Diameter Distributions and Yields Of Thinned Loblolly Pine Plantations

Publication No. FWS-1-82
School of Forestry and Wildlife Resources
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061
1982
DIAMETER DISTRIBUTIONS AND YIELDS
OF THINNED LOBLOLLY PINE PLANTATIONS

by

Quang V. Cao
Harold E. Burkhart
Ronald C. Lemin, Jr.

Publication No. FWS-1-82
School of Forestry and Wildlife Resources
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061
1982
ACKNOWLEDGMENTS

The work reported here was financed in part by the Loblolly Pine Growth and Yield Research Cooperative. We gratefully acknowledge the Virginia Division of Forestry for the plot data used in this study. The plots were installed and remeasured by numerous foresters and wardens of the Virginia Division of Forestry who, in all cases, were assisted and supervised by a member of the Applied Research Branch of the Division.

AUTHORS

The authors are, respectively, Assistant Professor in the School of Forestry and Wildlife Management, Louisiana State University, Baton Rouge, LA 70803, and Thomas M. Brooks Professor and former Graduate Research Assistant in the Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.

ABSTRACT

A growth and yield model for thinned loblolly pine plantations was developed using data from 128 0.2-acre permanent plots in the Virginia Piedmont and Coastal Plain. The Weibull function, used to characterize stand diameter distributions, was searched to insure that the resulting total basal area and average dbh estimates were identical to those predicted from stand variables using regression equations. Program WITHIN was written in standard FORTRAN to provide stand and stock tables for thinned old-field loblolly pine plantations.

Trials with different thinning intensities indicated reasonable trends, as compared with published studies.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Tables</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>PREVIOUS WORK</td>
<td></td>
</tr>
<tr>
<td>Whole Stand and Diameter Distribution Models</td>
<td>1</td>
</tr>
<tr>
<td>Modeling Thinned Loblolly Pine Stands</td>
<td>3</td>
</tr>
<tr>
<td>DEVELOPING THE THINNED-STAND MODEL</td>
<td>4</td>
</tr>
<tr>
<td>Data</td>
<td>4</td>
</tr>
<tr>
<td>Model for Thinned Loblolly Pine Plantations</td>
<td>4</td>
</tr>
<tr>
<td>Stand-level model</td>
<td>8</td>
</tr>
<tr>
<td>Deriving diameter distribution from stand attributes</td>
<td>8</td>
</tr>
<tr>
<td>RESULTS AND DISCUSSION</td>
<td>12</td>
</tr>
<tr>
<td>Program WITHIN</td>
<td>12</td>
</tr>
<tr>
<td>Prediction of the present stand</td>
<td>12</td>
</tr>
<tr>
<td>Thinning</td>
<td>13</td>
</tr>
<tr>
<td>Projection</td>
<td>13</td>
</tr>
<tr>
<td>Diameter distribution of a previously low-thinned stand</td>
<td>14</td>
</tr>
<tr>
<td>Effect of Thinning Regimes on Yield</td>
<td>15</td>
</tr>
<tr>
<td>Comparison with Published Information on Thinning</td>
<td>15</td>
</tr>
<tr>
<td>Coile and Schumacher's (1964) model</td>
<td>15</td>
</tr>
<tr>
<td>Yields reported by Goebel et al. (1974)</td>
<td>20</td>
</tr>
<tr>
<td>Possible Modifications and Refinements</td>
<td>23</td>
</tr>
<tr>
<td>LITERATURE CITED</td>
<td>24</td>
</tr>
<tr>
<td>APPENDICES</td>
<td>29</td>
</tr>
</tbody>
</table>

iii
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Description of plots immediately before and after thinning and amount of thinning.</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Distribution of all observations by site index (base age 25 years), age, basal area, and number of trees per acre.</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>Regression equations that form a whole stand model for thinned loblolly pine plantations.</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option.</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>Comparison of predicted yields of Coile and Schumacher (1964) and those from program WTHIN on a per acre basis for thinned loblolly pine plantations.</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>Comparison of observed yields of Goebel et al. (1974) and predicted yields from program WTHIN on a per acre basis for thinned loblolly pine plantations.</td>
<td>22</td>
</tr>
</tbody>
</table>
DIAMETER DISTRIBUTIONS AND YIELDS
OF THINNED LOBLOLLY PINE PLANTATIONS

Quang V. Cao, Harold E. Burkhart, and Ronald C. Lemin, Jr.

INTRODUCTION

Growth and yield predictions are essential to forest management planning. Reliable growth and yield models assist managers in analyzing alternative management strategies. For loblolly pine (Pinus taeda L.), a myriad of yield information for unmanaged stands has accumulated over the years. On the other hand, yield models for thinned loblolly pine plantations still seem inadequate, and flexible models that supply information about diameter distributions are needed.

Different probability density functions (pdf's) have been used to characterize diameter distributions; most recently the beta, Weibull, and Johnson's S_d distributions have been employed to develop yield estimates. The so-called probability density function approach to yield modeling involves predicting the pdf parameters from stand variables (age, site, and density) using regression techniques, and then calculating the number of trees and yield per acre in each dbh class. The drawback of this approach is that the regression models for predicting the pdf parameters usually account for only a small percentage of the variation (i.e. low R^2 values). Recently, research has been conducted to develop methods for approximating the parameters in a theoretical diameter distribution (e.g. the beta or Weibull) from overall stand values such as total basal area and mean diameter (Hyink 1980, Frazier 1981, Matney and Sullivan 1982).

The objectives of this study were: (1) to develop a whole stand model for thinned loblolly pine plantations using regression techniques, and (2) to derive diameter distributions from the predicted stand attributes by assuming that the underlying dbh distribution is Weibull distributed.

PREVIOUS WORK

Whole Stand and Diameter Distribution Models

MacKinney and Chaiken (1939) used multiple linear regression techniques to predict the logarithm of yield as a function of stand variables (age, site, density, and composition). This approach, with
certain modifications, has been employed in more recent models for loblolly pine (such as Schumacher and Coile 1960, Coile and Schumacher 1964, Goebel and Warner 1969, Burkhart et al. 1972a, 1972b).

Growth and yield are not two separate attributes but are closely related to one another. Buckman (1962) developed a yield model for red pine where yield is obtained by mathematically integrating the growth equation over time. Clutter (1963) discussed this concept in detail and introduced a compatible growth and yield model which was later refined by Sullivan and Clutter (1972). A similar approach has been used by several other researchers including Breder and Clutter (1970), Bennett (1970), Beck and Della-Bianca (1972), Sullivan and Williston (1977), Murphy and Sternitzke (1979), and Murphy and Belft (1981).

Diameter distributions in even-aged stands have been modeled with various probability density functions, among them the Gram-Charlier series (Meyer 1928, 1930; Schumacher 1928, 1930; Schnur 1934), the modified Pearl-Reed growth curve (Osborne and Schumacher 1935, Nelson 1964), Pearsonnian curves (Schnur 1934), and the log-normal distribution (Bliss and Reinker 1964).

Bennett and Clutter (1968) developed a yield model to predict multiple-product yields for slash pine plantations by using the stand table generated from a beta pdf via the Clutter and Bennett (1965) diameter distribution model. In this yield model, the parameters of the beta function that approximated the diameter distribution were predicted from stand variables (age, site, and density). The number of trees and volume per acre in each diameter class were calculated and per acre yield estimates were obtained by summing over diameter classes of interest. A similar approach was applied to loblolly pine plantations by Lenhart and Clutter (1971), Lenhart (1972), and Burkhart and Strub (1974).

The main drawback of using the beta distribution is that its cumulative distribution function (cdf) does not exist in closed form. As a result, the proportion of trees in each diameter class has to be solved by numerical integration techniques. Bailey and Dell (1973) pointed out that the Weibull distribution fits diameter data well and its cdf exists in closed form. The Weibull function was applied in plantation yield models for loblolly pine (Smalley and Bailey 1974a, Feduccia et al. 1979), slash pine (Clutter and Belcher 1978, Dell et al. 1979), and shortleaf pine (Smalley and Bailey 1974b).

Strub and Burkhart (1975) presented a class-interval-free method for predicting whole stand yield per unit area from diameter distribution models:


\[ TV = N \int_L^U g(D) f(D) \, dD \]

where \( TV \) = expected stand volume per unit area,
\( N \) = number of trees per unit area,
\( D \) = diameter at breast height,
\( g(D) \) = individual tree volume equation,
\( f(D) \) = pdf for \( D \), and
\((L, U)\) = merchantability limits for the product described by 
\( g(D) \).

Using this relationship, Hyink (1980) introduced a method of solving for the parameters of the pdf approximating the diameter distribution, using attributes predicted from a whole stand model. The same concept was employed by Matney and Sullivan (1982) in their model for loblolly pine plantations. In the first phase of Matney and Sullivan's study, stand volume and basal area were predicted using compatible growth and yield equations. The second phase involved solving for two parameters of the Weibull pdf which characterized the diameter distribution such that the resulting stand volume and basal area per acre would be identical to those predicted in the first phase. Frazier (1981) investigated alternative formulations for estimating parameter values in the beta and Weibull distributions from stand attributes.

Modeling Thinned Loblolly Pine Stands

Coile and Schumacher (1964) included amount of thinning as input in their model. Different types of thinning (thinning by rows, from below, or by a combination of both) can be specified in Daniels and Burkhart's (1975) and Daniels et al.'s (1979) individual tree models. Other models based on data from thinned loblolly pine stands include Clutter (1963), Brender and Clutter (1970), Sullivan and Clutter (1972), and Sullivan and Williston (1977).

The Weibull function was used by Bailey et al. (1981) to describe diameter distribution of slash pine plantations before and after thinning. Matney and Sullivan (1982) also used the Weibull distribution to produce compatible stand and stock tables for thinned loblolly pine plantations. In addition to the models mentioned above, growth and yield of thinned loblolly pine stands have been reported by many researchers (such as Bassett 1966, Bruner and Goebel 1968, Andrulot et al. 1972, Shepard 1974, Goebel et al. 1974, Feduccia and Mann 1976, Burton 1980).
DEVELOPING THE THINNED-STAND MODEL

Data

The growth and yield model for thinned loblolly pine plantations developed in this study was based on data from the Virginia Division of Forestry (VDF). This data set consists of 128 0.2-acre permanent plots from old-field plantations in the Virginia Piedmont and Coastal Plain. Number of remeasurements varied from plot to plot, ranging from 1 to 7. There were a total of 490 plot measurements.

Diameter at breast height (dbh) was recorded to the nearest inch and total height was measured to the nearest foot. Trees in the 1- and 2-inch classes were not tallied separately but combined to form one class whose midpoint was arbitrarily set at 1.5 inches. In each plot, measurements of dbh of all trees were taken but only some tree heights were measured. Height corresponding to each dbh class was predicted for each plot measurement using a regression equation of the form

$$\log_e (H) = b_0 + b_1/D,$$

where $H =$ total tree height in feet, $D =$ diameter at breast height in inches, $b_0, b_1 =$ regression coefficients.

Site index was determined from the average height of the dominants and codominants in each plot, using a site index equation developed by Devan (1979). Total cubic-foot volume outside bark per acre was computed using Burkhart et al.'s (1972b) individual tree volume equation.

The stands were thinned up to 3 times and, for the most part, thinnings were from below. However, some codominants and dominants were removed to improve the quality of the leave stand. The thinnings carried out were done during routine, operational thinnings of the plantations in which the plots were located. Table 1 presents a description of plots in this data set immediately before and after thinning. The distribution of all observations by site index, age, basal area, and number of trees per acre is presented in Table 2.

Model for Thinned Loblolly Pine Plantations

The model for thinned loblolly pine plantations developed in this study consisted of two stages. In the first stage, stand-level
Table 1. Description of plots immediately before and after thinning and amount of thinning. *a/*

<table>
<thead>
<tr>
<th>Variable</th>
<th>First thinning</th>
<th>Subsequent thinnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
<td>Amount</td>
</tr>
<tr>
<td>Number of trees/acre</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>355</td>
<td>165</td>
</tr>
<tr>
<td>Mean</td>
<td>774</td>
<td>459</td>
</tr>
<tr>
<td>Maximum</td>
<td>1305</td>
<td>770</td>
</tr>
<tr>
<td>Basal area (sq.ft./acre)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>107</td>
<td>29</td>
</tr>
<tr>
<td>Mean</td>
<td>174</td>
<td>87</td>
</tr>
<tr>
<td>Maximum</td>
<td>227</td>
<td>148</td>
</tr>
<tr>
<td>Total outside-bark volume (cu.ft./acre)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>1700</td>
<td>475</td>
</tr>
<tr>
<td>Mean</td>
<td>3839</td>
<td>1910</td>
</tr>
<tr>
<td>Maximum</td>
<td>6235</td>
<td>3705</td>
</tr>
<tr>
<td>Average DBH (inches)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>4.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Mean</td>
<td>6.4</td>
<td>7.1</td>
</tr>
<tr>
<td>Maximum</td>
<td>9.5</td>
<td>10.1</td>
</tr>
<tr>
<td>Age (years)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Mean</td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>Maximum</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

*a/* Discrepancies in the plot description (e.g., the means of a stand attribute after thinning and amount of thinning do not sum to the mean of that attribute before thinning as expected) are due to missing observations either before or after thinning.
Table 2. Distribution of all observations by site index (base age 25 years), age, basal area, and number of trees per acre.

<table>
<thead>
<tr>
<th>Site Index</th>
<th>Basal Area (sq.ft) (feet) Age (years)</th>
<th>Number of trees per acre</th>
<th>≤ 300</th>
<th>501- 700</th>
<th>900- 1100</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>20 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>17</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>50 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>38</td>
<td>24</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>40</td>
<td>50 10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>10 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>50 20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>26</td>
<td>44</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>
Table 2. Distribution of all observations by site index (base age 25 years), age, basal area, and number of trees per acre (continued).

