successive stand tables from permanent plots are required to get the tree movement rates between diameter class. Rudra has used stand table data from several successive inventories to develop a stationary transition matrix for tree movements. They both implicitly assume that rate of tree movement by diameter class remains stable over time.

Practically all investigations into the stand structure have been based on stem frequency by diameter class. Only exception is the prediction of basal area by diameter classes using beta distribution by McGee and Della Bianca (1967). There are obvious advantages in working with basal area instead of number of stems. First, in fitting a mathematical model by regression, a better fit is obtained in the upper half of the diameter range if basal area is used. This happens because approximately one-third largest trees in a stand typically contain over half of its basal area (Figure 1). As larger trees contain proportionately higher amount of volume, working with basal area would provide a better representation of larger trees in the predicted stand table. Second, within a stand, the tree basal area is linearly related to its volume. Consequently, errors in predicted basal area distribution will have little impact on total stand volume.

Fig. 1. Relationship between cumulative percent basal area and cumulative percent number of trees in a typical evenaged stand. The smallest 67 percent of the trees are contributing only 45 percent to the stand basal area.



Primary use of a stand table is to provide a break-up of stand volume by narrow (or broad) size classes. The conventional stand table serves this need rather indirectly as the stem frequency from a stand table has to be converted into volume before it could be useful in management decisions. We develop here a methodology for directly predicting basal area for any given size class. If required, a stem frequency table could also be derived indirectly.

The Stand Structure Model

A number of function forms describe the stand structure of evenaged stands. Schreuder and Swank (1974) found Weibull function superior to several other function forms. They, however, tested this and other models on stem frequency distribution by diameter class basal area in place of class diameter.

Though the Weibull function was developed in an entirely different context (Fisher and Tippet 1928, Weibull 1939), it does an excellent job of quantifying stand structure of both evenaged and unevenaged stands (Bailey and Dell 1973). We propose to adopt this model for predicting basal area distribution in evenaged stands.

The general 3-parameter probability density function (p.d.f.) for the Weibull random variable (r.v.) X is:

$$f(x) = c/b \left\{(x-x_o)/b\right\}^{c-1} \exp\left[-\left\{(x-x_o)/b\right\}^{c}\right] \qquad (1)$$

$$x > x_o;\ x_o > 0;\ b > 0;\ c > 0$$

Where b and c are Weibull function parameters and x_o is the minimum value the r.v. can take.

The parameters b and c define the curve. b is called the scale parameter and implies that 63 percent of the curve area is to the left of b on the horizontal axis (Bailey and Dell 1973). The shape of the curve depends on c (Harter 1964). If $c > 1$, the curve is unimodel characterizing structure of evenaged stand. if $c < 1$, the curve is reversed J shaped and characterizes unevenaged stand structure (Figures 2a and b).

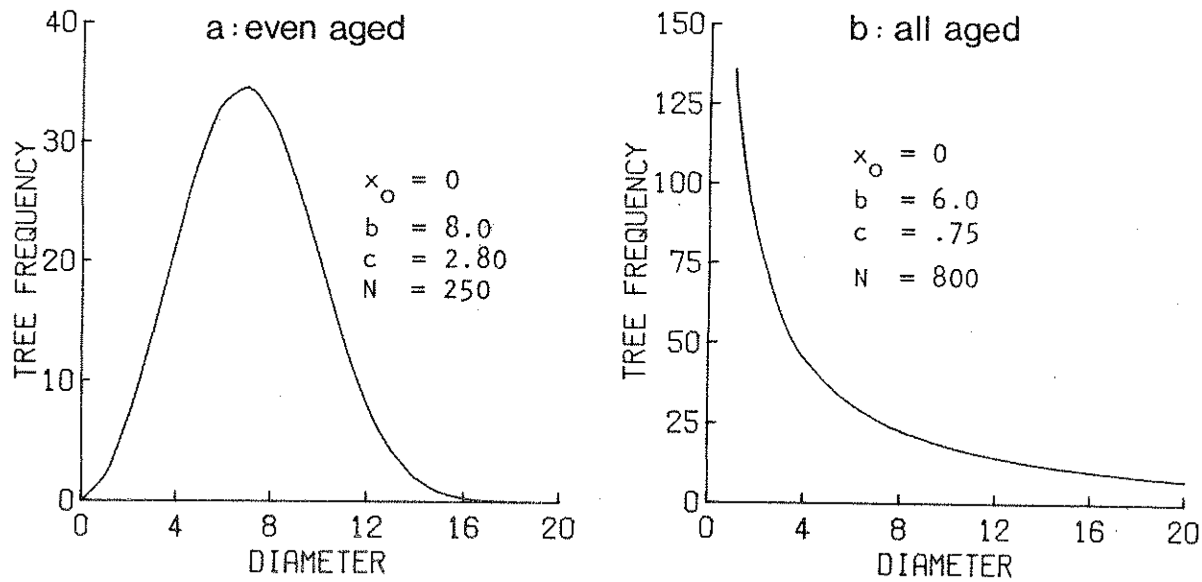


Fig. 2a,b: Distribution of stem frequency in a: evenaged and b: all-aged stand using Weibull probability density function.

The cumulative distribution function form of the 3-parameter Weibull random variable X is:

$$F(x) = 1 - \exp\left[-\left\{(x-x_o)/b\right\}^c\right] \tag{2}$$

for $x > x_o$

$o \leq F(x) \leq 1$,

where F(x) measures the area under the curve between $X = x_o$ and $X = x$. If we replace F(x) by the ratio of cumulative basal area (Bx) of all trees up to diameter x to the total stand basal area (B), we have:

$$B_x/B = 1 - \exp\left[-\left\{(x-x_o)/b\right\}^c\right] \tag{3a}$$

Or

$$B_x = B\left[1 - \exp\left\{-\left[(x-x_o)/b\right]^c\right\}\right] \tag{3b}$$

Thus, if we knew the minimum stand diameter X_o, the Weibull function parameters b and c and the total stand basal area B, the cumulative basal area up to the diameter x can be computed from (3b). The basal area included within the diameters x_1 and x_2 $(x_1 < x_2)$ in that case is given by:

$$B_{(x_1 - x_2)} = B\left[\exp\left\{-\left[(X_1 - x_o)/b\right]^c\right\} - \exp\left\{-\left[(x_2 - x_o)/b\right]^c\right\}\right] \quad (4)$$

Quantifying Basal Area Distribution

If the stand structure is already known, the estimation of the parameters b and c is relatively simple. Bailey and Dell (1973) provide an excellent summary of the maximum likelihood and percentile estimators of b and c. Besides these, statistical regression may also be used to estimate these parameters.

The issue, however, is not of estimating the Weibull function parameters when basal area (or stem frequency) distribution is known. Instead, the issue is of defining basal area distribution of a stand of known attributes with the added knowledge that the stand structure follows Weibull distribution.

We accomplish basal area prediction in evenaged stands in two steps. First, we fit the Weibull function to the basal area distribution data of a number of stands and thus obtain the parameter values. Second, we attempt multiple correlation of the parameters with stand attributes. This gives us the estimates of Weibull function parameters in terms of the stand attributes.

Finally, we validate the model on some stands by comparing the actual and estimated basal area distribution.

Fitting the Model

Basal area distribution in evenaged stand does not always begin at zero. If the minimum stand diameter x_o is known, the following three parameter Weibull distribution function may be used to describe basal area distribution.

$$B_x = B\left\{1 - \exp\left[-\left\{(x-x_o)/b\right\}^c\right]\right\} \quad (5)$$

$$x > x_o; \quad o < B_x > B.$$

There are two problems in fitting this model. First, in forest measurement practices, trees are not always measured down to the smallest diameter. It would therefore be erroneous to fit this model to such truncated data. A stand which has been thinned from below would also present the same problem. Second, the determination (or updating) of the minimum diameter would not always be simple. Because of suppression mortality, and thinnings, keeping track of the smallest diameter x_o would be difficult. Smallest diameter is probably the most inconsistent of all stand attributes.

It is possible to eliminate x_o from the model. Figure 3 illustrates the logic used in the derivation of a modified Weibull distribution model from which x_o has been eliminated.

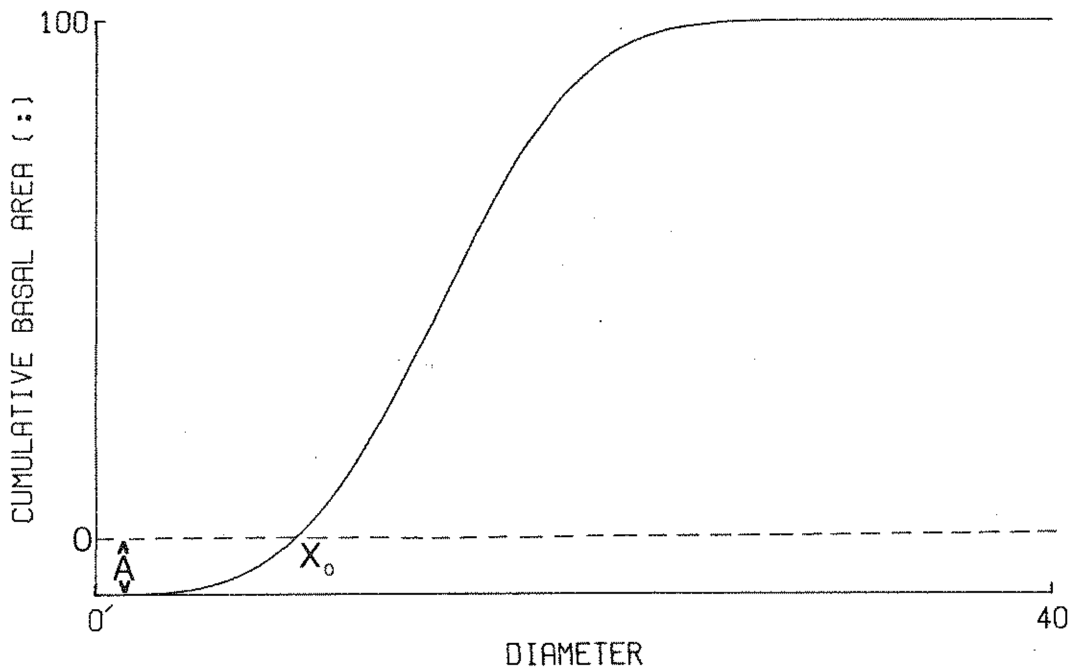


Fig. 3: The cumulative basal area distribution using 3 parameter Weibull function is replaced by modified 2 parameter Weibull function by shifting the horizontal axis downwards. The total cumulative percent basal area in the modified case is 100+A.

Suppose we fit a 3-parameter Weibull distribution function over cumulative basal area in a stand with minimum diameter x_o. We can extend this distribution backwards till it meets the vertical axis at 0' which will always be below the origin. If we now shift our origin downwards to 0', we can fit a 2-parameter Weibull distribution function over the stand data after adding amount A(=00') to the cumulative basal area for each diameter class. This modified 2-parameter Weibull distribution would become:

$$\frac{B_x + A}{B + A} = 1 - \exp\left[-\left(\frac{x}{b}\right)^c\right] \qquad (6a)$$

which leads to:

$$B_x = \left\{B + A\right\}\left\{1 - \exp\left[-\left(\frac{x}{b}\right)^c\right]\right\} - A \qquad (6b)$$

where

B_x - the cumulative basal area from diameter x_o to x.

B - total basal area in stand in trees above x_o.

A - theoretical basal area in trees below x_o. This is in addition to the measured basal area B.

Though x_o of (5) has been replaced by another unknown A in (6b), there is one major advantage in working with this modified two parameter form. The available truncated stand table data can be used to derive the model parameters using regression technique. If required, x_o -- the minimum diameter to which basal area data pertains -- can be derived if A, b and c are known. It can be shown that

$$x_o = b^* \left[\ln \left(\frac{B + A}{B} \right)^{1/c} \right] \tag{7}$$

The modified 2-parameter Weibull function (6b) should give us the distribution of the total basal area (B) in evenaged stands once the two parameters (b and c) and the basal area (A) of trees in unmeasured diameter range is known. In fact, it should also be possible to estimate basal area distribution in diameter classes below the minimum measured diameter x_o as well.

Estimation of Model Parameters

A two-step estimation procedure was adopted for A, b and c in terms of stand attributes. Data from fully-stocked Douglas fir plots -- used originally by McArdle and Meyer (1930) in their yield tables studies -- was employed to test the validity of the model and the estimation of its parameters. This initial phase of stand structure study was limited to fully-stocked stands as complete data for a wide range of age by site was available. We also felt that initial work with these stands may provide insight into estimated procedures for model parameters in other than fully-stocked stands.

Data from 67 plots was used. For each stand starting with the smallest diameter, cumulative basal area percent was computed by diameter classes. The cumulative basal area for each stand to the upper limit of the largest diameter class was thus 100. Though not essential to the analysis, conversion to cumulative percent basal area made stand structure data comparable.

The model parameters: b,c and A of the modified Weibull distribution (7) were obtained for each plot using non-linear regression. Excellent fit, as measured by the residual root mean square (rrmsq), was obtained in every plot.

The rrmsq varied from a low of .30 to a high of 2.31 percent. Out of 67, 38 computed rrmsqs were smaller than 1.00 percent and only two were 2.00 percent or larger. Considering the variations in stand structure from plot to plot, it can be stated that the modified 2-parameter Weibull distribution function fits well over the cumulative basal area.

The value of b (scale parameter) ranged from a low of 6.07 to a high of 35.00. As expected, b increased with increase in average stand diameter.

The value of c (shape parameter) ranged between 2.31 and 5.43. Though c increased with increase in average diameter, it also appears to be influenced by the spread of diameter distribution.

The value of A (the basal area below the minimum measured diameter)-- expressed as percent of B (the measured plot basal area)-- ranged from a low of -1.99 to a high of 10.65 percent. In all, six plots gave negative A which is not theoretically possible. Examination of the stand structure data showed that in each case, the tree frequency data was other than normal. In these plots it was either multi-modal or the individual frequencies were low and more or less flat over the entire diameter range.

The cumulative basal area from these six plots was regressed again with the added stipulation that A = 0. The revised computed values of b and c were only marginally different and were used instead of the original values in further analysis.

Excellent fit was also obtained when the same model was used with the cumulative tree frequency data. The b and c values were consistently smaller and that of A consistently larger than the corresponding basal area derived values. However, in view of arguments presented earlier, no further analysis of the estimated model parameters of tree frequency data was done.

In order to develop a prediction model for modified Weibull function parameters, their linear correlation with stand attributes was examined. This is summarized in Table 1a. Table 1b summarizes the linear and non-linear correlation of A with b,c, and their functions.

Table 1a: The Coefficients of Correlation of Model Parameters with Stand Attributes

Stand Attributes	Model Parameters		
	A	b	c
Age	-.38682	.82398	.68003
Number of Trees	.59325	-.78210	-.65492
Basal Area	-.37110	.80903	.62138
Average DBH	-.55353	.99526	.75792
Dominant Height	-.52838	.95944	.65858
Site Index	-.36201	.44674	.17234
Max Diameter	-.50587	.95601	.60945
Ave/Max Diameter	-.44219	.62310	.81648
Inverse Trees	-.53781	.95271	.71238
Inverse BA	.34039	-.80181	-.62352
Inverse Age	.33822	-.74811	-.66503
BA/Trees	-.50336	.96832	.71926
Inverse DBH	.58087	-.89780	-.72965
Inverse Height	.53817	-.87951	-.65817

Table 1b: The Coefficient of Correlation of A with parameters b and c and their Functions

b, c and Their Functions	A
b	-.58161
b^2	-.53004
$1/b$	.62104
$1/b^2$	.60898
c	-.63846
$1/c$	.69249
$1/c^2$	.70819
$1/c^3$	.71443

These tables are very revealing. The scale parameter b is most strongly correlated with stand attributes followed by c and A. b is positively correlated with c, and not surprisingly, A is negative correlated with both b and c. Though these tables show that it may be possible to predict the model parameters from stand attributes, there is distinct indication of considerable multi-colliniarity among the stand attributes themselves. Thus, adding attributes to the prediction models may improve goodness of the fit as expressed by R-square, but only at the cost of decrease in the precision in the estimation of the coefficients associated with these parameters.

The scale parameter b is almost perfectly correlated with the average diameter. It is also very strongly correlated with most of the other stand attributes. It is obvious that in undisturbed stands average stand diameter alone may provide a precise estimate of b. The following linear model for estimating b is therefore selected:

$$\hat{b} = 1.0478 + 1.1813 * \text{AVDBH} \quad (r = .99526) \qquad (8)$$

The shape parameter c is most strongly correlated with the ratio of average to maximum diameter (r = .81648). It is also strongly correlated with the difference of maximum and average diameter (r = .77148), the average diameter (r = .75792) and the inverse of number of stems per acre (r = .74648). Absence of stronger correlation in this case is perhaps due to the lack of pronounced unimodel distribution in some of the plots. The prediction model finally selected is:

$$\hat{c} = -.78867 + 8.29915 * \text{AVBYMAX} + .04509 * \text{MXMINAV} \quad (R = .87316) \qquad (9)$$

AVBYMAX - Ratio of average to maximum diameter

MXMINAV - Difference between maximum and average diameter

A is really hard to predict in terms of stand attributes. This is because its value essentially depends on the distribution of the basal area in the lowest diameter classes. The distribution of basal area in these diameter classes in 67 plots was rather erratic resulting in inconsistency in the values of A including six which were negative. Fortunately, A influences basal area distribution in the smallest diameter classes only and approaches zero with stand maturity. Thus, even substantial errors in prediction of A will have little impact on the distribution of basal area over most of the stand. For example, with b = 16 and c = 4, changing value of A from 0 to 4 percent would cause a drop of only 0.25 of one percent in the basal area among trees above 8 inch diameter.

The best correlation of A is with the inverse of c, c-squared and c-cubed (r = .69249, .70819, 71443). This suggested that a non-linear model may provide suitable prediction of A in terms of c. Linear prediction models were tried but rejected as some of the predicted values of A were negative.

The model finally chosen to predict A is:

$$\hat{A} = 115.31 * c^{-3.1625} \qquad (rrmsq = 1.4965) \qquad (10)$$

The coefficient of correlation is not computed for non-linear regression. However, indirect determination gave an r = .75132.

The basal area distribution prediction model for undisturbed evenaged stands is defined by equations (6b), (8), (9) and (10) and may be summarized as follows:

$$B_x = B\left[(100 + A) * (1\text{-exp}\left\{- (x/b) ** c\right\}) - A\right]$$

$$b = a_o + a_1 * \text{AVDBH}$$

$$c = a_2 + a_3 * \text{AVBYMAX} + a_4 \text{ MXMINAV}$$

$$A = a_5 * c ** a_6$$

B_x - cumulative basal area of trees smaller than diameter X in the stand.

B - total basal area in the stand.

b and c - Weibull function parameters

A - theoretical basal area in trees below the minimum measured diameter X_o expressed as percent of the observed stand basal area B.

$a_o,..,a_6$ - linear and nonlinear regression coefficients.

AVDBH - average (quadractic mean) stand diameter.

MXDIAM - maximum diameter within the stand.

AVBYMAX - the ratio of average to maximum diameter.

MXMINAV - difference between the maximum and average diameter.

Validation of the Model

Our model has been developed in two stages using data from 67 plots. The next step is to see how good this model is in predicting basal area distribution in undisturbed evenaged stands.

We tested our model on 10 plots. Five were selected from the original 67 plots and five new plots were added. The fitted values of model parameters (obtained by non-linear regression) and their predicted values (obtained by equations 8, 9, and 10) are given in Table 2.

Table 2. Fitted and Estimated Values of Modified Weibull Function Parameters for 10 Plots

		PARAMETER VALUES					
		Fitted			Estimated		
Plot #		b	c	A	b	c	A
5	(NEW)	12.2396	4.1266	3.7116	13.2156	3.8410	1.6351
36	(NEW)	25.4316	4.5819	.6119	25.8559	4.3938	1.0688
37	(NEW)	33.9524	5.0710	.000	33.6527	4.8262	.7943
46	(NEW)	26.8841	3.8810	1.9753	26.5647	4.3716	1.0861
207	(NEW)	20.3218	4.5547	.8681	20.8943	4.2092	1.2241
39	(ORIGINAL)	32.9871	5.4309	.000	33.5346	4.8904	.7618
40	(ORIGINAL)	9.8027	2.6965	1.8258	8.8446	2.7684	4.6063
50	(ORIGINAL)	12.6079	3.3789	1.8297	12.3886	3.3919	2.4231
148	(ORIGINAL)	11.4777	2.3180	10.6470	11.4436	3.1262	3.1362
184	(ORIGINAL)	24.6955	4.1173	2.2065	24.9108	3.9866	1.4537

Comparison of the fitted and estimated parameter values indicates that the model is consistent. In three original and four new plots the predicted values of b and c are quite close. In plots #5 (new) and #40 (original) the estimated value of b is off by about an inch. In plot #148 (original) the estimated value of c is off by about 0.8 and that of A by 7.5. There is nothing in this table to suggest that the predicted values are better in original plots than in the new ones.

The estimated and actual cumulative basal area values were compared next (Figures 4-5). In most of the plots, the estimated values were close to the actual. In plot 148, in which the predicted values of c and A were significantly off, the predicted values came quite close (Figure 5d). The predicted values were off by 10 percent or more in plots 5 and 40 (Figure 4a and 5b). In plots 37, 46, 39, 148 (Figures 4c, 4d, 5a, 5d), the predicted values were off in the upper tail.