<table>
<thead>
<tr>
<th>Site Index</th>
<th>Basal Area (sq.ft.)</th>
<th>Number of trees per acre</th>
<th>300</th>
<th>500</th>
<th>700</th>
<th>900</th>
<th>1100</th>
<th>1100</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 30</td>
<td>50</td>
<td>6</td>
<td>6</td>
<td>99</td>
<td></td>
<td>41</td>
<td></td>
<td>2</td>
<td>148</td>
</tr>
<tr>
<td>60 100</td>
<td>88</td>
<td>11</td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>60 150</td>
<td>19</td>
<td>20</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>60 200</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>113</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>43</td>
<td></td>
<td></td>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 100</td>
<td>23</td>
<td></td>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 150</td>
<td>20</td>
<td></td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>43</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 100</td>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 150</td>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 200</td>
<td>3</td>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70 10</td>
<td>50</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70 100</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>70 150</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>23</td>
</tr>
<tr>
<td>20 100</td>
<td>7</td>
<td>11</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21</td>
</tr>
<tr>
<td>20 150</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>20 200</td>
<td>2</td>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
<td>17</td>
<td>6</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>33</td>
</tr>
<tr>
<td>30 100</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30 150</td>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>276</td>
<td>140</td>
<td>28</td>
<td>29</td>
<td>15</td>
<td>2</td>
<td></td>
<td></td>
<td>490</td>
</tr>
</tbody>
</table>
attributes were predicted using regression techniques. The second stage involved determining the Weibull parameters so that the resulting diameter distribution would produce stand basal area and average dbh estimates identical to those predicted from regression equations in the first stage. By linking these two stages, the size-class distribution information produced is conditioned to provide aggregate values that are consistent with the predicted overall stand attributes.

Stand-Level Model

The stand-level model consisted of regression equations that predict (1) stand attributes (such as number of trees, basal area, minimum, and average diameters), and (2) density of a stand in the future (age A2) based on stand information at present (age A1). Also needed was a mean height equation that predicts total height corresponding to a given dbh. Table 3 shows the equations that form a whole stand model for thinned loblolly pine plantations.

Individual tree volume equations developed by Burkhart et al. (1972b) and Burkhart's (1977) volume ratio model were employed for estimating merchantable volumes. The site index equation developed by Devan (1979) was used to predict the average height of the dominants and codominants (HD) from site index and stand age, or to estimate site index from HD and stand age.

Deriving Diameter Distribution from Stand Attributes

The three-parameter Weibull pdf employed here to approximate diameter distribution is:

\[ f(x) = \frac{c}{b} \left( \frac{x-a}{b} \right)^{c-1} \exp \left\{ -\left( \frac{x-a}{b} \right)^c \right\} , \quad x \geq a, \]

where \( b, c \) = positive scale and shape parameters, respectively, 
\( a \) = nonnegative location parameter, 
\( x \) = diameter random variable.

The location parameter was predicted from a regression equation. The scale and shape parameters were searched for such that the resulting Weibull distribution would produce stand basal area and arithmetic mean dbh estimates identical to those predicted from regression equations. In other words, \( b \) and \( c \) were solutions of the following system of two equations:
Table 3. Regression equations that form a whole stand model for thinned loblolly pine plantations.

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Equation a/</th>
</tr>
</thead>
</table>
| 1               | \[ \ln(B_2) = 5.40816 + 0.0032121 S - \left(\frac{A_1}{A_2}\right) [5.40816 \
|                 | \quad + 0.0032121 S - \ln(B_1)] \] \[ n = 207; \quad \overline{\ln(B_2)} = 4.7230; \quad s_{y.x} = 0.0860 \] \[ R^2 = 99.34\%; \quad R^2(B_2) = 80.47\% \] |
| 2               | \[ N_2 = [N_1^{-0.65808} + 0.0000075795 (A_2^{1.78019} - A_1^{1.78019})]^{-1/0.65808} \] \[ n = 207; \quad \overline{N_2} = 253.02; \quad s_{y.x} = 18.64 \] \[ R^2 = 97.07\%; \quad R^2(N_2) = 97.07\% \] |
| 3               | \[ \ln(B) = -4.39181 + 0.19054 \ln(HD) + 0.63902 \ln(N) \] \[ n = 490; \quad \overline{\ln(B)} = 4.7149; \quad s_{y.x} = 0.1407 \] \[ R^2 = 75.48\%; \quad R^2(B) = 77.01\% \] |
| 4               | \[ \ln(N) = 7.79805 + 2.10495 \ln(HD) + 1.16744 \ln(B) \] \[ n = 490; \quad \overline{\ln(N)} = 5.6732; \quad s_{y.x} = 0.1902 \] \[ R^2 = 87.19\%; \quad R^2(N) = 85.78\% \] |
| 5               | \[ \ln(H) = 0.46152 + 0.43275 \ln(HD) - 0.93333 \ln(N) - 0.08583 \ln(B) + 0.07596 \ln(N) - 2.15312 \ln(D) \] \[ n = 3559; \quad \overline{\ln(H)} = 4.0404; \quad s_{y.x} = 0.0422 \] \[ R^2 = 96.76\%; \quad R^2(H) = 97.62\% \] |
Table 3. Regression equations that form a whole stand model for thinned loblolly pine plantations (continued).

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Equation</th>
</tr>
</thead>
</table>
| 6               | \[\ln(D_{\text{min}}) = 1.10835 + 5.10755/A + 0.50531 \ln(HD) + 0.28544 \ln(B) - 0.57131 \ln(N)\]  
|                 | n = 427; \(\bar{\ln(D_{\text{min}})} = 1.5253; s_{y,x} = 0.2972\)  
|                 | \(R^2 = 46.84\%; R^2(D_{\text{min}}) = 51.02\%\) |
| 7               | \[\ln(D_q - \bar{D}) = -9.05733 + 0.89274 \ln(HD) + 0.58151 \ln(N)\]  
|                 | n = 489; \(\bar{\ln(D_q - \bar{D})} = -2.1316; s_{y,x} = 0.6206\)  
|                 | \(R^2 = 11.50\%; R^2(\bar{D}) = 99.80\%) |

\(\bar{D}\) = Notation:

\(\ln(x)\) = Natural logarithm of \(x\),  
\(R^2(x)\) = Percent variation of \(x\) explained by the model,  
\(A\) = Stand age in years,  
\(B\) = Basal area in square feet per acre,  
\(D\) = Tree diameter at breast height (dbh) in inches,  
\(\bar{D}\) = Arithmetic mean dbh in inches,  
\(D_{\text{min}}\) = Minimum dbh in inches,  
\(D_q\) = Quadratic mean dbh in inches,  
\(H\) = Total height in feet of a tree having dbh \(D\),  
\(HD\) = Average height in feet of the dominants and codominants,  
\(N\) = Number of surviving trees per acre,  
\(S\) = Site index in feet (base age 25 years).

Subscript \(i\) denotes that the measurement is taken at time \(i\).
\[ \hat{D} = \int_a^\infty x f(x) \, dx \]  
\[ \hat{B} = 0.005454 N \int_a^\infty x^2 f(x) \, dx \]  

where \( \hat{D} \) = predicted arithmetic mean dbh in inches, 
\( \hat{B} \) = predicted basal area in square feet per acre, 
\( N \) = number of surviving trees per acre, 
\( f(x) \) = Weibull pdf with parameters \( a \), \( b \), and \( c \).

Equation (8) can be rewritten as

\[ \hat{D} = a + b \Gamma(1 + 1/c) \]  
\[ b = (\hat{D} - a) / \Gamma(1 + 1/c) \]  

where \( \Gamma(x) \) = gamma function evaluated at \( x \).

In most diameter distribution models, stand volume and basal area are often obtained by first computing these attributes for each dbh class and then summing over diameter classes of interest. Equation (9) can be approximated in a similar manner by replacing the integral sign with a summation sign:

\[ B = 0.005454 N \sum_{x_i=1}^\infty x_i^2 f_i \]  

where \( x_i \) = midpoint of the \( i \)th dbh class, 
\( f_i = F(x_i + 0.5) - F(x_i - 0.5) \) = proportion of trees in the \( i \)th dbh class, 
\( F(x) = 1 - \exp \left\{ -\left(\frac{x-a}{b}\right)^c \right\} \) = Weibull cumulative distribution function with parameters \( a \), \( b \), and \( c \).

Starting with a guess for \( c \), parameter \( b \) can be computed from (11) given \( a \) and \( c \). All three parameters \( (a, b, \text{ and } c) \) then specify a Weibull distribution. If equation (12) is not satisfied, a refined estimate for \( c \) will be computed and the procedures are repeated until both sides of equation (12) are almost equal. This method reduces the problem to that of solving one nonlinear equation (equation 12) whose unknown is the shape parameter \( c \) of the Weibull pdf.
RESULTS AND DISCUSSION

Program WTHIN

All of the techniques described earlier were incorporated into program WTHIN, which was written in standard FORTRAN. This program can generate stand and stock tables for different combinations of site, stand age, and density. It is also able to simulate a loblolly pine stand for a specified period during which thinning options are available at any point in time.

Prediction of the Present Stand

The inputs needed are:

1. age of the present stand,
2. site index (or average height of the current dominants and codominants),
3. two measures of density (total basal area and number of trees per acre).

If only one measure of density is available, the other can be estimated by employing the appropriate equation (3 or 4) of Table 3. Equations (6, 7) of Table 3 predict the minimum and arithmetic mean dbh of the stand. The Weibull location parameter $a$ is computed from $D_{min}$ as follows:

$$a = \text{FLOOR} (D_{min} - 0.5) - 0.49,$$

where $\text{FLOOR}(x) = \text{integer portion of } x$.
This adjustment simply sets $D_{min}$ at the lower end of its 1-inch dbh class and then decreases it by 1 inch.

The Weibull parameters $b$ and $c$ are obtained by solving equation (12). As a result, number of trees and basal area per acre for each dbh class can be computed. The mean height equation (equation 5 of Table 3) predicts total height corresponding to the midpoint of each dbh class. Total volumes outside and inside bark can be obtained from the individual tree volume equations published by Burkhart et al. (1972b). Merchantable volumes can also be calculated using the volume ratio methods developed by Burkhart (1977) and Cao and Burkhart (1980).
Thinning

Inputs for the thinning option include age of the stand when thinning occurs and type of thinning. Thinning can be carried out by rows, from below, or a combination of both.

It is assumed that the diameter distribution does not change due to row thinning. Thus the number of trees, basal area, and volume per acre in each dbh class are reduced by the proportion of trees removed in thinning.

Thinning from below is defined here as removing all trees with dbh values less than a specified diameter. Input for this type of thinning can be either this diameter limit or a residual basal area. A combination of row and low thinning involves first a row thinning followed by a thinning from below.

Alternative thinning algorithms can be easily substituted for those included in this model if one has information on removal patterns for the operations of interest.

Projection

Basal area and number of trees per acre at some age in the future can be projected using equations (1) and (2) of Table 3 for thinned stands, or the following equations from Coile and Schumacher (1964) for unthinned loblolly pine plantations:

\[
\log_{10}(N) = \log_{10}(N_0) + [2.1346 - 1.1103 \log_{10}(N_0)] + 0.1384 \log_{10}(N_0) A/100
\]

\[
\log_{10}(B) = 1.4366 \log_{10}(S) - 0.7084 (10/A) + 0.4888 \log_{10}(N) + 0.0585 (OF) - 1.4436
\]

where

- \( A \) = age in years,
- \( B \) = stand basal area in square feet per acre at age \( A \),
- \( N \) = number of surviving trees per acre at age \( A \),
- \( N_0 \) = number of trees planted per acre,
- \( OF \) = +1 if old-field origin, and -1 otherwise,
- \( S \) = site index in feet (base age 25 years).

Procedures similar to those for predicting the present stand are then employed to produce stand and stock tables for the future stand.
Diameter Distribution of a
Previously Low-Thinned Stand

Suppose that in a previous thinning from below, all trees having dbh below Dthin were cut. If the predicted Weibull location parameter (a) for the present stand is greater than or equal to Dthin, then the complete Weibull function is used to characterize the current diameter distribution. On the other hand, when a is less than Dthin, a left-truncated Weibull pdf is more appropriate where Dthin is the truncation point.

When the truncated Weibull is employed, equation (10) is replaced with:

\[ \hat{a} = a + \frac{\int_{(Dthin-a)}^{\infty} x(c/b)(x/b)^{c-1} \exp[-(x/b)^c]}{1 - F(Dthin)} \, dx \]

\[ \hat{b} = a + \frac{b}{1 - F(Dthin)} \int_{(Dthin-a)}^{\infty} \frac{y^{1/c} \exp(-y)}{(Dthin/b)^c} \, dy \]

or

\[ \hat{b} = a + \frac{b}{1 - F(Dthin)} \left[ (1 + 1/c) - \int_{0}^{(Dthin-a)/b} y^{1/c} \exp(-y) \, dy \right] \]

(13)

where \( F(x) = 1 - \exp \{-[(x-a)/b]^c} \).

The procedures for deriving the parameters of the truncated Weibull pdf are similar to those of the complete Weibull described earlier. The shape parameter c is solved from equation (12); for each estimated value of c, the scale parameter b is obtained from equation (13) (instead of from equation (11) as in the case of the complete Weibull pdf). The proportion of trees in the i\textsuperscript{th} dbh class of the truncated distribution is given by:

\[ f_i = \frac{F(i+0.5) - F(i-0.5)}{1 - F(Dthin)} \]
Effect of
Thinning Regimes on Yield

In order to demonstrate the effect of thinning type and intensities on yield, different thinning options were applied to loblolly pine plantations on site index 60 soil. These hypothetical stands had 800 trees and 130 sq.ft. per acre of basal area at age 15, and would be harvested at age 30. Option D was the control where no thinning was applied. In the rest of the thinning options, the stands were thinned repeatedly at ages 15, 20, and 25 to a specified residual basal area. Residual basal areas were arbitrarily set at 80, 95, and 110 sq.ft. per acre for options A, B and C, respectively. Three types of thinning were considered for each residual density: (1) row thinning, (2) low thinning, and (3) a combination of row and low thinnings, where 25% of the basal area removed was first cut in a row thinning and then the remainder from a thinning from below. Option Bl, for example, means row thinning to 95 sq.ft./acre of residual basal area.

Yields of these stands under different regimes are presented in Table 4. Total cubic-foot volume production (amount removed in thinnings plus final harvest volume) did not differ much from row to low thinning for a given thinning level. Note that thinning level is to a specified residual basal area and that number of trees remaining therefore varies by thinning type. Stand average diameter, however, was lowest in row thinning, highest in low thinning, and somewhere between these two extremes in the combination of row and low thinnings, as expected. As found by other researchers (such as Feduccia and Mann 1976, Sullivan and Williston 1977), cubic-foot volume production increased with higher residual basal area. On the other hand, average dbh increased as the thinnings were more severe, which implies an increase in board-foot volume production.

Although only total cubic-foot volume is presented in Table 4, users can readily develop yield tables in other units (cords, board feet, pounds, etc.) and for any specified portion of the stand by substituting appropriate volume or weight equations and specifying desired threshold diameters in the model.

Comparison with
Published Information on Thinning

Coile and Schumacher's (1964) Model

Program WTHIN was compared with the model for thinned loblolly pine plantations developed by Coile and Schumacher (1964); results
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Number of trees</th>
<th>Basal area (sq.ft.)</th>
<th>Average DBH (inches)</th>
<th>Total Volume (cu.ft.)</th>
<th>Number of trees</th>
<th>Basal area (sq.ft.)</th>
<th>Average DBH (inches)</th>
<th>Total Volume (cu.ft.)</th>
<th>Volume removed, Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Before thinning</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>After thinning</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
<td>492</td>
<td>80</td>
<td>5.3</td>
<td>1369</td>
<td>856</td>
</tr>
<tr>
<td>20</td>
<td>466</td>
<td>108</td>
<td>6.4</td>
<td>2375</td>
<td>343</td>
<td>80</td>
<td>6.4</td>
<td>1751</td>
<td>624</td>
</tr>
<tr>
<td>25</td>
<td>326</td>
<td>102</td>
<td>7.4</td>
<td>2643</td>
<td>255</td>
<td>80</td>
<td>7.4</td>
<td>2071</td>
<td>572</td>
</tr>
<tr>
<td>30</td>
<td>242</td>
<td>98</td>
<td>8.5</td>
<td>2860</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4912</td>
</tr>
</tbody>
</table>

**OPTION A1:** Row thinning — Residual basal area = 80 sq.ft./acre

**OPTION A2:** Low thinning — Residual basal area = 80 sq.ft./acre

**OPTION A3:** 25% row thinning and 75% low thinning — Residual basal area = 80 sq.ft./acre

15          | 800              | 130                 | 5.3                  | 2225                  | 367              | 80                  | 6.3                  | 1378                  | 847                              | 2225                              |
| 20          | 351              | 108                 | 7.4                  | 2376                  | 221              | 80                  | 8.1                  | 1770                  | 606                              | 3223                              |
| 25          | 212              | 102                 | 9.3                  | 2652                  | 149              | 80                  | 9.9                  | 2094                  | 558                              | 4105                              |
| 30          | 143              | 98                  | 11.1                 | 2868                  |                  |                     |                      |                       | 4879                              |
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Number of trees</th>
<th>Basal Area (sq.ft.)</th>
<th>Average DBH (inches)</th>
<th>Total Volume (cu.ft.)</th>
<th>Number of trees</th>
<th>Basal Area (sq.ft.)</th>
<th>Average DBH (inches)</th>
<th>Total Volume (cu.ft.)</th>
<th>Volume removed Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before thinning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>After thinning</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
<td>585</td>
<td>95</td>
<td>5.3</td>
<td>1625</td>
<td>590</td>
</tr>
<tr>
<td>20</td>
<td>550</td>
<td>123</td>
<td>6.3</td>
<td>2700</td>
<td>423</td>
<td>95</td>
<td>6.3</td>
<td>2078</td>
<td>622</td>
</tr>
<tr>
<td>25</td>
<td>398</td>
<td>117</td>
<td>7.2</td>
<td>3028</td>
<td>323</td>
<td>95</td>
<td>7.2</td>
<td>2456</td>
<td>572</td>
</tr>
<tr>
<td>30</td>
<td>304</td>
<td>113</td>
<td>8.1</td>
<td>3294</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5078</td>
</tr>
<tr>
<td><strong>OPTION B1:</strong> Row thinning -- Residual basal area = 95 sq.ft./acre</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
<td>454</td>
<td>95</td>
<td>6.2</td>
<td>1633</td>
<td>592</td>
</tr>
<tr>
<td>20</td>
<td>430</td>
<td>123</td>
<td>7.1</td>
<td>2700</td>
<td>274</td>
<td>95</td>
<td>7.9</td>
<td>2104</td>
<td>596</td>
</tr>
<tr>
<td>25</td>
<td>261</td>
<td>117</td>
<td>9.0</td>
<td>3038</td>
<td>188</td>
<td>95</td>
<td>9.6</td>
<td>2485</td>
<td>553</td>
</tr>
<tr>
<td>30</td>
<td>180</td>
<td>113</td>
<td>10.6</td>
<td>3305</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5046</td>
</tr>
<tr>
<td><strong>OPTION B2:</strong> Low thinning -- Residual basal area = 95 sq.ft./acre</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
<td>470</td>
<td>95</td>
<td>6.0</td>
<td>1631</td>
<td>594</td>
</tr>
<tr>
<td>20</td>
<td>446</td>
<td>123</td>
<td>7.0</td>
<td>2699</td>
<td>293</td>
<td>95</td>
<td>7.6</td>
<td>2098</td>
<td>601</td>
</tr>
<tr>
<td>25</td>
<td>279</td>
<td>117</td>
<td>8.6</td>
<td>3037</td>
<td>201</td>
<td>95</td>
<td>9.2</td>
<td>2483</td>
<td>554</td>
</tr>
<tr>
<td>30</td>
<td>192</td>
<td>113</td>
<td>10.3</td>
<td>3305</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5054</td>
</tr>
<tr>
<td><strong>OPTION B3:</strong> 25% row thinning and 75% low thinning -- Residual basal area = 95 sq.ft./acre</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
<td>470</td>
<td>95</td>
<td>6.0</td>
<td>1631</td>
<td>594</td>
</tr>
<tr>
<td>20</td>
<td>446</td>
<td>123</td>
<td>7.0</td>
<td>2699</td>
<td>293</td>
<td>95</td>
<td>7.6</td>
<td>2098</td>
<td>601</td>
</tr>
<tr>
<td>25</td>
<td>279</td>
<td>117</td>
<td>8.6</td>
<td>3037</td>
<td>201</td>
<td>95</td>
<td>9.2</td>
<td>2483</td>
<td>554</td>
</tr>
<tr>
<td>30</td>
<td>192</td>
<td>113</td>
<td>10.3</td>
<td>3305</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5054</td>
</tr>
</tbody>
</table>
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Area</td>
<td>Number of Area</td>
<td>Number of Area</td>
</tr>
<tr>
<td></td>
<td>Basal Area (sq.ft.)</td>
<td>Basal Area (sq.ft.)</td>
<td>Basal Area (sq.ft.)</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>677</td>
<td>504</td>
</tr>
<tr>
<td>20</td>
<td>632</td>
<td>504</td>
<td>394</td>
</tr>
<tr>
<td>25</td>
<td>472</td>
<td>394</td>
<td>394</td>
</tr>
<tr>
<td>30</td>
<td>368</td>
<td>394</td>
<td>394</td>
</tr>
</tbody>
</table>

**OPTION C1:** Row thinning -- Residual basal area = 110 sq.ft./acre

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Area</td>
<td>Number of Area</td>
<td>Number of Area</td>
</tr>
<tr>
<td></td>
<td>Basal Area (sq.ft.)</td>
<td>Basal Area (sq.ft.)</td>
<td>Basal Area (sq.ft.)</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>564</td>
<td>357</td>
</tr>
<tr>
<td>20</td>
<td>531</td>
<td>357</td>
<td>246</td>
</tr>
<tr>
<td>25</td>
<td>338</td>
<td>246</td>
<td>246</td>
</tr>
</tbody>
</table>

**OPTION C2:** Low thinning -- Residual basal area = 110 sq.ft./acre

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Area</td>
<td>Number of Area</td>
<td>Number of Area</td>
</tr>
<tr>
<td></td>
<td>Basal Area (sq.ft.)</td>
<td>Basal Area (sq.ft.)</td>
<td>Basal Area (sq.ft.)</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>573</td>
<td>372</td>
</tr>
<tr>
<td>20</td>
<td>539</td>
<td>372</td>
<td>264</td>
</tr>
<tr>
<td>25</td>
<td>352</td>
<td>264</td>
<td>264</td>
</tr>
</tbody>
</table>

**OPTION C3:** 25% row thinning and 75% low thinning -- Residual basal area = 110 sq.ft./acre
Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Before thinning</th>
<th>After thinning</th>
<th>Total Volume removed</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of trees</td>
<td>Basal Area (sq.ft.)</td>
<td>Average DBH (inches)</td>
<td>Total Volume ob (cu.ft.)</td>
</tr>
<tr>
<td>15</td>
<td>800</td>
<td>130</td>
<td>5.3</td>
<td>2225</td>
</tr>
<tr>
<td>30</td>
<td>540</td>
<td>186</td>
<td>7.8</td>
<td>5387</td>
</tr>
</tbody>
</table>

OPTION D: No thinning

2225
5387
are presented in Table 5. Both row and low thinning options were tried, for the thinning in practice would likely be somewhere between these two cases. Care was taken such that cord volume removed in each thinning was identical to that specified by Coile and Schumacher. Examination of the residual stands at age 30 revealed that the number of surviving trees from Coile and Schumacher's model was between the predicted values from the two types of thinning of program WTHIN. Residual basal area, quadratic mean dbh, and volume from Coile and Schumacher's predictions were consistently higher than those from WTHIN.

Coile and Schumacher's predicted total volume production of thinned stands far exceeded that of unthinned counterparts. On the other hand, total volume predictions (i.e., volume removed in thinnings plus residual volume) of thinned stands at age 30 from program WTHIN were close to volumes of unthinned stands at age 30 from Coile and Schumacher's model. This agrees well with what other investigators have found, namely, that total cubic-foot volume production is generally little affected by thinning (Smith 1962, Andruslot et al. 1972, Goebel et al. 1974).

Yields Reported by Goebel et al. (1974)

Goebel et al. (1974) reported yields of 9 old-field loblolly pine stands; each had been thinned 4 to 5 times to a specified residual basal area per acre. Site indices were determined from curves developed by Goebel and Shipman (1964). Goebel and Warner (1969) recognized a significant site-age bias in these site index curves and revised their yield model using Clutter and Lenhart's (1968) polymorphic site index curves. Devan's (1979) site index equation was replaced with that of Clutter and Lenhart (1968) in program WTHIN when simulating the stands based on the guidelines set forth by Goebel et al. (1974). Data for total cubic-foot volumes reported by Goebel et al. (1974) were based on volume tables prepared by MacKinney and Chaiken (1939). Thus MacKinney and Chaiken's (1939) individual tree volume equation was used in this simulation.

The observed number of trees per acre and average dbh in each plot fell between values predicted from WTHIN using the row and low thinning options (Table 6). Comparison of total volume production in these 9 stands shows that the mean relative difference between observed and predicted yields (averages of yields from the row and low thinning options) is -2.52%.
Table 5. Comparison of predicted yields of Coile and Schumacher (1964) and those from program WTHIN on a per acre basis for thinned loblolly pine plantations.

<table>
<thead>
<tr>
<th>Source</th>
<th>Site Index</th>
<th>Number of trees at age 5</th>
<th>Basal Area (sq.ft.) at age 5</th>
<th>Age when thinned (years)</th>
<th>Amount of thinning</th>
<th>Residual stand at age 30</th>
<th>Total Volume Production (cords)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Quadratic mean DBH (inches)</td>
<td>Number of trees</td>
</tr>
<tr>
<td>G&amp;S 5/</td>
<td>50</td>
<td>600</td>
<td>9.9</td>
<td>20</td>
<td>68</td>
<td>10</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>800</td>
<td>11.4</td>
<td>20</td>
<td>82</td>
<td>12</td>
<td>13.4</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>600</td>
<td>12.9</td>
<td>17,22</td>
<td>45,36</td>
<td>7,7</td>
<td>13.6</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>800</td>
<td>14.8</td>
<td>17,22</td>
<td>58,47</td>
<td>9,9</td>
<td>14.6</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>600</td>
<td>16.1</td>
<td>15,20,25</td>
<td>37,37,39</td>
<td>6,8,10</td>
<td>15.1</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>800</td>
<td>18.5</td>
<td>15,20,25</td>
<td>43,47,51</td>
<td>7,8,13</td>
<td>14.7</td>
</tr>
</tbody>
</table>

a/ Site index at base age 25 years.
b/ Cord volume to a 6-inch top, converted from cubic-foot volume outside bark to a 6-inch top, using ratios from Burkhart et al. (1972).

c/ Coile and Schumacher (1964).
d/ Row thinning, program WTHIN.
e/ Low thinning, program WTHIN.
f/ Numbers in parentheses are for unthinned stands.
Table 6. Comparison of observed yields of Goebel et al. (1974) and predicted yields from program WTHIN on a per acre basis for thinned loblolly pine plantations.