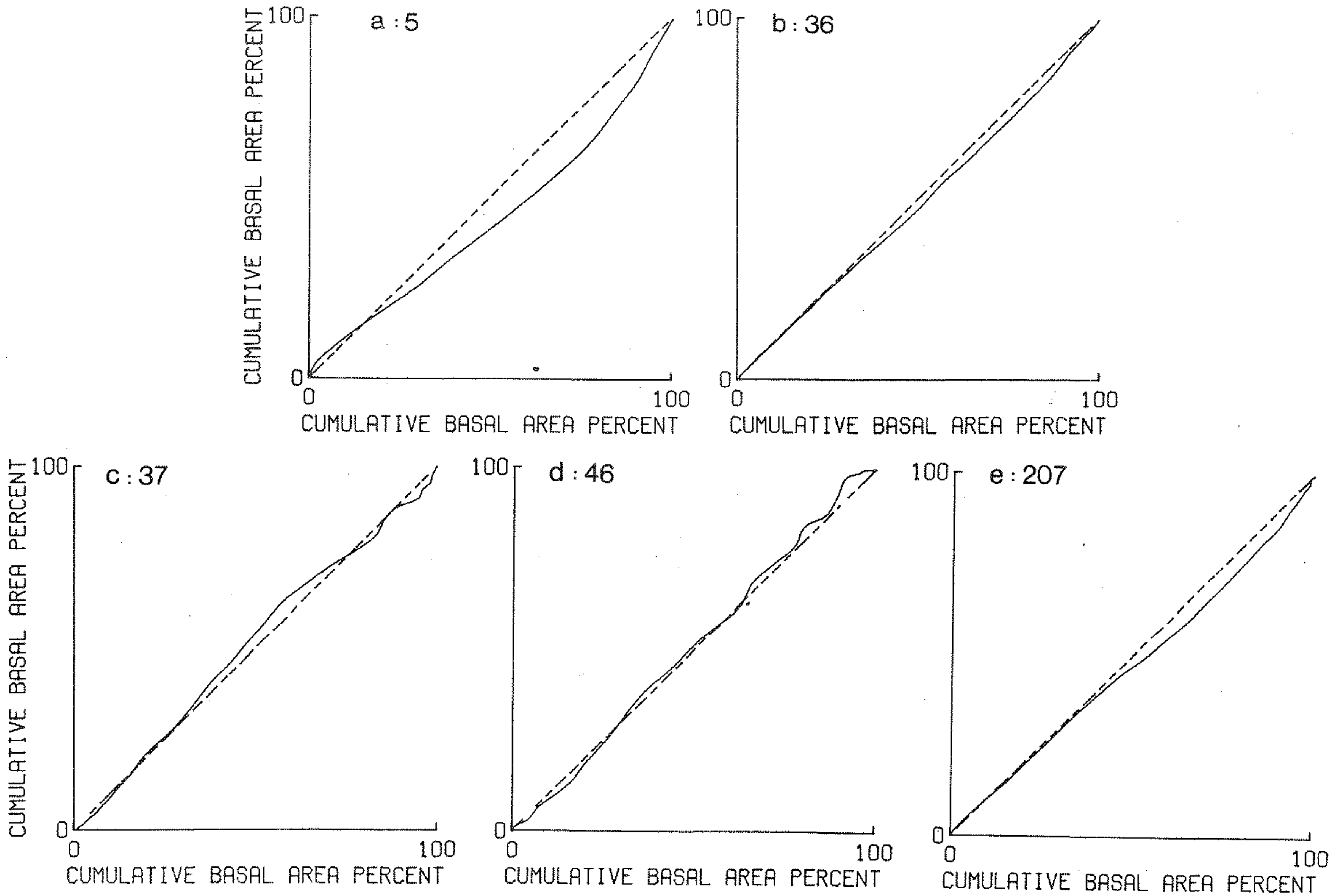


Fig. 4a-e: Comparison of the estimated with the actual cumulative basal area percent in the five new plots used in model validation.

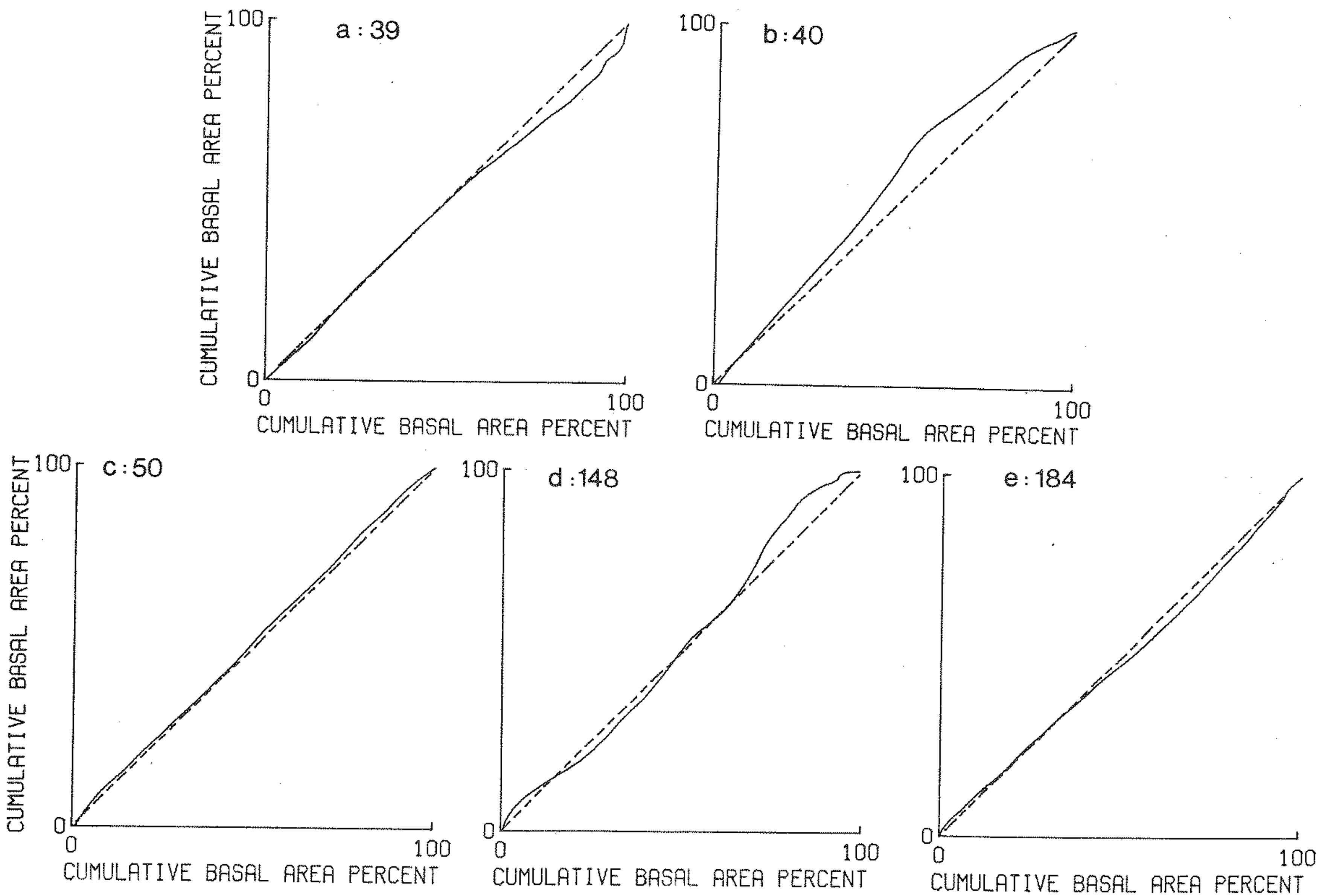


Fig. 5a-e: Comparison of the estimated with the actual cumulative basal area percent in the five original plots used in model validation.

Figures 4a, 5b and 5d indicate that errors in prediction of b have far more serious impact on the basal area distribution than the errors in prediction of c or A. There is very simple explanation for this. All smaller trees up to diameter equalling b contribute 63.21 percent to the stand basal area. Therefore, any error in prediction of b will cause errors in basal area distribution below and over b. In both plots 5 and 40, there was an error of about one inch in the prediction of parameter b. This error was perhaps caused by abnormality in stand structure.

Fortunately the prediction of b in most instances is extremely precise. Thus, the model should perform well in undisturbed stands except in situations where the distribution of trees in the stand is way out of line.

Discussion

We have presented a modified cumulative Weibull distribution function for quantifying basal area distribution in undisturbed natural Douglas fir stands. We have also shown that reasonable and consistent estimates of the model parameters can be obtained from average and maximum diameter of the stand.

Description of stand structure by basal area distribution instead of stem frequencies have obvious advantages. Statement such as "the stand has 100 square feet basal area in trees above 16 inch diameter is more meaningful to a forest manager than a statement that "there are 50 trees over 16 inch diameter in the stand." A basal area of 100 square feet can be directly translated into volume whereas further details are needed to obtain a reasonable estimate of volume in these 50 trees.

Elimination of minimum diameter as model parameter has implications beyond just fitting the model over truncated data. X_o and b determine two points between which 63.21 percent of the stand basal area is distributed. Prespecifying X_o essentially amounts to influencing the shape of basal area distribution. It should therefore be preferable to let basal area distribution determine X_o. That is exactly what our model does through A which is determined by the average and maximum stand diameter.

This study shows that average stand diameter is an important attribute in defining stand structure. Unfortunately, the average diameter within a stand is not consistent as its value is affected by the lower diameter limit used in inventory. The stand structure, on the other hand, is independent of the lower diameter limit, suggesting that the stand structure should be predicated on some other stand attribute which is not influenced by the lower diameter limit.

Further investigation is required to see if the average diameter of a fixed number of trees per acre (or hectare) may be substituted for average stand diameter in parameter prediction. Unlike average stand diameter, this average is not likely to be affected by suppression mortality and thinning thus increasing the applicability of the model to understocked or disturbed stands and to stands which have been measured down to different minimum limits.

Summary

Prediction of stand structure from its attributes is of great practical value to forest managers. Further, distribution of basal area by diameter classes as opposed to stem distribution is of more direct use because of the linear relationship between tree volume and its basal area in evenaged stands. A modified two parameter Weibull function is discussed which quantifies structure within an evenaged stand in terms of its basal area distribution. The average stand diameter and the maximum diameter provide satisfactory estimates of the Weibull function parameters.

Keywords: Weibull function, stand structure, basal area distribution.

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STATISTICAL ANALYSIS OF LINEAR GROWTH AND YIELD MODELS WITH CORRELATED OBSERVATIONS FROM PERMANENT PLOTS REMEASURED AT FIXED INTERVALS

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Most forest growth and yield models use linear models to describe the growth or yield functions. Sullivan and Clutter (1972) assumed that two repeated measurements of the logarithm of the volume had a bivariate normal distribution. They derived the maximum likelihood equations and solved the likelihood equations by an iterative procedure. Their estimator is known as an iterated Aitken estimator. Other authors--for example, Curtis (1967)--recognized that volume measurements are probably correlated, but they use ordinary least squares procedures to estimate the model parameters. Sullivan and Clutter (1972) discussed the problem of correlated errors. To date, foresters have not had a general procedure for estimating parameters of linear models with correlated observations.

The purpose of this paper is to derive the maximum likelihood (ML) estimators of parameters for a linear model when the error components are correlated due to having repeated measurements on plots. The resulting ML equations cannot be solved explicitly, so two numerical, iterative methods are tried using test data from plots in managed oak-hickory forests.

A YIELD MODEL

We use Clutter's (1963) growth and yield model to describe the plot volumes. The model is

$$\mathcal{E}(\ln v(y_{ti})) = \beta_0 + \beta_1 S_i + \beta_2 A_{ti}^{-1} + \beta_3 \ln B_{ti}$$

$$\mathcal{E}(\ln B_{ti}) = (A_{1i}/A_{ti}) \ln B_{1i} + \alpha_1(1-A_{1i}/A_{ti}) + \alpha_2(1-A_{1i}/A_{ti})S_i$$

where $\ln v_{tt}$ = natural logarithm of the cubic foot volume per acre on the i^{th} plot at time t,

S_i = site index of i^{th} plot (in feet),

A_{ti} = stand age of i^{th} plot at time t,

and B_{ti} = basal area per acre of i^{th} plot at time t (in square feet).

The time index t runs from 1 to T.

The equations for volume and basal area constitute a set of simultaneous equations. Systems of equations for predicting forest growth and yield were discussed by Furnival and Wilson (1971). Sullivan and Clutter (1972) replaced $\ln B_t$ in the yield equation with the functional form of its expected value. The equation is

$$\mathcal{E}(\ln v_{ti}) = \beta_0 + \beta_1 S_i + \beta_2 A_{ti}^{-1} + \beta_3 (A_{1i}/A_{ti}) \ln B_{1i} + \beta_3\alpha_1 (1-A_{1i}/A_{ti}) + \beta_3\alpha_2 (1-A_{1i}/A_{ti})S_i$$

which is a "reduced form" of the two-equation system. The yield equation is nonlinear in its parameters. But if we set $\beta_4 = \beta_3\alpha_1$ and $\beta_5 = \beta_3\alpha_2$, we have a linear model from which we can estimate the β's and the α's.