<table>
<thead>
<tr>
<th>Source</th>
<th>Site Index (feet)</th>
<th>Age (years)</th>
<th>Number of Basal Area Trees (sq.ft.)</th>
<th>Average DBH (inches)</th>
<th>Total Volume 1b (cu.ft.)</th>
<th>Number of Basal Area Trees (sq.ft.)</th>
<th>Average DBH (inches)</th>
<th>Residual Volume (cu.ft.)</th>
<th>Age when thinned (years)</th>
<th>Basal area limit (sq.ft.)</th>
<th>Volume removed in thinning (cu.ft.)</th>
<th>Total Volume Production (cu.ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed 51</td>
<td>13</td>
<td>790</td>
<td>121</td>
<td>5.3</td>
<td>1476</td>
<td>34</td>
<td>140</td>
<td>75</td>
<td>9.9</td>
<td>1870</td>
<td>13,21, 75</td>
<td>2325</td>
</tr>
<tr>
<td>Row 60</td>
<td>b/</td>
<td>5.2</td>
<td>1491</td>
<td>6.8</td>
<td>1479</td>
<td>68</td>
<td>75</td>
<td>14.2</td>
<td>1967</td>
<td>27,34</td>
<td>2644</td>
<td>4631</td>
</tr>
<tr>
<td>Low 60</td>
<td>b/</td>
<td>5.2</td>
<td>1491</td>
<td>68</td>
<td>1479</td>
<td>75</td>
<td>14.2</td>
<td>1967</td>
<td>27,34</td>
<td>2644</td>
<td>4631</td>
<td>4539</td>
</tr>
<tr>
<td>Observed 51</td>
<td>13</td>
<td>800</td>
<td>116</td>
<td>5.0</td>
<td>2116</td>
<td>34</td>
<td>160</td>
<td>84</td>
<td>9.8</td>
<td>2075</td>
<td>13,21, 85</td>
<td>2188</td>
</tr>
<tr>
<td>Row 60</td>
<td>b/</td>
<td>5.0</td>
<td>1422</td>
<td>89</td>
<td>2142</td>
<td>89</td>
<td>84</td>
<td>13.2</td>
<td>2224</td>
<td>27,34</td>
<td>2836</td>
<td>4680</td>
</tr>
<tr>
<td>Low 60</td>
<td>b/</td>
<td>5.0</td>
<td>1422</td>
<td>89</td>
<td>2142</td>
<td>85</td>
<td>13.2</td>
<td>2224</td>
<td>27,34</td>
<td>2836</td>
<td>4680</td>
<td>4585</td>
</tr>
<tr>
<td>Observed 51</td>
<td>13</td>
<td>780</td>
<td>129</td>
<td>5.3</td>
<td>3579</td>
<td>34</td>
<td>160</td>
<td>94</td>
<td>10.4</td>
<td>2349</td>
<td>13,21, 95</td>
<td>2189</td>
</tr>
<tr>
<td>Row 60</td>
<td>b/</td>
<td>5.4</td>
<td>1600</td>
<td>101</td>
<td>2502</td>
<td>101</td>
<td>95</td>
<td>13.1</td>
<td>2485</td>
<td>27,34</td>
<td>2498</td>
<td>4973</td>
</tr>
<tr>
<td>Low 60</td>
<td>b/</td>
<td>5.4</td>
<td>1600</td>
<td>101</td>
<td>2502</td>
<td>95</td>
<td>13.1</td>
<td>2485</td>
<td>27,34</td>
<td>2498</td>
<td>4973</td>
<td>4538</td>
</tr>
<tr>
<td>Observed 51</td>
<td>13</td>
<td>1916</td>
<td>124</td>
<td>4.6</td>
<td>1409</td>
<td>34</td>
<td>132</td>
<td>80</td>
<td>10.5</td>
<td>2065</td>
<td>13,18, 20, 80</td>
<td>2261</td>
</tr>
<tr>
<td>Row 60</td>
<td>b/</td>
<td>4.6</td>
<td>1494</td>
<td>184</td>
<td>2089</td>
<td>80</td>
<td>80</td>
<td>11.3</td>
<td>2110</td>
<td>25,34</td>
<td>2536</td>
<td>4625</td>
</tr>
<tr>
<td>Low 60</td>
<td>b/</td>
<td>4.6</td>
<td>1494</td>
<td>100</td>
<td>2376</td>
<td>80</td>
<td>80</td>
<td>11.3</td>
<td>2110</td>
<td>25,34</td>
<td>2536</td>
<td>4625</td>
</tr>
<tr>
<td>Observed 51</td>
<td>13</td>
<td>1004</td>
<td>122</td>
<td>4.6</td>
<td>1350</td>
<td>34</td>
<td>148</td>
<td>89</td>
<td>10.5</td>
<td>2436</td>
<td>13,18, 20, 90</td>
<td>2431</td>
</tr>
<tr>
<td>Row 60</td>
<td>b/</td>
<td>4.6</td>
<td>1469</td>
<td>224</td>
<td>2365</td>
<td>90</td>
<td>90</td>
<td>12.8</td>
<td>2376</td>
<td>25,34</td>
<td>2388</td>
<td>4733</td>
</tr>
<tr>
<td>Low 60</td>
<td>b/</td>
<td>4.6</td>
<td>1469</td>
<td>100</td>
<td>2258</td>
<td>90</td>
<td>90</td>
<td>12.8</td>
<td>2376</td>
<td>25,34</td>
<td>2258</td>
<td>4635</td>
</tr>
<tr>
<td>Observed 51</td>
<td>13</td>
<td>924</td>
<td>105</td>
<td>4.5</td>
<td>1133</td>
<td>34</td>
<td>176</td>
<td>103</td>
<td>10.4</td>
<td>2934</td>
<td>13,18, 20, 100</td>
<td>2707</td>
</tr>
<tr>
<td>Row 60</td>
<td>b/</td>
<td>4.4</td>
<td>1254</td>
<td>281</td>
<td>2595</td>
<td>100</td>
<td>100</td>
<td>11.4</td>
<td>2647</td>
<td>25,34</td>
<td>2034</td>
<td>4629</td>
</tr>
<tr>
<td>Low 60</td>
<td>b/</td>
<td>4.4</td>
<td>1254</td>
<td>141</td>
<td>1896</td>
<td>100</td>
<td>100</td>
<td>11.4</td>
<td>2647</td>
<td>25,34</td>
<td>1896</td>
<td>4542</td>
</tr>
<tr>
<td>Observed 55</td>
<td>17</td>
<td>1180</td>
<td>196</td>
<td>5.3</td>
<td>2784</td>
<td>30</td>
<td>252</td>
<td>85</td>
<td>7.8</td>
<td>2107</td>
<td>17,20, 85</td>
<td>2401</td>
</tr>
<tr>
<td>Row 61</td>
<td>b/</td>
<td>5.3</td>
<td>3164</td>
<td>161</td>
<td>2106</td>
<td>85</td>
<td>85</td>
<td>12.2</td>
<td>2142</td>
<td>24,30</td>
<td>2534</td>
<td>5140</td>
</tr>
<tr>
<td>Low 61</td>
<td>b/</td>
<td>5.3</td>
<td>3164</td>
<td>104</td>
<td>2094</td>
<td>85</td>
<td>85</td>
<td>12.2</td>
<td>2142</td>
<td>24,30</td>
<td>2534</td>
<td>5036</td>
</tr>
<tr>
<td>Observed 55</td>
<td>17</td>
<td>1220</td>
<td>187</td>
<td>5.4</td>
<td>3054</td>
<td>30</td>
<td>280</td>
<td>111</td>
<td>8.6</td>
<td>2854</td>
<td>17,20, 110</td>
<td>2192</td>
</tr>
<tr>
<td>Row 61</td>
<td>b/</td>
<td>5.4</td>
<td>3000</td>
<td>178</td>
<td>2704</td>
<td>110</td>
<td>110</td>
<td>7.2</td>
<td>2704</td>
<td>24,30</td>
<td>2464</td>
<td>5151</td>
</tr>
<tr>
<td>Low 61</td>
<td>b/</td>
<td>5.1</td>
<td>3000</td>
<td>181</td>
<td>2280</td>
<td>110</td>
<td>110</td>
<td>7.2</td>
<td>2704</td>
<td>24,30</td>
<td>2464</td>
<td>5051</td>
</tr>
<tr>
<td>Observed 55</td>
<td>17</td>
<td>1212</td>
<td>180</td>
<td>5.3</td>
<td>2884</td>
<td>30</td>
<td>372</td>
<td>129</td>
<td>8.0</td>
<td>3212</td>
<td>17,20, 135</td>
<td>1896</td>
</tr>
<tr>
<td>Row 61</td>
<td>b/</td>
<td>5.0</td>
<td>2880</td>
<td>162</td>
<td>3302</td>
<td>135</td>
<td>135</td>
<td>6.8</td>
<td>3302</td>
<td>24,30</td>
<td>1662</td>
<td>5144</td>
</tr>
<tr>
<td>Low 61</td>
<td>b/</td>
<td>5.0</td>
<td>2880</td>
<td>165</td>
<td>1658</td>
<td>135</td>
<td>135</td>
<td>6.8</td>
<td>3302</td>
<td>24,30</td>
<td>1658</td>
<td>5048</td>
</tr>
</tbody>
</table>

Legend:
- **b/**: Site Index (base age 25 years) from Goebel and Shipman (1964).
- **3/**: Row thinning, program WTHIN.
- **4/**: Low thinning, program WTHIN.
Possible Modifications and Refinements

In this study, a growth and yield model for thinned loblolly pine plantations was developed in which the parameters of the Weibull function that characterized the diameter distribution were searched for to insure that the resulting stand basal area and average dbh estimates were identical to those predicted from stand variables using regression techniques. Although the model gave logical results that agreed well with past work on thinning, there is still room for improvement.

Two specific areas for further investigation are:

(1) Various methods for deriving a dbh distribution from stand attributes for thinned stands need to be more fully evaluated.

(2) More realistic removal patterns for thinning from below should be developed. One possibility is to establish stochastic models in which trees in each dbh class are assigned probabilities of being removed, and are cut or left in each thinning operation depending on values of the random numbers generated.
LITERATURE CITED


### APPENDICES

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A numerical example.</td>
<td>30</td>
</tr>
<tr>
<td>Input variable formats and description for program WTHIN.</td>
<td></td>
</tr>
<tr>
<td>2a. Subprogram identification card (first card).</td>
<td>36</td>
</tr>
<tr>
<td>2b. Subprogram INPUT1.</td>
<td>37</td>
</tr>
<tr>
<td>2c. Subprogram INPUT2.</td>
<td>40</td>
</tr>
<tr>
<td>Input example for program WTHIN.</td>
<td></td>
</tr>
<tr>
<td>3a. Simulate a stand through time.</td>
<td>41</td>
</tr>
<tr>
<td>3b. Stand and stock tables for specified combinations of site index, age, and density.</td>
<td>42</td>
</tr>
<tr>
<td>Generalized flowchart of program WTHIN.</td>
<td>43</td>
</tr>
<tr>
<td>Source listing for program WTHIN.</td>
<td>45</td>
</tr>
</tbody>
</table>
Appendix 1. A numerical example.

The following example is chosen to illustrate the techniques employed in program WTHIN. Consider a loblolly pine plantation on soil of site index 60 feet (base age 25 years), with 600 trees and 150 sq.ft. of basal area per acre at age 20. The stand is thinned to 100 sq.ft. per acre at age 20; the thinning method is a combination of 25% row thinning and 75% low thinning (i.e. a row thinning removes 25% of the total basal area scheduled to be thinned, and then a thinning from below removes the remaining 75%). No minimum diameter for removal in the low thinning is specified in this example. The stand is then left to grow until it is harvested at age 40. The card input needed by program WTHIN to simulate this particular stand is presented in Appendix 3a. Figures A1 to A4 show the outputs of this simulation from program WTHIN. The computational steps (on a per acre basis) are outlined as follows.

**Step 1:** Yield prediction of the stand before thinning.

Stand variables: Site index = 60 feet, $A = 20$ years, $N = 600$ trees, $B = 150$ sq.ft. (variable names are defined in Table 3).

From Devan's (1979) site index equation, average height of the dominants and codominants at age 20 is 49.55 feet. Substituting the values into the appropriate stand variables in equations (6, 7) of Table 3 gives: $D_{\text{min}} = 3.04$ inches and $\bar{D} = 6.61$ inches.

The Weibull location parameter is adjusted from $D_{\text{min}}$ as follows:

$$a = \text{FLOOR} (D_{\text{min}} - 0.5) - 0.49 = 1.51,$$

where $\text{FLOOR} (x)$ = integer portion of $x$.

The remaining parameters defining a Weibull distribution which produces a total basal area of 150 sq.ft./acre and an average dbh of 6.61 inches are found to be

$$b = 5.6274 \quad \text{and} \quad c = 4.0385.$$  

Per acre number of trees, basal area, and volume for each dbh class can be computed. For example, number of trees in the 6-inch class is $600 \quad F(6.5) - F(5.5) = 143.3$ trees, where $F(x)$ is the Weibull cdf evaluated at $x$. Basal area in the 6-inch class:

$$143.3 \times (0.005454) \times (6)^2 = 28.1 \text{ sq.ft.}$$

Average height of a tree with a 6-inch dbh in this plantation is calculated from equation (5) of Table 3 to be 45.7 feet. Burkhart et al.'s (1972b) tree volume equation is applied on 143.3 trees of dbh 6 inches and total height 45.7 feet, resulting in a volume of 597.4 cu.ft. outside bark in the 6-inch dbh class. Summing volume
25% row, 75% low thinning down to 100 sqft/acre. Harvest age = 40.

Inputs
---
Site = 60.00
Age = 20.00
Number of trees = 600.00
Basal area = 150.00

Predicted
---
HD = 49.55
Average DBH = 6.61
Minimum DBH = 3.04

| DBH Class of Trees | Number of Trees | Average Height | Total Basal Area | Total Volume O.B. | Total Volume I.B. | Volume to 4.1
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.5</td>
<td>22.3</td>
<td>0.0</td>
<td>0.3</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>8.4</td>
<td>31.9</td>
<td>0.4</td>
<td>6.5</td>
<td>5.5</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>36.0</td>
<td>38.2</td>
<td>3.1</td>
<td>63.6</td>
<td>44.9</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>87.5</td>
<td>42.6</td>
<td>11.9</td>
<td>246.7</td>
<td>182.6</td>
<td>165.8</td>
</tr>
<tr>
<td>6</td>
<td>143.3</td>
<td>45.7</td>
<td>28.1</td>
<td>597.4</td>
<td>453.3</td>
<td>477.9</td>
</tr>
<tr>
<td>7</td>
<td>158.6</td>
<td>48.1</td>
<td>42.4</td>
<td>923.2</td>
<td>710.6</td>
<td>801.6</td>
</tr>
<tr>
<td>8</td>
<td>111.3</td>
<td>50.0</td>
<td>38.8</td>
<td>865.6</td>
<td>672.3</td>
<td>786.2</td>
</tr>
<tr>
<td>9</td>
<td>44.7</td>
<td>51.6</td>
<td>19.7</td>
<td>448.2</td>
<td>350.2</td>
<td>418.3</td>
</tr>
<tr>
<td>10</td>
<td>9.0</td>
<td>52.8</td>
<td>4.9</td>
<td>112.9</td>
<td>86.6</td>
<td>107.3</td>
</tr>
<tr>
<td>11</td>
<td>0.8</td>
<td>53.8</td>
<td>0.5</td>
<td>11.9</td>
<td>9.3</td>
<td>11.4</td>
</tr>
<tr>
<td>12</td>
<td>0.0</td>
<td>54.7</td>
<td>0.0</td>
<td>0.4</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

600.0 = 150.0 = 3278.7 = 2517.8 = 2768.8 = 2105.4

Average DBH = 6.61 Based on 1-inch DBH Classes
Cord Volume to 4.1 = 31.43

Weibull Parameters
A = 1.5100
B = 5.6274
C = 0.0385

Convergence attained

Figure A1. Example output from program WTHIN --
Step 1: Yield prediction of the stand before thinning.
25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

ROW THINNING AT AGE 20.

8.33% OF TREES IN ALL DIAMETER CLASSES ARE CUT

BEFORE ROW THINNING

\[ \begin{array}{cccc}
\text{SITE} & = & 60.00 \\
\text{AGE} & = & 20.00 \\
\text{NUMBER OF TREES} & = & 600.00 \\
\text{BASAL AREA} & = & 150.00 \\
\text{AVERAGE DBH} & = & 6.61 \\
\end{array} \]

AFTER ROW THINNING

\[ \begin{array}{cccccccc}
\text{DBH CLASS} & \text{NUMBER OF TREES} & \text{AVERAGE HEIGHT} & \text{BASAL AREA} & \text{TOTAL VOLUME 0.B.} & \text{TOTAL VOLUME I.B.} & \text{VOLUME TO 4.1IN 0.B.} & \text{VOLUME TO 4.1IN I.B.} \\
2 & 0.5 & 22.3 & 0.0 & 0.3 & 0.1 & 0.0 & 0.0 \\
3 & 7.7 & 31.9 & 0.4 & 7.8 & 5.0 & 0.0 & 0.0 \\
4 & 33.0 & 38.2 & 2.9 & 58.3 & 41.1 & 0.0 & 0.0 \\
5 & 80.2 & 42.6 & 10.9 & 226.1 & 167.4 & 151.9 & 108.9 \\
6 & 131.4 & 45.7 & 25.8 & 547.6 & 415.5 & 436.7 & 326.6 \\
7 & 145.4 & 48.1 & 38.8 & 846.3 & 651.4 & 734.8 & 558.4 \\
8 & 102.0 & 50.0 & 35.6 & 793.4 & 616.3 & 720.7 & 554.4 \\
9 & 40.9 & 51.6 & 18.1 & 410.8 & 321.0 & 383.4 & 297.4 \\
10 & 8.2 & 52.8 & 4.5 & 103.5 & 81.2 & 98.3 & 76.7 \\
11 & 0.7 & 53.8 & 0.5 & 10.9 & 8.6 & 10.5 & 8.2 \\
12 & 0.0 & 54.7 & 0.0 & 0.4 & 0.3 & 0.4 & 0.3 \\
\hline
550.0 & 137.5 & 3005.4 & 2308.0 & 2538.1 & 1930.0 \\
\end{array} \]

\[ \begin{array}{cccc}
\text{SITE} & = & 60.00 \\
\text{AGE} & = & 20.00 \\
\text{NUMBER OF TREES} & = & 550.00 \\
\text{BASAL AREA} & = & 137.50 \\
\text{AVERAGE DBH} & = & 6.61 \text{ BASED ON 1-INCH DBH CLASSES} \\
\end{array} \]

AMOUNT REMOVED IN ROW THINNING

\[ \begin{array}{cccc}
\text{NUMBER OF TREES} & = & 50.00 \\
\text{BASAL AREA} & = & 12.50 \\
\text{TOTAL CU.FT. VOLUME 0.B.} & = & 273.22 \\
\text{CU.FT. VOLUME 0.B. TO 4.1IN} & = & 230.73 \\
\text{CORD VOLUME TO 4.1IN} & = & 2.62 \\
\end{array} \]

Figure A2. Example output from program WTHIN --
Step 2: Row thinning at age 20.
25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

LOW THINNING AT AGE 20.
THIN TO 100.00 SQ.FT. RESIDUAL BASAL AREA

BEFORE LOW THINNING

--- --- ---

SITE = 60.00
AGE = 20.00
NUMBER OF TREES = 550.00
BASAL AREA = 137.50
AVERAGE DBH = 6.61

AFTER LOW THINNING

--- --- ---

<table>
<thead>
<tr>
<th>DBH CLASS</th>
<th>NUMBER OF TREES</th>
<th>AVERAGE HEIGHT</th>
<th>AVERAGE DBH</th>
<th>TOTAL VOLUME O.B.</th>
<th>VOLUME TO 4 IN O.B.</th>
<th>TOTAL VOLUME 1.B.</th>
<th>VOLUME TO 4 IN 1.B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12.7</td>
<td>45.7</td>
<td>2.5</td>
<td>53.1</td>
<td>40.3</td>
<td>42.4</td>
<td>31.6</td>
</tr>
<tr>
<td>7</td>
<td>145.4</td>
<td>48.1</td>
<td>38.8</td>
<td>846.3</td>
<td>651.4</td>
<td>734.8</td>
<td>558.1</td>
</tr>
<tr>
<td>8</td>
<td>102.0</td>
<td>50.0</td>
<td>35.6</td>
<td>793.4</td>
<td>616.3</td>
<td>720.7</td>
<td>554.4</td>
</tr>
<tr>
<td>9</td>
<td>40.9</td>
<td>51.6</td>
<td>18.1</td>
<td>410.8</td>
<td>321.0</td>
<td>383.4</td>
<td>297.4</td>
</tr>
<tr>
<td>10</td>
<td>8.2</td>
<td>52.8</td>
<td>4.5</td>
<td>103.5</td>
<td>81.2</td>
<td>98.3</td>
<td>76.7</td>
</tr>
<tr>
<td>11</td>
<td>0.7</td>
<td>53.8</td>
<td>0.5</td>
<td>10.9</td>
<td>8.6</td>
<td>10.5</td>
<td>8.2</td>
</tr>
<tr>
<td>12</td>
<td>0.0</td>
<td>54.7</td>
<td>0.0</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>310.0</td>
<td>100.0</td>
<td>2218.4</td>
<td>1719.0</td>
<td></td>
<td></td>
<td>1990.5</td>
<td>1526.7</td>
</tr>
</tbody>
</table>

--- --- ---

SITE = 60.00
AGE = 20.00
NUMBER OF TREES = 309.97
BASAL AREA = 100.00
AVERAGE DBH = 7.64 BASED ON 1-INCH DBH CLASSES

AMOUNT REMOVED IN LOW THINNING

--- --- ---

NUMBER OF TREES = 240.03
BASAL AREA = 37.50
TOTAL CU.FT. VOLUME O.B. = 787.06
CU.FT. VOLUME O.B. TO 4 IN = 547.57
CORD VOLUME TO 4 IN = 6.46

Figure A3. Example output from program WITHIN --
Step 3: Low thinning at age 20.
25% ROW, 75% LOW THINNING DOWN TO 100 SQFT/ACRE. HARVEST AGE = 40.

INPUTS
-----
SITE = 60.00
AGE = 40.00
NUMBER OF TREES = 245.26
BASAL AREA = 164.52

PREDICTED
---------
HD = 81.14
AVERAGE DBH = 10.95
MINIMUM DBH = 5.87

THIS STAND WAS PREVIOUSLY THINNED FROM BELOW
ALL TREES UNDER 5.5 INCHES IN DBH WERE CUT

<table>
<thead>
<tr>
<th>DBH CLASS</th>
<th>NUMBER OF TREES</th>
<th>AVERAGE HEIGHT</th>
<th>BASALAREA</th>
<th>TOTAL VOLUME O.B.</th>
<th>TOTAL VOLUME I.B.</th>
<th>VOLUME TO 4.1N O.B.</th>
<th>VOLUME TO 4.1N I.B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.3</td>
<td>66.5</td>
<td>0.2</td>
<td>7.4</td>
<td>5.7</td>
<td>5.9</td>
<td>4.5</td>
</tr>
<tr>
<td>7</td>
<td>5.7</td>
<td>69.9</td>
<td>1.5</td>
<td>47.0</td>
<td>36.5</td>
<td>40.8</td>
<td>31.3</td>
</tr>
<tr>
<td>8</td>
<td>15.1</td>
<td>72.7</td>
<td>5.3</td>
<td>169.6</td>
<td>132.0</td>
<td>153.1</td>
<td>118.7</td>
</tr>
<tr>
<td>9</td>
<td>29.6</td>
<td>74.9</td>
<td>13.1</td>
<td>428.9</td>
<td>335.6</td>
<td>398.4</td>
<td>310.9</td>
</tr>
<tr>
<td>10</td>
<td>45.0</td>
<td>76.7</td>
<td>24.5</td>
<td>816.5</td>
<td>643.8</td>
<td>775.6</td>
<td>608.1</td>
</tr>
<tr>
<td>11</td>
<td>53.3</td>
<td>78.2</td>
<td>35.1</td>
<td>1188.0</td>
<td>938.8</td>
<td>1142.1</td>
<td>898.3</td>
</tr>
<tr>
<td>12</td>
<td>47.6</td>
<td>79.5</td>
<td>37.4</td>
<td>1281.1</td>
<td>1013.9</td>
<td>1241.9</td>
<td>979.1</td>
</tr>
<tr>
<td>13</td>
<td>30.5</td>
<td>80.6</td>
<td>28.1</td>
<td>975.8</td>
<td>773.2</td>
<td>951.8</td>
<td>751.8</td>
</tr>
<tr>
<td>14</td>
<td>13.2</td>
<td>81.6</td>
<td>14.1</td>
<td>492.4</td>
<td>390.6</td>
<td>482.5</td>
<td>381.7</td>
</tr>
<tr>
<td>15</td>
<td>3.5</td>
<td>82.4</td>
<td>4.3</td>
<td>152.3</td>
<td>120.9</td>
<td>149.8</td>
<td>118.6</td>
</tr>
<tr>
<td>16</td>
<td>0.5</td>
<td>83.2</td>
<td>0.7</td>
<td>26.4</td>
<td>21.0</td>
<td>26.0</td>
<td>20.6</td>
</tr>
<tr>
<td>17</td>
<td>0.0</td>
<td>83.8</td>
<td>0.1</td>
<td>2.3</td>
<td>1.8</td>
<td>2.3</td>
<td>1.8</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------</td>
<td>----------------</td>
<td>-----------</td>
<td>-------------------</td>
<td>-------------------</td>
<td>--------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>245.3</td>
<td>164.5</td>
<td>5584.7</td>
<td>4413.8</td>
<td>5370.3</td>
<td>4225.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

AVERAGE DBH = 10.95 BASED ON 1-INCH DBH CLASSES
CORD VOLUME TO 4.1N = 57.51

WEIBULL PARAMETERS
A = 4.5100
B = 7.0872
C = 4.1068

CONVERGENCE ATTAINED

Figure A4. Example output from program WITHIN --
Step 4: Project to age 40.
estimates over dbh classes gives a stand volume value of 3279 cu.ft. per acre.

Step 2: Row thinning at age 20.

In this example, 25% of the basal area removed is due to row thinning. Total basal area removed in two thinnings: 150 - 100 = 50 sq.ft. Residual basal area after row thinning:

\[ 150 - 0.25 \times 50 = 137.5 \text{ sq.ft.} \]

Let Q be the ratio of basal area after row thinning and basal area before thinning, \( Q = \frac{137.5}{150} = 0.9167 \). The stand and stock table after row thinning is constructed by multiplying the residual ratio Q by the entries in the stand and stock table before row thinning.

Number of trees in the 6-inch class: \( 0.9167 \times 143.3 = 131.4 \) trees. Basal area in the 6-inch class: \( 0.9167 \times 28.1 = 25.76 \) sq.ft. Volume in the 6-inch class: \( 0.9167 \times 597.4 = 547.6 \) cu.ft.

Step 3: Low thinning at age 20.

Basal area removed in low thinning: \( 0.75 \times 50 = 37.5 \) sq.ft. The diameter limit (Dthin) is searched for by summing basal area in each dbh class, starting from the lowest class, until the total is closest to but not greater than 37.5 sq.ft. Basal area of cut trees having dbh's of 5.5 inches and below:

\[ 0.4 + 2.9 + 10.9 = 14.2 \text{ sq.ft.} \]

Basal area of trees in the 6-inch class that are removed in low thinning: \( 37.5 - 14.2 = 23.3 \) sq.ft., which corresponds to:

\[ 131.4 \times 23.3 / 25.76 = 118.7 \text{ trees.} \]

Residual number of trees in the 6-inch class: \( 131.4 - 118.7 = 12.7 \) trees/acre. Trees in the 7-inch class and above are left in this low thinning.