The reduced model equation for the i^{th} plot can be written in matrix notation as

$$\mathcal{E}(y_{ti}) = x_{ti} \underline{\beta}.$$

Let $\underline{y}_t = (y_{t1} \dots y_{tn})'$ be the vector of dependent variables and $\underline{X}_t = (x'_{t1} \dots x'_{tn})'$ be the matrix of independent variables, both measured on the n plots at time t. The n equations for the observations at time t can be written as

$$\mathcal{E}(y_t) = X_t\beta.$$

It should be noted that the expected yield, $\mathcal{E}(y_t)$, depends on the time through X_t. The vector of regression parameters β is constant for all time periods.

The vectors of dependent variables can be strung out to form the composite vector $y = (y_1' \ldots y_T')'$. The T matrices of independent variables are composited to form $X = (X_1' \ldots X_T')'$. The model for the composited observations can now be written as

$$\mathcal{E}(y) = X\beta.$$

The covariance matrix of composite vector y is assumed to be

$$\Psi = \Sigma \otimes I$$

where Σ is a TxT general covariance matrix giving the variances and covariances among the T measurements on each plot. The symbol $\otimes$ denotes the direct product of two matrices. The matrix I is a nxn identity matrix. The form of the covariance matrix of y shows that the covariance matrix of the yields on each plot is assumed to be the same for all plots, and that yields on different plots are mutually independent.

Our model differs from the well-known multivariate regression model (Anderson 1958: chapter 8) which can be written

$$\mathcal{E}(y_t) = X\beta_t.$$

In this case, the design matrix X is the same for all time periods and the expected value of y_t depends on the time through β_t.

THE LIKELIHOOD FUNCTION

Let's assume that the T measurements on the n plots have a multivariate normal distribution. The likelihood function for the vector y is

$$L = (2\Pi)^{-nT/2}|\Sigma|^{-n/2}e^{-1/2Q}$$

$$\text{where } Q = (y - X\beta)' \ \Psi^{-1} \ (y - X\beta).$$

The logarithm of the likelihood function is

$$\ln L = K - n/2 \ \ln|\Sigma| - 1/2Q.$$

Since $\ln|\Sigma| = - \ln|\Sigma^{-1}|$, we can write the log likelihood as

$$\ln L = K + n/2 \ \ln|\Sigma^{-1}| - 1/2Q.$$

THE LIKELIHOOD EQUATIONS

The likelihood equations are the derivatives of the log likelihood function in respect to the unknown parameters. We found that matrix derivatives were most helpful in deriving the likelihood equations. Matrix derivatives were discussed by Dwyer (1969) and Graybill (1969). Theorems by these authors were used to derive the likelihood equations and we use matrix derivatives to solve the equations.

Likelihood equation for β

As is well known, the derivative of the log likelihood function in respect to β is

$$\frac{d \ \ln L}{d\beta} = X'\Psi^{-1}y - X'\Psi^{-1}X\beta.$$

If Σ were known, an estimator of β is

$$\hat{\beta} = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}y.$$

The vector $\hat{\beta}$ is the generalized least squares estimator of β. But Σ is not known, so Σ and β must be estimated from the data.

Denote the tu^{th} element of Σ^{-1} as σ^{tu}. The two terms in the derivative can be written

$$X'\Psi^{-1}y = \sum_{t=1}^{T} \sigma^{tt}X_t'y_t + \sum_{t=1}^{T}\sum_{u>t}^{T} \sigma^{tu}(X_t'y_u + X_u'y_t),$$

$$X'\Psi^{-1}X = \sum_{t=1}^{T} \sigma^{tt}X_t'X_t + \sum_{t=1}^{T}\sum_{u>t}^{T} \sigma^{tu}(X_t'X_u + X_u'X_t),$$

which are convenient forms for computing purposes.

The likelihood equations for elements of Σ

To derive the likelihood equations for the variances and covariances, we can differentiate with respect to elements of either Σ or Σ^{-1}. We differentiate with respect to Σ^{-1} because of the simple form of the resulting likelihood equations. The derivatives we need are

$$\frac{d \ln L}{d\sigma^{tu}} = \frac{n}{2}\frac{d \ln|\Sigma^{-1}|}{d\sigma^{tu}} - \frac{1}{2}(y - X\beta)'\ \frac{d\Sigma^{-1}}{d\sigma^{tu}} \otimes I \ \ (y - X\beta).$$

It follows from Graybill's theorem 10.8.8 that

$$\frac{d \ln|\Sigma^{-1}|}{d\sigma^{tt}} = \sigma_t^2$$

and

$$\frac{d \ln|\Sigma^{-1}|}{d\sigma^{tu}} = 2\sigma_{tu}.$$

Also, we use

$$\frac{d\ \Sigma^{-1}}{d\sigma^{tu}} = \Delta_{tu}^*$$

where Δ_{tu}^* is a TXT matrix where all the elements are 0 except the tu^{th} and ut^{th} elements which are equal to 1. It follows that the likelihood equations for the variances are

$$\frac{d \ln L}{d\sigma^{tt}} = \frac{n}{2}\sigma_t^2 - \frac{1}{2}(y - X\beta)'\ (\Delta_{tt}^* \otimes I)\ (y - X\beta)$$

$$= \frac{n}{2}\sigma_t^2 - \frac{1}{2}(y_t - X_t\beta)'(y_t - X_t\beta),$$

and that the equations for the covariances are

$$\frac{d \ln L}{d^{tu}} = n\ \sigma_{tu} - \frac{1}{2}\ (y - X\beta)'(\Delta^*_{tu} \otimes I)\ (y - X\beta)$$

$$= n\ \sigma_{tu} - (y_t - X_t\beta)'(y_u - X_u\beta).$$

More convenient forms for computing purposes can be derived.

THE MAXIMUM LIKELIHOOD ESTIMATORS

The maximum likelihood estimators of β and Σ, must satisfy the equations

$$\frac{d \ln L}{d\beta} = 0$$

$$\frac{d \ln L}{d\sigma^{tt}} = 0 \qquad \text{for } t = 1, \ldots, T$$

$$\frac{d \ln L}{d\sigma^{tu}} = 0 \qquad \text{for } t = 1, \ldots, T$$

$$u > t.$$

The likelihood equations are nonlinear and must be solved by iterative procedures. We solve the likelihood equations by the Newton-Raphson method. This means we have to have the second order derivatives of the log likelihood function.

The second derivatives in respect to β

The second derivatives of the log likelihood function in respect to β are

$$\frac{d^2 \ln L}{d\beta d\beta} = \frac{d(-1/2\ \beta'X'\Psi^{-1}X\beta)}{d\beta}$$

$$= -X'\Psi^{-1}X.$$

The partial derivatives of ln L

The partial derivatives we need are

$$\frac{d^2 \ln L}{d\sigma^{tu} d\beta} = X' \frac{d\Sigma^{-1}}{d\sigma^{tu}} \otimes I\, y - X' \frac{d\Sigma^{-1}}{d\sigma^{tu}} \otimes I\, X\beta$$

$$= X'(\Delta^*_{tu} \otimes I)y - X'(\Delta^*_{tu} \otimes I)X\beta$$

which can be written

$$\frac{d^2 \ln L}{d\sigma^{tu} d\beta} = X_t' y_t - X_t' X_t \beta \qquad \text{for } u = t,$$

$$= X_t' y_u + X_u' y_t - (X_t' X_u + X_u' X_t)\beta \qquad \text{for } u > t.$$

The results are stored in a pxT(T+1)/2 matrix, whose columns are the px1 vectors ($d^2 \ln L/d\sigma^{tu} d\beta$). The matrix formed from the vectors of partial derivatives ($d^2 \ln L/d\beta d\sigma^{tu}$) is the transpose of the matrix formed from the vectors of partial derivatives ($d^2 \ln L/d\sigma^{tu} d\beta$).

The second order derivatives in respect to elements of Σ^{-1}

To derive the second derivatives of the log likelihood function elements of Σ^{-1}, we use Graybill's theorem 10.8.10 which shows that

$$\frac{d\,\Sigma}{d\sigma^{tu}} = -\,\Sigma\, \Delta^*_{tu}\, \Sigma.$$

The derivatives are

$$\frac{d^2 \ln L}{d\sigma^{rs} d\sigma^{tt}} = -\frac{n}{2}\,(\Sigma\, \Delta^*_{rs} \Sigma)_{tt}$$

$$\frac{d^2 \ln L}{d\sigma^{rs} d\sigma^{tu}} = -\,n\,(\Sigma\, \Delta^*_{rs} \Sigma)_{tu}.$$

The notation $(X)_{tu}$ denotes the tu^{th} element of the matrix X.

SOLVING THE LIKELIHOOD EQUATIONS

To solve the likelihood equations by the Newton method, we start with the ordinary least squares estimators $\tilde{\beta}_0 = (X'X)^{-1}X'y$ and $\tilde{\sigma}_{tu0} = (y_t - X_t\tilde{\beta}_0)'(y_u - X_u\tilde{\beta}_0)$ for $t = 1, \ldots, T$ and $u \geq t$. The estimates are stored in vector, which we denote by θ. The first derivatives are evaluated at θ and stored in a vector, known as the scores, which we denote by S. The second order derivatives are evaluated at θ and stored in a matrix denoted by Δ.

The first iterated value of θ is $\theta_1 = \theta_0 - \Delta_0^{-1} S_0$. The first and second order derivatives are then evaluated at θ_1. The iteration continues until the vector of scores S approaches 0. At this point, the values of θ are accepted as the maximum likelihood estimates of the parameters.

A second method of solving the likelihood equations is the iterated Aiken procedure. The first step is to calculate the ordinary least squares estimator $\tilde{\beta}_0$. The second step is to calculate the empirical variances and covariances $(\tilde{\sigma}_{tu0})$ whose values are stored in matrix $\tilde{\Sigma}_0$. The first iterated Aiken estimator is $\tilde{\beta}_1 = (X'\tilde{\Psi}_0^{-1} X)^{-1} X'\tilde{\Psi}_0^{-1}y$. New estimates of the variances and covariances are calculated with $\tilde{\beta}_1$. The iteration continues until successive estimates of β and Σ are not appreciably different from one another. Malinvaud (1970:340) said that we can expect the iterations to converge, and that the iteration procedures seem convenient for finding the maximum likelihood estimators.

If the two methods converge, but the estimates differ, we accept as maximum likelihood estimates the set for which the determinant of $\tilde{\Sigma}$ is the smallest, because the maximum likelihood estimator $\tilde{\beta}$ minimizes $|\tilde{\Sigma}|$ considered as a function of β. In most cases, the two methods should result in the same estimates.

THE ASYMPTOTIC COVARIANCE MATRIX OF β

In maximum likelihood theory, the asymptotic covariance matrix of the estimators is the inverse of the negative of the expected value of the matrix of second derivatives of the log-likelihood function. Malinvaud (1970) showed that the asymptotic covariance matrix of $\tilde{\beta}$ is

$$\Sigma(\tilde{\beta}) = (X'\Psi^{-1}X)^{-1}.$$

The sample estimator of $\Sigma(\beta)$ is

$$\tilde{\Sigma}(\tilde{\beta}) = (X'\tilde{\Psi}^{-1}X)^{-1}.$$

VOLUME YIELD PREDICTIONS AND THEIR VARIANCES

The primary purpose of a growth and yield study is to predict the volume for various combinations of site, age, and basal area.