Step 4: Project to age 40.

Stand attributes at age 40 are predicted from those at age 20 after thinning. The procedures for constructing the stand and stock table are similar to those described earlier in Step 1, except that a Weibull distribution left-truncated at a diameter of 5.5 inches is used in this case.
Appendix 2a. Input variable formats and description for program
WITHIN -- Subprogram identification card (first card).

<table>
<thead>
<tr>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
</table>
| 1      | I1     | IPROG    | = 1 = Call INPUT1: project a stand through time.  
           |        |          | = 2 = Call INPUT2: stand and stock tables for specified combinations of age,  
           |        |          | site, and density.  |
Appendix 2b. Input variable formats and descriptions for program
WITHIN -- Subprogram INPUT1.

<table>
<thead>
<tr>
<th>Card Type</th>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-3</td>
<td>F3.0</td>
<td>SI1</td>
<td>Site index in feet (base age 25 years).</td>
</tr>
<tr>
<td>4-6</td>
<td>F3.0</td>
<td>AGE1</td>
<td></td>
<td>Age in years of the present stand.</td>
</tr>
<tr>
<td>7-10</td>
<td>F4.0</td>
<td>XN1</td>
<td></td>
<td>Number of trees per acre at AGE1.</td>
</tr>
<tr>
<td>11-16</td>
<td>F6.2</td>
<td>BA1</td>
<td></td>
<td>Basal area in square feet per acre at AGE1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(Either XN1 or BA1 has to be specified).</td>
</tr>
<tr>
<td>17-18</td>
<td>I2</td>
<td>INDEX</td>
<td></td>
<td>= 1 = XN1 and BA1 are both inputs.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 2 = Only XN1 is input for density.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 3 = Only BA1 is input for density.</td>
</tr>
<tr>
<td>19-23</td>
<td>F5.2</td>
<td>DTHIN1</td>
<td></td>
<td>= 0 = This stand has never been thinned from below.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 0 = All trees having dbh below DTHIN1 were cut in a previous low thinning.</td>
</tr>
<tr>
<td>24-26</td>
<td>F3.0</td>
<td>AGE2</td>
<td></td>
<td>Age at the next input or decision period.</td>
</tr>
<tr>
<td>27-28</td>
<td>I2</td>
<td>NDEC</td>
<td></td>
<td>Number of decision cards, each card describes management routine (thinning or not) at a specified age.</td>
</tr>
<tr>
<td>29-30</td>
<td>I2</td>
<td>IOPT</td>
<td></td>
<td>= 0 = No title card for this stand.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 1 = Title card immediately follows this card.</td>
</tr>
<tr>
<td>31-32</td>
<td>I2</td>
<td>MORE</td>
<td></td>
<td>= 0 = No other stand. Stop when this stand is finished.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 1 = Another stand follows.</td>
</tr>
</tbody>
</table>
Appendix 2b. Input variable formats and descriptions for program
WITHN -- Subprogram INPUT1 (continued).

<table>
<thead>
<tr>
<th>Card Type</th>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>DECISION CARD</td>
</tr>
<tr>
<td>1-3</td>
<td>F3.0</td>
<td>AGE1</td>
<td></td>
<td>Current stand age, equal to AGE2 specified in the previous card.</td>
</tr>
<tr>
<td>4-6</td>
<td>F3.0</td>
<td>AGE2</td>
<td></td>
<td>Age at the next input or decision period (harvest age if this is the last decision card of this stand).</td>
</tr>
</tbody>
</table>
| 7-8       | I2     | ITHIN  |          | = 1 = No thinning at AGE1.  
            |        |        |          | = 2 = Row thinning at AGE1.  
            |        |        |          | = 3 = Low thinning at AGE1.  
            |        |        |          | = 4 = Row thinning followed by low thinning at AGE1. |
| 9-10      | I2     | JOPT   |          | (Needed only when IROW=2 or ILOW=2)  
            |        |        |          | = 1 = BTHIN is specified.  
            |        |        |          | = 2 = BRESR or BRES is specified. |
| 11-12     | I2     | IROW   |          | (Needed only when ITHIN=2 or 4).  
            |        |        |          | = 1 = Specify residual ratio (Q).  
            |        |        |          | = 2 = Residual ratio not specified. |
| 13-17     | F5.2   | Q      |          | = Residual ratio (after / before thinning), when ITHIN=2 and IROW=1.  
            |        |        |          | = Ratio of basal area removed in row thinning and total basal area removed, when ITHIN=4 and IROW=2. |
| 18-23     | F6.2   | BRESR  |          | (Needed only when JOPT=2 and IROW=2)  
            |        |        |          | Residual basal area per acre after row thinning. |
| 24-29     | F6.2   | BTHINR |          | (Needed only when JOPT=1 and IROW=2)  
            |        |        |          | Basal area per acre removed in row thinning. |
Appendix 2b. Input variable formats and descriptions for program
WTHIN -- Subprogram INPUT1 (continued).

<table>
<thead>
<tr>
<th>Card Type</th>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
</table>
| 2         | 30-31  | I2     | ILOW     | (Needed only when ITHIN=3 or 4)
|           |        |        |          | = 1 = All trees below a specified diameter limit (DTHIN) are cut.
|           |        |        |          | = 2 = Thin to a specified residual basal area (BRES). |
| 32-36     | F5.2   | DTHIN  |          | (Needed only when ILOW=1)
|           |        |        |          | All trees having dbh below DTHIN are cut. |
| 37-42     | F6.2   | BRES   |          | (Needed only when JOPT=2 and ILOW=2)
|           |        |        |          | Residual basal area per acre after low thinning. |
| 43-48     | F6.2   | BTHIN  |          | (Needed only when JOPT=1 and ILOW=2)
|           |        |        |          | Basal area per acre removed in low thinning. |
Appendix 2c. Input variable formats and description for program
WTIN -- Subprogram INPUT2.

<table>
<thead>
<tr>
<th>Column</th>
<th>Format</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>I4</td>
<td>ISB</td>
<td>Site index: Begin</td>
</tr>
<tr>
<td>5-8</td>
<td>I4</td>
<td>ISE</td>
<td>End</td>
</tr>
<tr>
<td>9-12</td>
<td>I4</td>
<td>ISI</td>
<td>Increment</td>
</tr>
<tr>
<td>13-16</td>
<td>I4</td>
<td>IAB</td>
<td>Stand age: Begin</td>
</tr>
<tr>
<td>17-20</td>
<td>I4</td>
<td>IAE</td>
<td>End</td>
</tr>
<tr>
<td>21-24</td>
<td>I4</td>
<td>IAI</td>
<td>Increment</td>
</tr>
<tr>
<td>25-28</td>
<td>I4</td>
<td>INB</td>
<td>Trees/acre: Begin</td>
</tr>
<tr>
<td>29-32</td>
<td>I4</td>
<td>INE</td>
<td>End</td>
</tr>
<tr>
<td>33-36</td>
<td>I4</td>
<td>INI</td>
<td>Increment</td>
</tr>
<tr>
<td>37-40</td>
<td>I4</td>
<td>IBB</td>
<td>Basal area: Begin</td>
</tr>
<tr>
<td>41-44</td>
<td>I4</td>
<td>IBE</td>
<td>End</td>
</tr>
<tr>
<td>45-48</td>
<td>I4</td>
<td>IB1</td>
<td>Increment</td>
</tr>
<tr>
<td>49-52</td>
<td>I4</td>
<td>INDEX</td>
<td>= 1 = Number of trees (IN) and basal area (IB) per acre are both inputs.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 2 = Only IN is input for density.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 3 = Only IB is input for density.</td>
</tr>
<tr>
<td>53-56</td>
<td>I4</td>
<td>IOPT</td>
<td>= 0 = No title card.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 = Title card immediately follows this card.</td>
</tr>
</tbody>
</table>
Appendix 3a. Input example for program WTHIN -- simulate a stand through time.

Stand 1:
Site index = 60 feet (base age 25 years).
Density at age 5 = 600 trees/acre.

Thinning: Age = 17. Amount = 38 sq.ft./acre. Type = ROW. 
Age = 22. Amount = 29 sq.ft./acre. Type = ROW.

Harvest age = 30 years.

Title: COILE AND SCHUMACHER (1964)

Stand 2:
Site index = 60 feet (base age 25 years).
Density at age 20 = 600 trees and 150 sq.ft./acre.

Thinning: Age = 20. Thin to 100 sq.ft./acre. Type = 25% ROW, 75% LOW.
Harvest age = 40 years.

Title: 25% ROW, 75% LOW THINNING

Card Input:

<table>
<thead>
<tr>
<th>Column:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1234567890...5...0...5...0...5...0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>60 5 600</td>
<td>2</td>
<td>17 2 1 1</td>
<td></td>
</tr>
<tr>
<td>COILE AND SCHUMACHER (1964)</td>
<td>17 22 2 1 2</td>
<td>38.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 30 2 1 2</td>
<td>29.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 20 600150.00</td>
<td>20 1 1 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25% ROW, 75% LOW THINNING</td>
<td>20 40 4 2 2 0.25100.00</td>
<td>2</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 3b. Input example for program WTHIN -- stand and stock tables for specified combinations of site index, age, and density.

Combinations:

Site index = 50 feet (base age 25 years).
Stand age = 10, 15, 20, 25, 30 years.
Number of trees = 200, 400, 600, 800 trees/acre.
Basal area = 50, 100, 150, 200 sq.ft./acre.
No title wanted.

Card input:

<table>
<thead>
<tr>
<th>Column:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1234567890...5...0...5...0...5...0...5...0...5...0...5...0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50 50 10 10 30 5 200 800 200 50 200 50 1 0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 4. Generalized flowchart of program WITHIN.
Appendix 4. Generalized flowchart of program WITHIN (continued).

```
GROW

Stand variables: Site, Age1, N1, B1

Call YIELD

Thinning? No

Yes

Change N1 and B1

Project to Age2
Site, Age2, N2, B2

Call YIELD

Return

YIELD

Whole stand model
Predict Dmin, D

Call DIST

No

Call OUTPUT

Yes

Call FINDB

Previously low-thinned?

Call FINDB

Compute FCN(c)

Solve FF(b)=0

Return

PCN

No

Yes

Call FINDB

Solve FCN(c)=0

Return

Return

Return
```
Appendix 5. Source listing of program WTHIN.

```
C
C
C ***************
C *
C * PROGRAM WTHIN PRODUCES STAND AND STOCK TABLES  *
C * FOR THINNED LOBOLLY PINE PLANTATIONS.    *
C * *
C * DEVELOPED BY QUANG V. CAO          *
C * VPI & SU.  AUGUST 1, 1981          *
C *
C ***************
C
CALL ERRSET(208,256,-1,1)
CALL ERRSET(207,256,-1,1)
CALL ERRSET(209,256,-1,1)
CALL ERRSET(262,256,-1,1)
CALL ERRSET(263,256,-1,1)
READ(5,500) Iprog
500 FORMAT(11) IF(Iprog.EQ.1) CALL INPUT1
            IF(Iprog.EQ.2) CALL INPUT2
            RETURN
            END
SUBROUTINE INPUT1

C
C
C ***************
C *
C * SUBROUTINE INPUT1 READS THE NECESSARY INPUTS *
C * FOR SUBROUTINE GROW.                        *
C *
C ***************
C
IMPLICIT REAL*8 (A-H, O-Z)
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DHED,DHAX,DDBAR,IMAX,IMIN
COMMON /TWO/ SI1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTW
          COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDBLCT,TVOB1,TVOB41
          COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
          DATA IBLANK/.TRUE./
C----- READ STAND DESCRIPTION CARD.
C 1 READ(5,500,END=999) SI1,AGE1,XN1,BA1,INDEX,DTHIN1,AGE2,NDEC,IOPT
C       MORE
500 FORMAT(2F3.0,F4.0,F6.2,I2,F5.2,F3.0,3I2)
      ITHIN=1
      JJJ=0
```
Appendix 5. Source listing of program WTHIN (continued).

C------  READ TITLE CARD IF ANY.  
C     DO 2 I=1,20  
2  ITITLE(I)=IBLANK  
   IF(JOFT.EQ.1) READ(5,501) (ITITLE(I),I=1,20)  
501  FORMAT(20A4)  
   CALL GROW  
C------  READ DECISION CARDS.  
C     IF(MORE.EQ.1.AND.NDEC.EQ.0) GO TO 1  
   IF(MORE.EQ.1.AND.NDEC.EQ.0) RETURN  
   DO 3 I=1,NDEC  
502  READ(5,502) AGE2,IITHN,JOFT,IROW,Q,BRESR,BTHINR,ILOW,DTHIN,BRES  
   :  
   BTHIN  
   ATE=AGE  
   XN1=XN  
   BA1=BA  
   IF(JOFT.EQ.1.AND.IROW.EQ.2) BRESR=BA1-BTHINR  
   IF(JOFT.EQ.1.AND.ILLOW.EQ.2) BRES=BA1-BTHIN  
   IF(JOFT.EQ.1.AND.ILLOW.EQ.2.AND.IITHN.EQ.4) BRES=BRESR-BTHIN  
   INDEX=1  
3  CALL GROW  
   IF(MORE.EQ.1) GO TO 1  
999  RETURN  
END  
SUBROUTINE INPUT2  

*****************************************************************  
* SUBROUTINE INPUT2 READS THE NECESSARY INPUTS  
* FOR SUBROUTINE YIELD.  
*****************************************************************  
C
IMPLICIT REAL*8 (A-H,O-Z)  
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DLOG,DMAX,DMIN,IA,IMAX,IMIN  
COMMON /TWO/ SI1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN  
     :  
     ,INDEX,ITHTN,ILLOW,IROW  
 COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOC,MDLOG,TVOB1,TVOB41  
     ,CVOB41,JOFT,NN  
 COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA  
 DATA IBLANK/*  
1  READ(5,500,END=999) ISB,ISE,ISI,ISB,IAE,IAI,INB,INE,INI,IBB  
     :  
     ,IBE,IBI,INDEX,JOFT  
500  FORMAT(14I4)  
   DO 2 I=1,20  
2  ITITLE(I)=IBLANK  
   IF(JOFT.EQ.1) READ(5,501) (ITITLE(I),I=1,20)  
501  FORMAT(20A4)  

Appendix 5. Source listing of program WTHIN (continued).

C----- DO LOOPS. CHECK INDEX FOR INPUTS FOR STAND DENSITY.
C
DO 40 IS=1SB, ISE, ISI
SI=DFLOAT(IS)
DO 30 IA=1AB, IAE, IAI
AGE=DFLOAT(IA)
AINV=1.0/AGE
CALL HEIGHT
AHI=AINV/HD
GO TO (13, 11, 12), INDEX
11 IBB=100
IBF=IBB
IBI=50
GO TO 13
12 INB=100
INE=INB
INI=50
13 DO 20 IN=INB, INE, INI
GO TO (21, 22, 23), INDEX
21 XN=DFLOAT(IN)
XNLOG=DLOG(XN)
GO TO 23
22 XN=DFLOAT(IN)
XNLOG=DLOG(XN)
BLOG=4.39180687D0 + 0.19054366D0*AINV
+ 1.34753473D0*HDLOG + 0.63902092D0*XNLOG
BA=DEXP(BLOG)
23 DO 10 IB=1BB, IBF, IBI
GO TO (31, 33, 32), INDEX
31 BA=DFLOAT(IB)
BLOG=DLOG(BA)
GO TO 33
32 BA=DFLOAT(IB)
BLOG=DLOG(BA)
XNLOG=7.79805237D0 + 2.10495039D0*AINV
- 1.8990831100*HDLOG + 1.16743646D0*BLOG
XN=DEXP(XNLOG)
33 CONTINUE

C----- SOLVE FOR DIAMETER CDF.
C
CALL YIELD
10 CONTINUE
20 CONTINUE
30 CONTINUE
40 CONTINUE
GO TO 1
999 RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

SUBROUTINE GROW

*******************************************************************************
*  SUBROUTINE GROW PRODUCES A STAND AND STOCK TABLE AT AGE1. THE STAND IS THEN SUBJECT TO  
*  THINNING (OR NO THINNING), AND THEN PROJECTED TO AGE2.  
*  TO AGE2.  
*******************************************************************************

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ S11,AGE1,XN1,BA1,DTH1N1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN

COMMON /THREE/ INDEX,I1TH1N,1LOW,IROW

COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
DATA B1/0.0227300/, B2/-0.0110300/

IF(AGE1.EQ.AGE.AND.XN1.EQ.XN.AND.BA1.EQ.BA) GO TO 5
DTHIN=DTHIN1+0.5D0
DTHIN1=DFLOAT(I1TH1N)-0.5D0
SI=S11
AGE=AGE1
AINV=1.0D0/AGE
CALL HEIGHT
GO TO (1,2,3), INDEX

C----- INDEX = 1 = BOTH XN1 AND BA1 ARE INPUTS FOR STAND DENSITY.
C
1 XNLOG=DLOG(XN1)
   BLOG=DLOG(BA1)
   GO TO 4
C----- INDEX = 2 = ONLY XN1 IS INPUT FOR STAND DENSITY.
C
2 XNLOG=DLOG(XN1)
   IF(JJJ.EQ.0) BLOG=DLOG(10.0D0)*(1.4366D0*DLOG10(SI)-7.0840D0*AINV
      +0.4888D0*DLOG10(XN1)+1.3851D0)
   IF(JJJ.EQ.1) BLOG=-4.39180687D0 + 0.19054366D0*AINV
      + 1.34753473D0*DLOG + 0.63902092D0*XNLOG
   BA1=DEXP(BLOG)
   GO TO 4
C----- INDEX = 3 = ONLY BA1 IS INPUT FOR STAND DENSITY.
C
3 BLOG=DLOG(BA1)
   IF(JJJ.EQ.0) XNLOG=DLOG(10.0D0)*(1.4366D0*DLOG10(SI)-7.0840D0*AINV
      -DLOG10(BA1)-1.3851D0)/(-0.4888D0)
   IF(JJJ.EQ.1) XNLOG=7.79805237D0 + 2.10495039D0*AINV
      -1.89908311D0*DLOG + 1.16743646D0*LOG
   XN1=DEXP(XNLOG)

WTH01570  WTH01580  WTH01590  WTH01600  WTH01610
WTH01620  WTH01630  WTH01640  WTH01650  WTH01660
WTH01670  WTH01680  WTH01690  WTH01700  WTH01710
WTH01720  WTH01730  WTH01740  WTH01750  WTH01760
WTH01770  WTH01780  WTH01790  WTH01800  WTH01810
WTH01820  WTH01830  WTH01840  WTH01850  WTH01860
WTH01870  WTH01880  WTH01890  WTH01900  WTH01910
WTH01920  WTH01930  WTH01940  WTH01950  WTH01960
WTH01970  WTH01980  WTH01990  WTH02000  WTH02010
WTH02020  WTH02030  WTH02040  WTH02050  WTH02060
WTH02070  WTH02080  WTH02090  WTH02100
Appendix 5. Source listing of program WTHIN (continued).

C
C-----  SOLVE FOR DIAMETER CDF.
C
4  BA=BA1
   XN=XN1
   CALL YIELD
C
C-----  THINNING AT AGE1.
C
5  CALL THIN
   IF(AGE.EQ.AGE2) RETURN
C
C-----  PROJECT TO AGE2.
C
AGE=AGE2
AINV=1.0D0/AGE2
CALL HEIGHT
C=5.40815546D0 + 0.321208D-2*S1
XNPLOG=DLOG10(XN1)
XNPLOG=(XNPLOG - B1*AGE1)/(1.0D0 + B2*AGE1)
IF(JJJ.EQ.0) XNLOG=DLOG(10.0D0)*(XNPLOG + AGE*
S (B1 + B2*XNPLOG))
IF(JJJ.EQ.1) XNLOG=DLOG(-0.658083D0*XNPLOG)+0.75795D-5
S *(AGE1**1.78018705D0+AGE2**1.78018705D0))/0.658083D0
XN=DEXP(XNPLOG)
IF(JJJ.EQ.0) BLOG=DLOG(10.0D0)*(1.4366D0*DLOG10(S1)-7.084D0)
S *AINV + 0.488800D0*DLOG10(XN) -1.385100)
IF(JJJ.EQ.1) BLOG=C1 + (BLOG-C1)*AGE1/AGE2
BA=DEXP(BLOG)
C
C-----  SOLVE FOR DIAMETER CDF.
C
CALL YIELD
RETURN
END
SUBROUTINE YIELD

**************************************************************************
*  *
* SUBROUTINE YIELD PRODUCES A STAND AND STOCK *
* TABLE FOR A SPECIFIED COMBINATION OF AGE, *
* SITE, AND DENSITY. *
* *
**************************************************************************

CALL MODEL
CALL DIST
CALL OUTPUT(1)
RETURN
END
Appendix 5. Source listing of program WITHIN (continued).

SUBROUTINE HEIGHT

*******************************************************************************
*                     * SUBROUTINE HEIGHT COMPUTES HEIGHT OF THE                  *
*                     * DOMINANTS AND CODEOMINANTS OF A STAND, GIVEN               *
*                     * SITE INDEX AND AGE.                                *
*                     * FROM JIM DEVAN'S THESIS (1979).                          *
*                     *                                           *
*******************************************************************************

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41
                  ,CVOB41,IOP7,JJJ
DATA X0/0.0400/, XL/0.200/, A/5.9617800/
     : B1/-5.2779400/, B2/19.9004700/, B3/-58.7612200/
X=AINV
Z=DEXP(A*(X-X0))
X0Z=X0*Z
Y0=DBLOG(S1)
HDLOG=Y0*Z+B1*(Z-1.D0)+B2*(X0Z-X)+B3*(X0Z*X0-X*X)
HD=DEXP(HDLOG)
RETURN
END

SUBROUTINE MODEL

*******************************************************************************
*                     * SUBROUTINE MODEL PREDICTS FROM THE STAND                  *
*                     * CHARACTERISTICS MINIMUM AND AVERAGE DIAMETERS.          *
*                     *                                           *
*******************************************************************************

IMPLICIT REAL*8 (A-H,O-Y)
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /THREE/ ITITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41
                  ,CVOB41,IOP7,JJJ
DATA DQ/(BA/(0.5454150-2*XN))**0.500
DMIN=DEXP(1.10834919D0 + 5.10754613D0*AINV + 0.50530582*HDLOG 
     : + 0.26543547D0*BLOG - 0.57131133D0*XNLOG)
DBAR=DQ + DEXP(-9.05733080D0 + 0.89273788D0*HDLOG 
     : + 0.58151144*XNLOG)
RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

```
SUBROUTINE DIST

* ***********************************************************************
* * SUBROUTINE DIST SOLVES FOR WEIBULL PARAMETERS                   *
* * FOR DBH, GIVEN BA, N, MINIMUM AND AVERAGE DBH.                    *
* * ***********************************************************************

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ SI,ACE,XN,BH,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /THREE/ TITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41
    ,CVOB41,OPT,JJJ
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
EXTERNAL FCN
DATA TOL/0.005/

C----- INITIALIZE VARIABLES.

CONST=LOG(2.5DO/XN)
I=DMIN=0.5DO
A=1.0-49DO
IF(A.LT.0.0D0) A=0.0D0
W1=0.5DO
IMIN=0.5DO+A
IF(IMIN.LE.0) IMIN=1

C----- SOLVE EQUATION: FCN(C) = 0, USING THE SECANT METHOD.

CALL SECAN1(FCN,TOL,W1,ITER,IER)
C=10.0D0*(1.0D0+DERF(W1))
RETURN
END

SUBROUTINE SECAN1(F,ERROR,SOL,ITER,IER)

* ***********************************************************************
* * SECANT METHOD
* * * FIND A ROOT OF A NONLINEAR EQUATION F(X) = 0.  
* * * INPUTS : F = FUNCTION. 
* * * ERROR = PROCEDURE IS STOPPED WHEN  
* * * IF(X) I < ERROR. 
* * * SOL = A GUESS OF THE SOLUTION TO 
* * * F(X) = 0. 
* *  
* * * OUTPUTS : SOL = SOLUTION TO F(X) = 0.  
* * * ITER = NUMBER OF ITERATIONS. 
* * * IER = 0 = A ROOT IS FOUND. 
* * * = 1 NO ROOT IS FOUND AFTER 
* * * 50 ITERATIONS. 
* * * 
* * ***********************************************************************

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
```

WTH03120 WTH03130 WTH03140 WTH03150 WTH03160 WTH03170 WTH03180 WTH03190 WTH03200 WTH03210 WTH03220 WTH03230 WTH03240 WTH03250 WTH03260 WTH03270 WTH03280 WTH03290 WTH03300 WTH03310 WTH03320 WTH03330 WTH03340 WTH03350 WTH03360 WTH03370 WTH03380 WTH03390 WTH03400 WTH03410 WTH03420 WTH03430 WTH03440 WTH03450 WTH03460 WTH03470 WTH03480 WTH03490 WTH03500 WTH03510 WTH03520 WTH03530 WTH03540 WTH03550 WTH03560 WTH03570 WTH03580 WTH03590 WTH03600 WTH03610 WTH03620 WTH03630 WTH03640 WTH03650 WTH03660 WTH03670 WTH03680 WTH03690 WTH03700 WTH03710 WTH03720
Appendix 5. Source listing of program WTHIN (continued).

C---- INITIALIZE.
C
IER=0
ITER=0
X0=SOL
F0=F(X0)
B0=B
X1=X0+0.5D0
F1=F(X1)
AFMIN=DAABS(F1)
XMIN=X1
BMIN=B
IF(AFMIN.LT.DABS(F0)) GO TO 1
C1=X0
C2=F0
X0=X1
F0=F1
X1=C1
F1=C2
AFMIN=DAABS(F1)
XMIN=X1
BMIN=B0
C---- START THE ITERATIVE PROCEDURE.
C
1 IER=IER+1
SOL=(X0*F1-X1*F0)/(F1-F0)
IF(DABS(SOL).GT.5.E0) GO TO 3
F2=F(SOL)
AF2=DAABS(F2)
IF(AF2.GE.AFMIN) GO TO 2
AFMIN=AF2
XMIN=SOL
BMIN=B
C---- CHECK CONVERGENCE.
C
2 IF(AF2.LE.ERROR) RETURN
IF(ITER.GE.50) GO TO 3
C---- REinitialize VARIABLES.
C
X0=X1
F0=F1
X1=SOL
F1=F2
GO TO 1
C---- NO SOLUTION AFTER 50 ITERATIONS.
C
3 IER=1
SOL=XMIN
B=BMIN
RETURN
END

WTH03730
WTH03740
WTH03750
WTH03760
WTH03770
WTH03780
WTH03790
WTH03800
WTH03810
WTH03820
WTH03830
WTH03840
WTH03850
WTH03860
WTH03870
WTH03880
WTH03890
WTH03900
WTH03910
WTH03920
WTH03930
WTH03940
WTH03950
WTH03960
WTH03970
WTH03980
WTH03990
WTH04000
WTH04010
WTH04020
WTH04030
WTH04040
WTH04050
WTH04060
WTH04070
WTH04080
WTH04090
WTH04100
WTH04110
WTH04120
WTH04130
WTH04140
WTH04150
WTH04160
WTH04170
WTH04180
WTH04190
WTH04200
WTH04210
WTH04220
WTH04230
WTH04240
WTH04250
WTH04260
WTH04270
WTH04280
Appendix 5. Source listing of program WTHIN (continued).