The predicted volume for a given vector x^* is

$$\tilde{v}^* = \exp\ (x^* \tilde{\beta}).$$

What is the variance of v^*? In most cases, there is no general formula for the variance of a nonlinear function of the random variables in terms of the covariance matrix of the random variables. However, a Taylor's expansion of the function can be used to obtain an approximate variance of the function. The first two terms of the Taylor's expansion for v^* about $\mathcal{E}(\tilde{\beta}) = \beta$ are

$$f(\tilde{\beta}) = \exp\ (x^*\beta) + \exp\ (x^*\beta)\ x^*\ (\tilde{\beta}-\beta).$$

The variance of $f(\tilde{\beta})$ is

$$\sigma^2[f(\tilde{\beta})] = [\exp\ (x^*\beta)]^2\ x^*\ \Sigma(\tilde{\beta})\ x^{*\prime}$$

which is known as the approximate variance of v^*. A sample estimator of $\sigma^2[f(\beta)]$ is

$$s^2[f(\tilde{\beta})] = \tilde{v}^{*2}\ x^*\ \tilde{\Sigma}(\tilde{\beta})\ x^{*\prime}.$$

TESTS OF LINEAR HYPOTHESIS AND THE LIKELIHOOD RATIO STATISTIC

An important statistic in maximum likelihood analysis is the value of the likelihood function evaluated at the maximum likelihood estimators. Many statistical textbooks discuss the likelihood ratio test for testing a q dimensional linear hypothesis about the regression parameter. The likelihood ratio is

$$\lambda = L(\tilde{\omega})/L(\tilde{\Omega})$$

where $L(\tilde{\Omega})$ = value of the likelihood statistic under the full model,

and $L(\tilde{\omega})$ = value of the likelihood statistic under the restricted model.

The asymptotic distribution of the statistic $-2\ln\lambda$ is χ^2 distribution with q degrees of freedom.

YIELD PREDICTIONS FOR A NEW PLOT

Suppose we observe a "new plot" and want to predict the yield on that plot at some future date. We denote the current measurements on the new plot as (y_1°, x_1°). What is the expected plot volume at time t in the future? In other words, what is the expected volume y_t° given the present volume y_1°?

We assumed that the multiple measurements on each plot have a multivariate normal distribution. It is well known that the conditional distribution of y_t given y_1 is also normally distributed with a mean

$$\mathcal{E}(y_t^\circ/y_1^\circ) = \mathcal{E}(y_t^\circ) + \sigma_{1t}/\sigma_1^2[y_1^\circ - \mathcal{E}(y_1^\circ)]$$

where σ_{1t} is the covariance between observations at time 1 and time t, and σ_t^2 is the variance at time t.

The expected value of y_1° is $x_1^\circ\,\beta$ and the future expected volume is $x_t^\circ\,\beta$.

The sample estimator is

$$\tilde{y}_t^\circ = x_t^\circ\,\tilde{\beta} + \tilde{s}_{1t}/\tilde{s}_1^2\,(y_1^\circ - x_1^\circ\,\tilde{\beta}).$$

Our estimator of the future volume per acre at time t is

$$\tilde{v}_t^\circ = \exp(\tilde{y}_t^\circ).$$

EXAMPLE

The example is based on part of a growth and yield study of managed hardwood stands being made by the Northeastern Forest Experiment Station. The analysis is presented only to illustrate the statistical methodology. Analysis of all of the data will be reported elsewhere. The data consists of three measurements on 64 permanent plots. The plots were first measured in 1961 (shortly after thinning), and again in 1967 and 1971. For convenience, the volumes, basal areas, and plot ages were adjusted to 5-year intervals between measurements.

The analysis was made with a computer program written in Fortran IV for use on an IBM 370 series. The program is available from the authors.

Estimates of regression coefficients and standard errors

The estimates of the regression coefficients, standard errors, and t ratios are

Variable	Coefficient	Standard error	t ratio
Intercept	3.1872	0.0596	53.48
S	0.0096	0.0008	12.00
A_t^{-1}	-23.4236	0.7140	-32.81
$(A_1/A_t)\ln B_1$	1.0094	0.0085	118.75
$1-A_1/A_t$	5.4694	1.2440	4.40
$(1-A_1/A_t)$ S	0.0075	0.0184	0.41.

The standard errors are the square roots of the diagonal elements of $\tilde{\Sigma}(\tilde{\beta}). = (X'\tilde{\Psi}^{-1}X)^{-1}$. The large t values show that each of the regression coefficients, except the last, are statistically significant from zero.

The estimates presented are based on 15 iterations. It should be noted that the quadratic form (Q) at the maximum likelihood solution is equal to n x T, which, in our example, equals 192. After 14 iterations, the value of $\tilde{Q}$ was 191.999384. On the 15th iteration, $\tilde{Q}$ was equal to 192.000000.

To estimate the coefficients of the basal-area equation, we note that $\beta_4 = \beta_3\alpha_1$ and $\beta_5 = \beta_3\alpha_2$. The estimates are:

$$\tilde{\alpha}_1 = \tilde{\beta}_4/\tilde{\beta}_3 = 5.4187$$
$$\tilde{\alpha}_2 = \tilde{\beta}_5\ \tilde{\beta}_3 = 0.0074.$$

If needed, the approximate variance-covariance matrix of $(\tilde{\alpha}_1, \tilde{\alpha}_2)$ can be found from the first two terms of the Taylor expansion for $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$.

Estimates of variances and covariances among the measurements

The sample variance-covariance matrix of the natural logarithm of the volume per acre is

$$\tilde{\Sigma} = \begin{bmatrix} 0.00192 & 0.00193 & 0.00404 \\ 0.00193 & 0.01103 & 0.01559 \\ 0.00404 & 0.01559 & 0.02564 \end{bmatrix}.$$

The correlation matrix is

$$r = \begin{bmatrix} 1.000 & 0.419 & 0.575 \\ 0.419 & 1.000 & 0.927 \\ 0.575 & 0.927 & 1.000 \end{bmatrix}.$$

We find that the variance at time 1 is small compared to times 2 and 3. The covariance between measurements are all positive. The correlation $r(2,3) = 0.927$ is especially high.

The large differences between variances and the high correlation among measurements suggest that the covariance matrix of the ordinary least squares estimator would have a large bias. The bias of the variance of ordinary least squares estimators based on correlated data is discussed by Sullivan and Reynolds (1977).

Hypothesis testing

One hypothesis of interest is that the volume is not a function of the expected basal area. The hypothesis in terms of the parameters of the model is

$$H_0 : \beta_3 = \beta_4 = \beta_5 = 0.$$

The model restricted by H_0 is

$$\mathcal{E}(y_{ti}) = \beta_0 + \beta_1 S_i + \beta_2 A_{ti}^2.$$

To test the hypothesis, we have to estimate the parameters of the restricted model and compute the natural logarithm of likelihood function for the restricted model, which is $\ln L(\tilde{\omega}) = 132.28$. Also, we need the natural logarithm of the likelihood function of the full model, which is $\ln L(\tilde{\Omega}) = 269.80$. The likelihood ratio statistic is $-2 \ln L(\tilde{\omega}) - \ln L(\tilde{\Omega})$ which is equal to 275.04.

The test statistic is distributed asymptotically as χ^2 with 3 degrees of freedom. The critical value of χ^2 is 7.81. The expected basal area is a highly significant variable for predicted volume.

Tests of the regression coefficients for site and the inverse of age could also be done with the likelihood ratio test. This is not necessary because we already have the standard errors and t values for each regression coefficient.

Growth projection given initial plot volume

To project the yield given the plot volume, we used data from one of the study plots. The plot has a site index of 67. The initial plot age was 47 years, and the basal area was 50.58 ft^2/acre. The expected initial volume (which is calculated from the volume equation) is 1469 ft^3/acre. The actual plot volume was 1489 ft^3/acre, which is 1.3 percent higher than the expected volume.

What is the 10-year yield projection conditional on the present plot volume? We need the sample values $x_1^\circ b = 7.2923$, $x_3^\circ b = 7.7336$, $y_1 = 7.3059$, and $s_{13}/s_1^2 = 2.1042$. The sample value of $\tilde{y}_t^\circ$ is 7.7621 and the projected yield for the plot is $\tilde{v}_3^\circ = \exp(7.7621) = 2350.1$ ft^3/acre. The expected volume is

$$\mathcal{E}(v_3) = \exp(7.7336) = 2283.8.$$

The ratio of the projected volume to the expected volume is 1.0290. In other words, the plot that started out with 1.3 percent more volume than predicted is expected--in 10 years--to have 2.9 percent more volume than the expected future volume.

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LONG-TERM YIELD FORECASTING MODELS; VALIDATION AND ITERATIVE ESTIMATION

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SUMMARY

Yield forecasting models, whether based on single tree or whole stand, commonly predict total volume or total volume per dbh class, by using stand variables together with a class or level of treatment; the tables or forecasts tend to be regarded as permanent and not subject to change. Validation, if conducted at all, is usually done once, and in terms of total yield, even in the case of single tree simulation models.

Examples of problems are discussed. It is proposed that a full forecasting system includes i) basic, conventional, yield model, ii) operational treatment effect sampling and feedback model, iii) climatic adjustment model, and iv) conversion model to provide log supply and product yield forecasts. The importance of validation on a continuing basis, coupled with monitoring for item ii) is emphasized. Single tree models should be validated against individual tree data rather than aggregate totals.

INTRODUCTION

For the purpose of this discussion we will consider a generalized yield model of the form

$$y = f_1(Sp, A, Si; S; Pt) + f_2 (Ot) + f_3 (Cl) + f_4 (Nc) + \varepsilon$$

where y = volume yield

Sp = species (and provenance), A = age, Si = site index or site variable

S = stocking density, or measure of crowding

Pc = prescribed treatment regime

Ot = Operational treatment regime

Cl = Climatic fluctuation

Nc = Non-timber management consideration (environment, etc.)

ε = random error

Conventionally, yield forecasting models include Sp, A, Si, S and Pt, and ε. Part of the concern of this paper is with the other terms in the yield model, and part is with quality control and validation procedures which appear to be needed.

THE PROBLEM

Until the advent of the computer, it was common to treat yield tables as "hewn in stone" ultimate answers which were not subject to change or challenge. Today yield tables, equations, and various simulation models are being produced in some profusion and each of these tends to be viewed as a sufficient and adequate source of yield information by itself. With few exceptions, these models are reasonably by the "fi" portion of the generalized model noted above.

Parenthetically, the term "crowding" rather than "competition" is used, as being more appropriate to the measures now being employed. Crowding is a state of condition, quite well measured by distances to neighboring trees, etc. Competition involves processes, of uptake, assimilation, etc. and constraints on the rates of these processes, which clearly are measured only in a very indirect way, if at all, by distances to neighbors and so on.

Returning to the main point, conventional models do not include the other elements, Ot, Cl, and Nc, and this deserves attention.

Two main objectives are sought through yield forecasting tools namely:

i) Estimate overall yield for whole management units (e.g. 50,000 - 100,000 hectares)

ii) Assist in obtaining an understanding of growth processes.

The function of the various forms of yield model and of the functions of Ot, Cl, and Nc can be examined relative to some operational situations.

Example 1. The "trend towards normality" has been accepted for years as a "fact" in using yield tables both in North America and Europe. However, most tables constructed in the 1910-1940 period were made from point data on volume at given age, rather than from historical trends of volume over a span of age. It was not until the more recent work (e.g. Pienaar and Turnbull, 1973, King, 1970), based on permanent plot growth records, that it appeared in some species (e.g. Pinus elliottii Engelmann) the trends for different initial stocking levels do converge, while for others (e.g. Pseudotsuga menziesii (Mirb.) Franco) they do not converge. The latter is also being discovered in Pinus taeda L. in Georgia (Pienaar, 1977). However, it is necessary to have not only "real growth series" data, but to have it for lengthy periods of growth, if one is to discover whether growth trends converge or not. How then can we expect yield forecasting models based only on short-term data to provide permanent and unalterable estimates? Periodic testing and validation against independent field data is needed.

Example 2. Yield tables for plantations of tropical hardwood species such as Albizzia falcata Becker and Gmelina arobrea show yields such as 400 m^3/ha at age eight years. These have been derived from carefully tended and protected test plots. In large-scale plantations, experience has shown that because of rough terrain, old stumps, insects, wildlife, and other influences, the actual yields are at best some 80 percent of the yield table forecasts. Clearly, some quality control sampling or growth monitoring system is needed to provide adjustment of forecasts to reflect operational conditions.

Example 3. Results of a regional nitrogen fertilization study (Turnbull and Peterson, 1976) have established that when 200 kg/ha of elemental nitrogen, in the form of synthetic urea, is applied by hand in experimental plots, then volume growth rate will increase by some 25 percent. However, in operational application some 40,000 hectares of Douglas-fir forest receives what is prescribed as 200 kg/ha of urea applied by helicopter. Limited field sampling has demonstrated wide variation in application rate between locations within a stand. The same is true in the case of thinning or spacing over extensive forest areas by contractors; the exact intensity of thinning and resulting density of remaining stand will vary from place to place and may well differ on the average from the prescribed thinning intensity. Clearly, one should not rely solely on forecasts given by experimental treatments made under experimental conditions. Some form of quality control sampling or growth monitoring system is needed to adjust estimates so that they include the effect of operational treatments to the extent that they differ from exact prescribed treatments.

Example 4. Single tree distance dependent growth simulator make use of distance measures to provide indices of "competition" or crowding.

The author participated in a study of a stem mapped Douglas-fir forest stand in which individual tree dbh was estimated by regression on measures of distance to neighboring trees; the objective of the study was to provide a first stage selection of plus-trees according to the magnitude of positive deviations from regression. The correlation between dbh and distance measures was relatively low ($R^2 = 0.1$ to 0.3). The point of the example is that the forest geneticist's view of "competition" was that trees differ in their capacity to grow under given conditions of crowding and that this difference is to some degree genotypic. This is only one example, but a review of published results of single tree distance dependent modelling shows that if such variables as dbh and height are already present, the distance measures increase correlation with growth rate, as the independent variable, by only 5 - 10 percent typically giving an $R^2 = 0.7$ without distance measures and $R^2 = 0.75$ with distance measures. This may be because such variables as dbh and height already include measures of the historical effect of crowding, together with genotype, so that distance measures alone have little more to contribute statistically.

Single tree distance dependent models clearly have a vast potential to provide a link between growth forecasting and physiological knowledge of growth processes. However, the present method of "validation" of forecasts by these models is to compare total volume predicted with total volume observed. This will never provide a direct check on the simulation of the individual trees within the aggregate, and indeed may obscure poor single tree estimates by aggregation. Validation in terms of individual trees seems to be obviously necessary if the effectiveness of distance measures is to be assessed fully, and these worthwhile models further developed.

The above discussion has focused primarily on need for validation generally, and the "operational treatment" term, Ot, in the generalized model. Because of the importance of increment core samples in the national forest survey in that country, there has been extensive study of the climatic term, Cl, in Sweden, some of which has been reported by Johsson and by Jonsson and Materu at this meeting; in the paper by Boyce, some aspects of dealing with the non-member terms, Nt, were discussed.

A further general observation is that commonly the growth and yield forecasting models provide estimates in terms of total volume/ha or total volume per dbh class/ha. Increasingly, it is being recognized that yield forecasts are needed in terms of volume composition according to diameter class of logs, log grade, product class or assortment. The conversion from stocktable by dbh to log stocktable can readily be accomplished by use of taper curves, while specific product yield estimates can be given by linked simulators. Examples of this are in the paper by Row and Norcross at this meeting and in the Tropical Forest Utilization System (1977) papers.

CONCLUSIONS

In general, growth and yield forecasting models are based on stand, or tree, variables and prescribed treatments. Rarely, if ever, is allowance made directly for the effect of difference between operation condtions and and experimental conditions, and between operational treatment and prescribed treatment. Most growth and yield models, aside from some in Sweden, make no provision for the effect of climatic fluctuation. Non-timber growing considerations are frequently omitted from the models also. In addition, growth and yield models tend to be regarded as being somewhat permanent and not subject to change.

The author proposes that yield models should be recognized as elements in forecasting systems which contain, generally,

i) "Basic" yield model based on stand parameters and prescribed treatment(s).

ii) Sampling and feedback model to adjust estimates due to fluctuation in operational treatment and conditions.

iii) Climatic model, if necessary, to adjust growth input data according to departures from average trends of climate.

iv) Conversion model, to render yield estimates in terms of log supply or product yield.

Since the elaboration to include non-timber production considerations is too broad to generalize here, we will simply note the potential importance of this here.

The author further proposes that much more attention should be given not only to validation, but to design of adequate procedures. Examples of validation were given in a number of papers at this meeting. The single most important characteristic of validation data is that it is separate and independent from the data used to estimate the parameters of the model being validated. The special sample described by Clutter is noteworthy. However, it appears that such a sample could well be instituted in some permanent form to supply data for item ii) above. An excellent example of such a system is given by Stage (1973). One of the points that this highlights is the need for less costly but sufficiently reliable sampling systems for monitoring operational treatments. It is possible that in the future there will be an increase in emphasis on monitoring and iterative improvement of yield forecasts, especially in view of the modelling complications created by regimes of successive and superimposed treatment.

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A MODEL TO GENERATE STAND STRATEGIES FOR INTENSIVELY MANAGED RADIATA PINE PLANTATIONS*

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SUMMARY

A stand model to generate alternative management strategies for a long-range planning model has been constructed for intensively managed radiata pine plantations in Victoria, Australia. The model can be used to evaluate different intensities of silviculture as well as to provide estimates of future production.

INTRODUCTION

Australian Paper Manufacturers, through its subsidiary A.P.M. Forests Pty. Ltd., is introducing heavy fertilization and chemical weed control into its intensively managed plantations in Victoria, Australia (Cromer et al., 1977). Company managers are vitally interested in such questions as whether intensive establishment techniques should be applied to all sites, which established stands should be fertilized, whether thinning practices should be different for high-yield stands, and what the trade-offs are among, for example, buying more land, increased fertilization, and investing in tree genetics research.

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*This paper is a slightly condensed version of an article to appear in Australian Forestry shortly.

The company aims to manage its plantations in the most economically efficient manner, to meet the raw material needs for its manufacture of pulp and paper, saw timber, and particle board. To plan mill expansions and to evaluate the profitability of more intensive silviculture, a new long-term (25-year) planning model was needed for the company's 40,000 hectares of radiata pine (*Pinus radiata* D. Don) plantations.

Following a review of possible planning models, the development team decided that the most appropriate type would be a linear programming (LP) formulation along the lines of Clutter's (1968) MAX-MILLION model. This formulation had several advantages:

(a) it has proved itself in similar industrial forest corporations in southern U.S.A. (Ware and Clutter, 1971);

(b) it has been the basis of many other planning models, for instance, Timber RAM used by the U.S. Forest Service (Novan, 1971), MASH developed by the Victorian Forests Commission (Gibson et al., 1974), and RADHOP developed by the N.S.W. Forestry Commission; and

(c) it used existing standard LP computer programs for optimization.

This type of formulation has four phases:

1. A stand generation phase which generates simulated alternative management strategies for each forest stand.

2. A matrix generation phase in which the output from the first phase together with information or constraints, such as supply levels to be attained, areas of each stand, and budgets, are put in a form suitable for input to a standard LP computer package.

3. An optimization phase in which the strategies which satisfy all constraints and optimize the objective are selected.

4. A report writing phase in which the optimal solution is tabulated and reported in a form suitable for managers to assimilate.

The purpose of the first phase dealt with in this paper is to generate a wide range of feasible alternative management strategies for each stand over the planning horizon so that the best combination can be chosen in the optimization phase.

PREVIOUS RELEVANT STAND MODELS

Models to simulate the growth of Pinus radiata in Australia have been developed by A.P.M. Forests (Hall, 1974), the Forests Commission of Victoria (Gibson et al., 1969), and the N.S.W. Forestry Commission. None of these allow for the evaluation of intensive silviculture as a management alternative to conventional practices.

Few of the many stand simulators developed in North America model the effects of intensive silviculture. Exceptions are those of Hegyi (1974), Ek and Monserud (1974), and Daniels and Burkhart (1975), all of which include fertilization as a management option. Thus, although several models exemplify the general structure, no single model incorporates weed control, tree breeding, thinning, fertilizing, and interactions as is required for modeling the management of radiata pine under alternative intensive regimes.

DATA BASE

Inventory and growth plots are installed in each stand on a grid system at 9 to 10 years of age. Records of the measurements of these, and research, plots as well as the areas of the stands are maintained on an information system (Dargavel et al., 1975).

The 1500 small stands were aggregated into 188 "coupes" each of which was relatively homogeneous with respect to age class, site, location, and slope class. For present purposes, we will define coupes which have plot data as "old" and those without as "young." Each coupe was classified into 1 of 32 "Silvicultural Types" which were defined on geology, soil type, and rainfall. These were aggregated into 6 groups reflecting differences in fertilization and weed control requirements.

The latest measurements of 4600 inventory plots were forecast to the start of the projection period in order to estimate the mean basal area, predominant height, stocking, and volume of each of the 83 old coupes. An additional 105 coupes were created for unplanted areas and for young areas in which plots had not yet been established. Only estimates of initial stocking and likely site index were available for the latter group.

SIMULATION OF GROWTH

This paper reports the methods used to simulate growth of stands on flat to steep country. Modifications were made to suit areas of very steep country.

Old Stands

Models were developed to describe the growth of the old stands from considerations of the biological relationships among the variables and examination of representative data. Where these resulted in non-linear forms, they were linearized by mathematical transformations or approximations and the coefficients estimated by unweighted or weighted linear regression using growth plot measurement data from the information system. Extensive error analysis using independent inventory plot data was conducted on all models and their performance within the range of these data was considered satisfactory.

For annual growth projection, predominant height is estimated by

$$H_t = \ln g(t) \cdot H_{t-1} / \ln g(t-1) \tag{1}$$

where

$g(t) = 0.06012\ A_t^{1.116} + 1$,

H_t = estimated predominant height (in meters) at time t,

H_{t-1} = estimated predominant height (in meters) at time t-1, and

A_t = age at time t.

Stand basal area is projected by a basal area increment model:

$$I = \exp(0.9194 - 2.032/B_{t-1} + 0.008298\ B_{t-1} - 0.03545\ A + 0.02132\ S) \cdot \Delta t \tag{2}$$

where

I = stand basal area increment (m^2/ha) between times t-1 and t,

B_{t-1} = stand basal area at time t-1 (m^2/ha),

A = average age during projection interval, and

S = site index (estimated or actual predominant height at age 15).

Thus, estimated basal area at time t is given by

$$B_t = B_{t-1} + I \tag{3}$$

unless a thinning is called for, in which case B_t becomes the prescribed residual basal area.

Stand volume is estimated from

$$V_t = -19.83 + 0.499\,B_t + 1.947\,H_t + 0.2868\,B_t H_t - 1.264\,S \tag{4}$$

where
V_t = estimated stand volume (m^3/ha).

The stand volume is reduced by one-third in the first thinning operation (a row thinning), and estimated for subsequent thinning by

$$V_r = 0.2982\,B_r H_t \tag{5}$$

where

V_r = estimated volume removed in thinning at time t; and

B_r = basal area removed in thinning, i.e., the difference between B_t and the prescribed residual basal area.

Stocking after thinning is estimated by a regression model:

$$N_a = 0.9143(N_b \cdot B_a/B_b) + 0.418\,A \tag{6}$$

where

N_a = estimated stocking after thinning (number of trees/ha),

N_b = stocking before thinning,

B_a = basal area after thinning, and

B_b = basal area before thinning

except for first thinnings, when the stocking is reduced by one-third.

Very few data were available on mortality trends since existing stands have been frequently thinned and little natural mortality has occurred. Mortality was erratic on the few unthinned growth plots and no significant relationship was found between mortality and other stand variables. However, inventory plot data from unthinned stands in nearby state plantations suggested that an upper limit on stand basal area could be empirically defined as a curvilinear increasing function of predominant height. It was assumed that mortality would occur when the limiting basal area was reached.

Records of volume losses due to past cataclysms (fire, wind-throw, hailstorms) were used to estimate a long-term average loss in yield due to such events. All yields were reduced by a factor derived from this to account for such probable future losses.

Young and Unplanted Stands

For young and unplanted stands the only existing data were estimates of site index (from comparison with nearby stands on similar soils, etc.) and the planned planting stocking. From a comparison of actual stocking just prior to thinning with planned initial stocking and consideration of data from counts of planting survival, a reduction factor was derived to estimate stocking at age 9.7 (a convenient reference age) from planned initial density. This varied with site preparation and weedicide regimes. From these data, stand basal area at age 9.7 ($B_{9.7}$) is estimated by

$$B_{9.7} = 19.48 + 1.377\ S + 0.007938\ N_{9.7} \qquad (7)$$

where

$N_{9.7}$ = estimated stocking at age 9.7

Predominant height at age 9.7 ($H_{9.7}$) is found by Eqn. (1) and stand volume from Eqn. (4).

EFFECTS OF INTENSIVE SILVICULTURE

Fertilization

Research into the effects on growth of fertilization with N, P, and K had been conducted locally for two decades. Many experiments showed how much initial growth could be increased by fertilizers applied at planting. Additionally, 22 independent experiments showed growth responses for fertilizers applied at ages up to 20 years. Most fertilizer applications had been made before 10 years of age and the main experiments had been measured up to 7 years after application. The experiments did not cover every Silvicultural Type but they did extend to each group of types.

As the ages of application and response measurement differed considerably between experiments, the plot data had to be transformed to a common base. In each main experiment the final measurements of height and basal area were first corrected for initial differences using an analysis of covariance procedure. The mean annual responses to fertilization were then expressed for all experiments as percentages of the values of the unfertilized control plots at 10 years of age.

When the stand parameters were plotted against time, most of the absolute differences in values between fertilized and control plots showed an increasing linear relationship with time following applications and none showed any pronounced reduction. From this evidence, the fertilizer effect was modeled as a constant annual increase to the annual height and basal area increments as estimated by Eqns. (1) and (2). From evidence elsewhere it was considered that these constant annual increases would eventually become smaller or cease. This was modeled by setting the annual increases to zero after a defined duration of response was reached. The effect of this on stand height is illustrated in Figure 1.

All the experimental data were used, together with some soil analysis data and results of surveys of the nutrient status of many of the plantations that had been obtained by foliar sampling, to define fertilization prescriptions. These specified the type of fertilizer, rate, method, cost, and age at application of each treatment for each Silvicultural Type. They included rates up to 1 ton per hectare at establishment at costs up to $A.220 per hectare and covered both young and mature stands.

The expected growth responses for each group were estimated from the main experiments and ranged from 1 percent in height increase and 6 percent in basal area 6 years after application to old stands on loamy soils, to 12 percent in height and 67 percent in basal area 10 years after application at planting on sandy soils. The expected durations of the responses were derived mainly from evidence elsewhere and a good deal of extrapolation. Estimates were made for each Silvicultural Type and ranged from 7 to 25 years.

The responses were reduced by 25 to 50 percent (depending on topography) to account for the difference between experimental and operating conditions. The reduction factors were derived mainly from a review of agricultural experience.

Weedicide Effects

Cromer et al. (1977) have demonstrated the necessity of weedicides at establishment to gain optimum value from fertilizer application. Thus, the effect of weedicides was included in the fertilizer effect. Additionally,

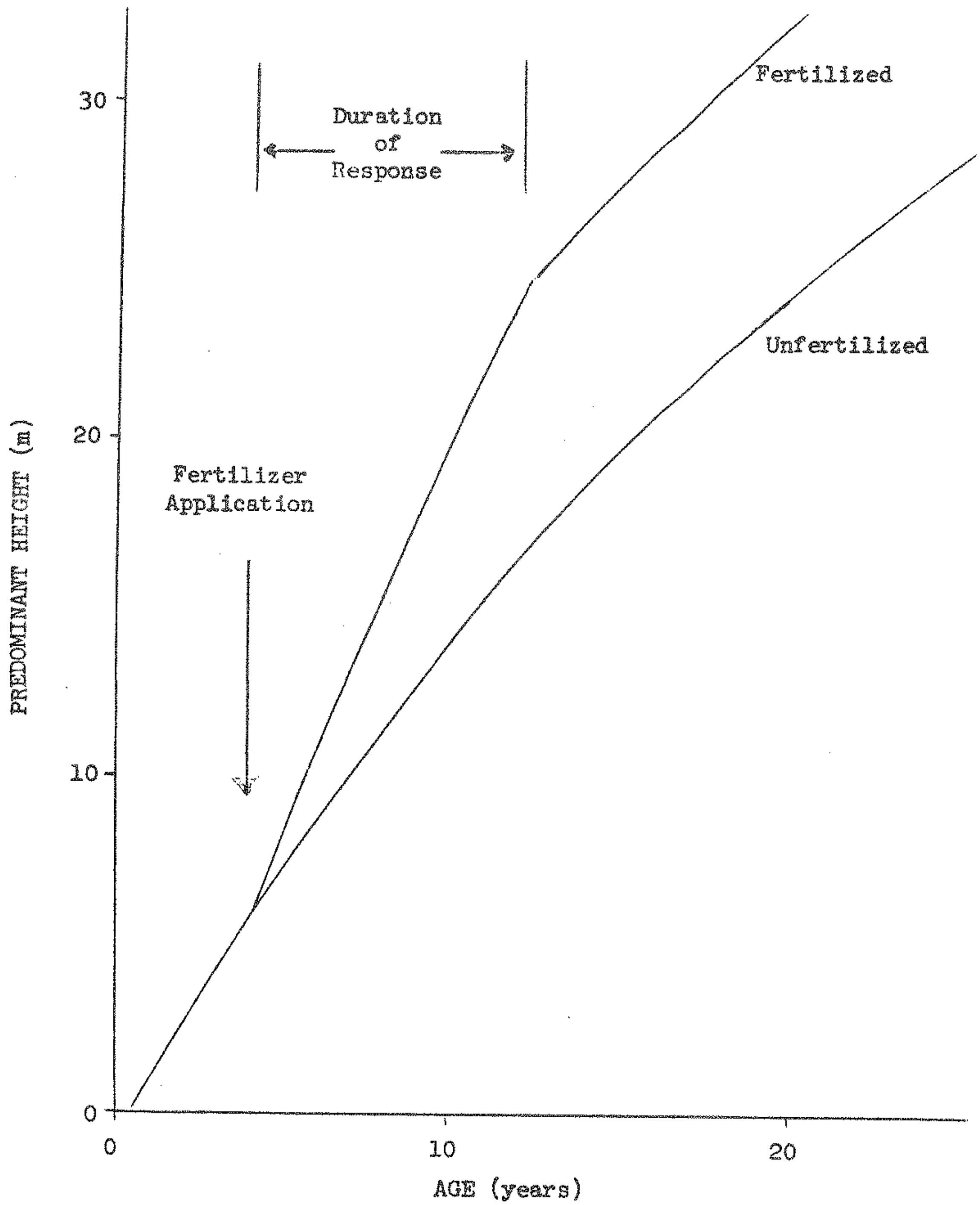

Figure 1. Predominant height-age curve for one site index value showing annual growth increased by 7 percent of the height at age 10 years for a duration of 8 years following fertilization at age 4 years.

weedicides improve planting survival so the estimated stocking at age 9.7 was increased by an empirically derived factor in regimes where weedicides were included. The type, rate, and cost of weedicide were prescribed for each Silvicultural Type.

Tree Breeding Effects

The tree breeding effect was handled in the simulator in the same way as fertilizers, the estimated constant annual increase in growth being added to the increments as estimated by Eqns. (1) and (2). The increase which lasts for the rotation, depends upon the progress of the tree breeding program. Experimental gains expected ranged from 5 percent improvement in stand basal area at age 10 for 1970 plantings to 23 percent for 1990 plantings. Corresponding values for predominant height were 2 and 7 percent. These values were reduced by 20 percent to allow for the difference between experimental and routine results.

Interaction Between Fertilizer and Tree Breeding Effects

Little experimental evidence existed on the nature of any possible interaction between the increases in growth due to fertilizing and those due to using genetically improved stock. It was assumed that suitable trees would be bred for the fertilizer regimes envisaged. The boosts to growth from fertilization and breeding were therefore modeled as additive.

Second Rotation Effects

Evidence from South Australia and elsewhere suggests that a drop in productivity in second rotation plantations on some soil types can be expected unless establishment techniques are improved. The only second rotations in A.P.M. Forests' estate are the result of recent premature salvage clear-felling. Good growth in these young stands shows that improved establishment practices more than compensated for nutritional losses or other factors which might contribute to decline. Since the gains from improved establishment techniques are modeled separately, a productivity loss for the second rotation is included in the simulator. The value of this percentage reduction in site index varied with Silvicultural Types from 0 to 15.

DEFINITION OF THINNING STRATEGIES

An important management option in plantation silviculture is the type and timing of thinnings. Different thinning types were defined by the minimum economic yield and upper and lower bounds on stand basal area. Data from experimental thinning trials were examined to determine stand basal areas at which volume increment was maximum. It was observed that this limiting stand density increased with age and site index, and since predominant height is related to both of these, it was assumed that a relationship between limiting stand density and predominant height would be independent of age and site.

A linear increasing function was derived by regression analysis to describe this relationship and then zones in which volume increment was a defined percentage of maximum increment were defined by similar linear increasing functions. Thus, the thinning strategy can be described by the values of the regression coefficients, a, b, c, and d, in the equations

$$B_b = a + bH \tag{9}$$

and

$$B_a = c + dH \tag{10}$$

where

B_b = stand basal area before thinning,

B_a = stand basal area after thinning, and

H = stand predominant height.

In the simulator, stands are thinned if the basal area exceeds B_b, to the residual basal area given by Eqn. (10), provided that the volume produced is more than the minimum yield (for most plantations, 35 m^3/ha).

MODELING THE ECONOMIC FACTORS

Unit Costs

Since the objective for optimization in the planning model was to maximize the contribution of the pine plantations' operations to corporate profit, it was necessary to incorporate into the simulator all costs which were dependent on the treatment given to individual coupes.

Costs incurred in the management of pine plantations are currently assigned to one of almost a hundred jobs in each of seven cost centers.

Cost records over the past 10 years were converted to present day values and projections made of future unit costs. After review and, where necessary, modification by plantation foresters, these estimates were incorporated into the simulator.

The major operating costs were logging and haulage to the Maryvale pulp mill. These were estimated for each operation in each coupe from standard schedules of rates for cutting, extraction, and haulage, according to topography and distance from mill.

Terminal Value

The asset value of a coupe at the end of the planning period is included in the contribution to profit calculated for each alternative. The method of valuation of this resource depends on what the likely future of the forest is assumed to be. One approach is to use the liquidation value which implies that all the timber will be removed immediately and the land converted to an alternate use. This value is easy to compute but the assumption is unlikely. The other extreme is to compute the discounted value of an infinite series of rotations, assuming that the forest is managed in the same way forever. This is a more tenable assumption but implies a longevity of stewardship more appropriate to public than private forest ownership.

We have adopted a position between these extremes; viz., that its value is as if the forest were to be sold as a going concern. Thus, mature stands (defined as older than 20 years) are valued at liquidation value, but immature stands are kept to maturity and their value then discounted back to the planning horizon.

Contribution to Profit

The contribution to profit is computed as the algebraic sum of the 25-year series of after-tax returns, costs, and terminal value, all discounted to the present at 9 percent interest rate. The returns used are notional interval transfer prices reflecting profit contribution of wood supplied for pulp manufacture.

CONSTRUCTING THE SIMULATOR

The computer program to link together the various models and assumptions described above and to produce outputs for the optimization model was written in PL/I for an IBM 370/135 computer. A major part of the programming effort revolved around the necessity to limit the number of strategies to about 10,000, or about 50 per coupe, to ensure that the LP problem could be solved in a reasonable length of computer time.

Strategies tested included 2 levels of intensity of establishment and 10 different years for planting unplanted stands, 3 different times for fertilizing established stands, 3 thinning levels, and 10 ages for clear-felling. It is expected that in subsequent runs some strategy levels which rarely occur will be eliminated and others (e.g., additional thinning strategies) will be added.

Some strategies were not permitted for certain coupes; e.g., intensive fertilization at establishment was not permitted until the estimated year that research results would be available for that Silvicultural Type. All stands were re-established 2 years after clear-felling.

The major output is a file of data passed to the matrix generation phase of the planning model. This file contains information identifying the coupe and the strategy generated, the string of annual costs and wood volumes produced by thinning and clear-felling, the terminal value, and the contribution to profit. Nominated strategies can be printed out to give the annual stand statistics, a breakdown of costs by jobs, and other intermediate calculations.

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MARKOV MODELS FOR STAND PROJECTION

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1. INTRODUCTION

A Markov model for stand projection of considerable generality has been presented by Peden, Williams, and Frayer (1973). The list of stand variables they considered for the model is the following:

$\omega_{h,i}$ is the number of live trees in class i at the beginning of year h.

$W_{h,i}$ is the number of live trees in class i in year h which increased in size to class $i+1$ during year h.

$\tilde{W}_{h,i}$ is the number of live trees in class i in year h which did not increase in size to class $i+1$ during year h.

$\chi_{h,i}$ is the cumulative number of live trees which in year 0 up to h were cut while in class i.

$X_{h,i}$ is the number of live trees in class i which during year h both increased to class $i+1$ and were cut.

$\tilde{X}_{h,i}$ is the number of live trees in class i which during year h did not increase in size to class $i+1$ and were cut.

$\psi_{h,i}$ is the number of standing dead trees in class i in year h.

$Y_{h,i}$ is the number of live trees in class i which during year h increased in size to class $i+1$, then died and remained uncut.

$\tilde{Y}_{h,i}$ is the number of live trees in class i which during year h did not increase in size to class $i+1$, died and remained uncut.

$\hat{Y}_{h,i}$ is the number of standing dead trees in class i which during year h remained uncut.

$\hat{\tilde{Y}}_{h,i}$ is the number of standing dead trees in class i which during year h were cut.

$\zeta_{h,i}$ is the cumulative number of dead trees which in years 0 up to h were cut while in class i.

$Z_{h,i}$ is the number of live trees in class i which during year h increased in size to class $i + 1$, then died and were cut.

$\tilde{Z}_{h,i}$ is the number of live trees in class i which during year h did not increase in size to class $i + 1$, died and were cut.

Subject to the restriction that a process be first order in both time and diameter class and time homogeneous, they successfully handled all of these variables by incorporating in the model: i) eight transition paths for any live tree; ii) two transition paths for any dead tree; and iii) any deterministic mode of ingrowth of new trees; and they derived; iv) closed-form expressions for each diameter class of the expectations of and transition probabilities associated with the yearly numbers of live trees, cumulative numbers of live trees cut, yearly numbers of standing dead trees and cumulative numbers of dead trees cut and v) the variances, covariances and approximate distributions of the numbers of live trees in the various diameter classes, the cumulative harvest counts and the yearly harvest counts. These results were applied to the two quite different problems of tolerance-interval stand prediction and the determination of effects of estimation errors on stand prediction. Now it would appear worthwhile to reexamine the class of models of which the one by Peden et. al. is a sophisticated example to determine how they could be modified and to make an initial assessment at least of how useful one might expect each modification to be for the improvement of stand-projection techniques.

2. MODIFICATIONS

The following five types of modifications will be considered:

1. Refinements of time and diameter-class scales.
2. Increases from first-to higher-order models in time and diameter-class dimensions.
3. Relaxation of time-homogeneity restrictions on transition probabilities.

4. Simplification of multi-step transition probabilities by modelling of single-step probabilities.

5. Substitution of stochastic ingrowth functions for deterministic functions.

Although these will be examined separately, it should be clear that they are related. In particular 1., 2., 3., and 2. and 5. are closely linked.

There is some trade-off involved in the choice of time and diameter-class units if a first-order model is to provide an adequate representation of stand dynamics. Clearly, whenever a satisfactory fit can be made, there is no need to refine the scales. However, if the scales must be refined, it will be very unlikely that time homogeneity of transition probabilities can be maintained. With highly refined scales, such as continuous ones, transition probabilities should be periodic functions of time with one cycle each year. A model with this feature which also incorporates a realistic set of transition paths would be very interesting but difficult to investigate because the results it would yield could not be obtained from a simple, discrete, time-homogeneous model by shortening scale increments or by limiting arguments. It would require an entirely new approach to the problem.

Higher-order models in which changes in one period depend on the state of the stand in several preceding periods or changes span several diameter classes in one period are attractive alternatives when scales cannot be balanced to obtain a satisfactory fit with a first-order model. Numerical analysis of such models is feasible for simple expectation predictors of stand composition. Theoretical analysis is less promising because the fundamental difference equations for expected values of basic stand characteristics become exceedingly complicated. Those given in Peden et. al. (1973, pp. 307-309) are first-order in both time and diameter class. Solutions are not difficult to obtain but are complicated in nature. Solutions for higher-order models will also be practicable, but the orders of complexity will go up so markedly there is little prospect that they will be useful. Neither numerical nor theoretical analysis can easily be used to calculate multistep transition probabilities, variances, covariances, and approximate distribution of stand variables. Thus some of the most attractive features of the first-order model for both stand prediction and the investigation of prediction methodology will be lost.

Time homogeneity of single-step transition probabilities is a very common feature of models which are investigated theoretically. However, it is not an essential feature. The real problem is to decide what patterns of nonhomogeneous transition probabilities would be meaningful replacements for time-homogeneous ones. One apparent selection would be the periodic transition probabilities which have already been discussed. These clearly exhibit a relaxed form of time homogeneity and therefore do not entail a complete loss of the simplicity of analytic results associated with homogeneity. Solutions for less patterned sets of transition probability will be complicated, and there is a question with these of what one expects to gain with a model in which transition probabilities are aperiodic functions of time. Such models could be used for numerical studies of stand dynamics. They cannot be used for stand prediction unless it is also possible to predict the form of the nonstationarities. Since nonstationarities are consequences of the interactions of stands with other components and inputs to an undelimited ecosystems in which the stands occur, it would appear impossible now to make such predictions except on very short-term bases. These would be valuable only for short-term stand predictions which could be obtained from updated input to a time-homogeneous model.

It should also be recognized that time nonhomogeneities of a stationary nature create only minor problems in the application of time-homogeneous models for stand prediction. The variation which affects growth transition probabilities from stationary nonhomogeneities, such as the effects of regularly repeated seasonal changes or transient fluctuations in the environment, tends to average out in long-range predictions. The principal effect of this type of variation is on the variances of predictors and not on their expected values. Nonhomogeneities of a nonstationary nature are the ones which should be accounted for with a nonhomogeneous model and are the ones which are currently exceedingly difficult to foresee.

Modelling of multistep transition probabilities has two attractive features. First, it could be used to simplify all the closed-form solutions which exist for a first-order, time-homogeneous model. This would make analysis of a model and prediction of stand dynamics based on it easier to investigate. Secondly, modelling results in an economy of parameters which must be estimated in order to make stand predictions. The number of transition probabilities in the Peden *et. al.* model is $8k - 5$ where k is the number of diameter-class classifications required. In the examples they considered k was 11, which illustrates that the total number of transition probabilities which must be estimated clearly can be considerable. Were it possible to model each type of transition probability by a single-parameter function of diameter-class index, the number of transition parameters in the Peden *et. al.* model would be reduced to no more than 11. Predictors derived from a good 11-parameter model should have appreciably smaller variances than those derived from a $(8k - 5)$-parameter model.

The deterministic ingrowth functions considered by Peden *et. al.* are less satisfactory than would be stochastic ones which are functions of numbers of potentially reproductive individuals and harvested trees which make space for new growth. For example, the ingrowth function might be

$$\sum_{g=1}^{h} \sum_{i=1}^{k} \alpha_{gi} W_{h-g,i} + \sum_{i=1}^{k} \beta_{1i}(\chi_{hi} - \chi_{h-1,i}) + \sum_{i=1}^{k} \beta_{2i}(\zeta_{hi} - \zeta_{h-1,i})$$

where α_{gi} is the rate of ingrowth due to reproduction in diameter class i in year $h - g$, β_{1i} is the rate of replacement with new growth of harvested live trees in diameter class i, and β_{2i} is a similar replacement rate for harvested dead trees. The process is no longer first order in time if $\alpha_{gi} \neq 0$ for any g greater than 1 and some i or first order in diameter class if any rate coefficient is nonzero for some i greater than 1. However, it is still possible to derive the expectations of all important stand variables, and to make stand predictions based on these expectations. Interval-type predictions for which variances and covariance are required do not appear to be feasible.

3. DISCUSSION

The modifications of a first-order, time-homogeneous model which are simplest to handle and therefore, most promising of success are 4. and 5. The first of these may be quite useful for theoretical investigation of stand dynamics. It would seem to be less applicable for stand projections because simple but realistic models of transition probabilities may be difficult to derive. The second modification appears promising for short- to moderate-range prediction of stand variables using expectations. The most useful applications would be to stands which are developing or to ones undergoing rapid changes due to abrupt shifts in management practices. Stands in which dynamics approximate steady-state conditions or ingrowth is controlled would be better modelled by deterministic ingrowth for which it is possible to make interval-type predictions.

The first three modifications are much more complicated in effects than the final two. Much of the simplicity of first-order, time-homogeneous models is lost, so that essentially, it will be necessary to formulate and solve the modelling problems from scratch. Theoretical solutions of parts or all for the problems will be very difficult and even numerical solutions may require considerable ingenuity.

It appears that there is very little that one can do to first-order time-homogeneous models to improve stand-prediction techniques which is both simple and productive. Better models appear to involve considerable increases in difficulty for their construction and analysis, particularly if one wishes to make the interval-type predictions possible with first-order time-homogeneous models. Therefore, the decision of when it will be advantageous to develop better models than ones currently in use will probably be strongly influenced by the economics of a trade-off between losses due to moderately large inaccuracies in predictions and costs of developing new models which will significantly reduce those inaccuracies.

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