DOUBLE PRECISION FUNCTION FCN(WI)

*******************************************************************************
*                      *                      *
* FUNCTION FCN IS CALLED BY SUBROUTINE SECAN1                           *
* TO EVALUATE THE LEFT-HAND SIDE OF EQUATION:                           *
* FCN(C) = 0.                                                           *
*
*******************************************************************************

IMPLICIT REAL*8 (A-H,O-Y)
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ S11,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
*: INDEX, ITHIN, ILOW, IROW
COMMON /FOUR/ A,B,MIN,C,CONST,CINV,GAMMA

C----- INITIALIZATION.
C
C = 10.0*(1.0+DERF(WI))
CINV = 1.0/C
GAMMA = DCMANM(1.0+CINV)
B = (DBAR+A)/GAMMA
IMAX = 1.5*D+A*B*CONST*(CINV)
FCN = 0.0
IF(A.LT.DTHIN1) GO TO 2
F1 = 0.0

C----- COMPUTE FCN.
C
DO 1 I = IMIN, IMAX
XI = DDFloat(I)
F2 = CDF(XI)*0.50
F = F2 - F1
IF(F.LT.0.00) F = 0.00
IF(F.LT.0.00) F = 0.00
F1 = F2
1 FCN = FCN + X1*XI*F
FCN = FCN*0.545415D-2*XN-BA
RETURN

C----- WHEN THE LOCATION PARAMETER (A) IS LOWER THAN DTHIN1.
C
2 CALL FINDB
F1 = CDF(DTHIN1)
FRES = 1.0-F1
IMIN = DTHIN1+0.5100
DO 3 I = IMIN, IMAX
XI = DDFloat(I)
F2 = CDF(XI)*0.50
F = (F2-F1)/FRES
IF(F.LT.0.00) F = 0.00
IF(F.LT.0.00) F = 0.00
F1 = F2
3 FCN = FCN + F*XI*F
FCN = FCN*0.545415D-2*XN-BA
RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

SUBROUTINE FINDB

******************************************************************************
* * SUBROUTINE FINDB SEARCHES FOR THE WEIBULL * *
* * PARAMETER B, GIVEN A AND C, IN CASE OF LEFT- * *
* * TRUNCATION DUE TO LOW THINNING. * *
******************************************************************************

IMPLICIT REAL*8 (A-H,O-Y)
COMMON /ONE/ S1,A,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMIN,IMAX
COMMON /TWO/ S11,AGET,XNT,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
:COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
EXTERNAL FF
DATA TOL/0.5D-2/
W2=-0.6D0
CALL SECAN2(F,F,TOL,W2,ITER,IER)
B=T1.O.D0*(1.D0+DERF(W2))
RETURN
END

SUBROUTINE SECAN2(F,ERROR,SOL,ITER,IER)

******************************************************************************
* * SECANT METHOD * *
* * FIND A ROOT OF A NONLINEAR EQUATION F(X) = 0. * *
* * INPUTS : * *
* * F = FUNCTION. * *
* * ERROR = PROCEDURE IS STOPPED WHEN * *
* * |F(X)| < ERROR. * *
* * SOL = A GUESS OF THE SOLUTION TO * *
* * F(X) = 0. * *
* * OUTPUTS : * *
* * SOL = SOLUTION TO F(X) = 0. * *
* * ITER = NUMBER OF ITERATIONS. * *
* * IER = 0 = A ROOT IS FOUND. * *
* * = 1 = NO ROOT IS FOUND AFTER * *
* * 50 ITERATIONS. * *
******************************************************************************
Appendix 5. Source listing of program WITHIN (continued).

```
IMPLICIT REAL*8 (A-H,O-Z)
C
C------  INITIATION.
C
IER=0
ITER=0
X0=SOL
F0=F(X0)
X1=X0+0.5D0
F1=F(X1)
AFMIN=DAABS(F1)
XMIN=X1
IF(AFMIN.LT.DABS(F0)) GO TO 1
C1=X0
C2=F0
X0=X1
F0=F1
X1=C1
F1=C2
AFMIN=DAABS(F1)
XMIN=X1
C
C------  START THE ITERATIVE PROCEDURE.
C
1  ITER=ITER+1
   SOL=(X0*F1-X1*F0)/(F1-F0)
   IF(DABS(SOL).GT.5.0D0) GO TO 3
   F2=F(SOL)
   AF2=DAABS(F2)
   IF(AF2.GE.AFMIN) GO TO 2
   AFMIN=AF2
   XMIN=SOL
C
C------  CHECK CONVERGENCE.
C
2  IF(AF2.LE.ERROR) RETURN
   IF(ITER.GE.50) GO TO 3
C
C------  REINITIALIZE VARIABLES.
C
   X0=X1
   F0=F1
   X1=SOL
   F1=F2
   GO TO 1
C
C------  NO SOLUTION AFTER 50 ITERATIONS.
C
3  IER=1
   SOL=XMIN
   RETURN
END
```
Appendix 5. Source listing of program WTHIN (continued).

DOUBLE PRECISION FUNCTION FF(W2)

*******************************************************************************
* FUNCTION FF IS CALLED BY SUBROUTINE SECAN2 TO *
* EVALUATE THE LEFT-HAND SIDE OF THE EQUATION: *
* FF(B) = 0. *
*******************************************************************************

IMPLICIT REAL*8 (A-H,O-Y)
COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ SI,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
COMMON /FOUR/ A,B,BMIN,C,CONST,CIKY,GAMMA
EXTERNAL Y
B=10.DO*(1.DO+DERF(W2))
FRES=1.DO-CDF(DTHIN1)

C----- EVALUATE THE INCOMPLETE GAMMA INTEGRAL.
C
ZA=0.DO
ZB=((DTHIN1-A)/B)**C
CALL GAUSS(Y,ZA,ZB,S)

C----- EVALUATE FF(B).
C
FF=A+B*(GAMMA-S)/FRES=DBAR
RETURN
END

SUBROUTINE GAUSS(F,XA,XB,S)

*******************************************************************************
* GAUSS QUADRATURE METHOD *
*******************************************************************************
* INPUTS:  F = FUNCTION TO BE INTEGRATED. *
* XA AND XB = LOWER AND UPPER LIMITS OF *
* INTEGRATION. *
* OUTPUT:  S = VALUE OF THE INTEGRAL. *
*******************************************************************************

C

C

C

C

C

C

C

C

C

C
Appendix 5. Source listing of program WTHIN (continued).

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION Y(5),W(5)
DATA Y/0.149074339000,
: 0.433953941000,
: 0.679497968300,
: 0.885063366700,
: 0.973906528500,
: 0.295524224700,
: 0.269266719300,
: 0.219086362500,
: 0.149451394200,
: 0.06661344300/, M/5/
C1=0.5D0*(X+X)
C2=0.5D0*(X-X)
S=0.00
DO 2 I=1,M
C3=C2*Y(I)
2 S=S+W(I)*(F(C1+C3)+F(C1-C3))
S=S*C2
RETURN
END

DOUBLE PRECISION FUNCTION Y(X)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON /FOUR/ A,B,MIN,C,CONST,CINV,GAMMA
Y=X**CINV*DEXP(-X)
RETURN
END

DOUBLE PRECISION FUNCTION CDF(XX)

**********************************************************************
** FUNCTION CDF EVALUATES THE WEIBULL CDF.                         **
**********************************************************************
IMPLICIT REAL*8 (A-H,O-Y)
COMMON /FOUR/ A,B,MIN,C,CONST,CINV,GAMMA
CDF=0.0
IF(XX.LE.A) RETURN
C1=C*LOG((XX-A)/B)
C2=0.0
IF(C1.GT.-50.00.AND.C1.LT.50.00) C2=-DEXP(C1)
IF(C1.GT.50.00) C2=-1.08
C3=0.0
IF(C2.GT.-50.00) C3=DEXP(C2)
CDF=1.00-C3
RETURN
END
Appendix 5. Source listing of program WTHIN (continued).

SUBROUTINE OUTPUT(111)  

*******************************************************************************
* * SUBROUTINE OUTPUT PRINTS THE STAND AND STOCK * *
* TABLE. * *
* * ***************************************************************************

IMPLICIT REAL*8 (A-H,O-Z)  
DIMENSION CF(20),ROB(3),RIB(3),ROB(2),BIB(2),BH(2)  
COMMON /ONE/ SI,AGE,XN,BA,RO,DIN,DMED,DMAX,DMIN,IMAX,IMIN  
COMMON /TWO/ SI1,AGE1,XN1,BA1,DTH1,AGE2,Q,DTH2,IMAX,IMIN  
: INDEX,ITION,LOW,ROW, 
COMMON /THREE/ TITLE(20),AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41  
: CVOB01,IOPT,JJJ  
COMMON /FOUR/ A,B,DIN,C,CONST,C INV,GMMA  
DATA ROB/-0.32354000, 3.157900, -2.711500/  
: RIB/-0.352000, 3.076300, -2.654000/  
: ROB/ 0.348600, 0.002300/  
: BIB/ 0.116900, 0.001850/  
: TOP/4,DO,KROWN,ROW,LOW, /  
COMMON /FIVE/ TITLE(20),A,TIT1,AINV,XNLOG,BLOG,HDLOG,TVOB1,TVOB41  
: CVOB01,IOPT,JJJ  
COMMON /SIX/ A,B,DIN,C,CONST,C INV,GMMA  
DATA ROB/-0.32354000, 3.157900, -2.711500/  
: RIB/-0.352000, 3.076300, -2.654000/  
: ROB/ 0.348600, 0.002300/  
: BIB/ 0.116900, 0.001850/  
: TOP/4,DO,KROWN,ROW,LOW, /  
DATA CF/0.0,0.0,0.0,84.85,86.87,90.91,92.93,94.95,96,97.98,99.99,99. /  
BH(1)=0.0615154000 + 0.4327425100*AINV + 0.9333381000*HDLOG  
BH(2)=2.1531122600  
TOPB=TOP**ROB(2)  
TOPB=TOP**RIB(2)  

**** WRITE HEADINGS. ****

C-----  
IF(I11,EQ,2) GO TO 11  
WRITE(6,666) (TITLE(I11),I11=1,20)  
666 FORMAT(11,'/10X,20A4)  
WRITE(6,599) SI,HD,AGE,DEBAR,XN,DMIN,BA  
599 FORMAT(3/3X, 'INPUTS',22X, 'PREDICTED')/
: 33X,6('.'),22X,9('.'),/
: 31X, 'SITE',F7.2,18X,'HD=',F6.2,/
: 32X, 'AGE=',F7.2,9X,'AVG DBH=',F6.2,/
: 20X, 'NDB',F7.2,9X,'MIN DBH=',F6.2,/
: 20X, 'BASAL AREA=',F7.2)  
GO TO 12  
11 KTYPE=KLLOW  
! I F I THIN.NE. 3 KTYPE=KROW  
WRITE(6,600) KTYPE  
600 FORMAT(15X, 'BEFORE ',AH, 'THINNING'/15X,6('.'),---,8('.'))  
WRITE(6,601) SI,AGE1,XN,BA,DEBAR  
601 FORMAT(42X, 'SITE=',F7.2,43X,'AGE=',F7.2,31X,'NUMBER OF '  
: 'TREES=',F7.2,36X,'BASAL AREA=',F7.2  
: \ 35X,'AVG DBH=',F7.2)  
12 IF(A,LT.DTH1) WRITE(6,602) DTH1  
602 FORMAT(15X,'THIS STAND WAS PREVIOUSLY THINNED FROM BELOW'  
: '/26X,'ALL TREES UNDER ',F5.1,' INCHES IN DBH WERE CUT')  
WRITE(6,603) KTYPE  
603 FORMAT(15X,'AFTER ',AH, 'THINNING'/15X,5,'---',8('.'))  
WRITE(6,604) TOP,TOP  
604 FORMAT(15X,'TOTAL',6X,'TOTAL',5X,'VOLUME',5X,'VOLUME'  
: '/9X,DBH',5X,'NUMBER',4X,'AVERAGE',5X,'BASAL',5X  
: \ 'VOLUME',5X,'AREA',7X,\ 'DBH',5X,'VOLUME',7X,\ 'AREA',7X  
: \ 'DBH',5X,'VOLUME',7X,'DBH',5X,'VOLUME',7X)  
! O.B.7X,'DBH'7X,'VOLUME',7X,'TREES',4X,'TO/',F3.0,'IN',4X,'TO',F3.0,'IN'//  
! O.B.7X,'DBH'7X,'VOLUME',7X,'TREES',4X,'TO/',F3.0,'IN',4X,'TO',F3.0,'IN'//  
}
Appendix 5. Source listing of program WTHIN (continued).

C
C----- INITIALIZATION.
C
C F1=0.0D0
B8=0.0D0
XRES=0.0D0
DAVG=0.0D0
TVOB=0.0D0
TVIB=0.0D0
TVOB4=0.0D0
TVIB4=0.0D0
CVOB4=0.0D0
XNT=XN
IMIN1=IMIN
IF(111.EQ.2) GO TO 13
IF(A.GE.DTHIN1) GO TO 3
F1=CDF(DTHIN1)
IMIN1=DTHIN1+0.51D0
XNT=XN/(1.0D0-F1)
GO TO 3
13 IF(TTHIN.EQ.1) GO TO 3
IF(TTHIN.EQ.3) GO TO 2
IF(A.LT.DTHIN1) GO TO 1
C
C----- ROW THINNING. NO PREVIOUS LOW THINNING.
C
XNT=XN*Q
GO TO 3
C
C----- ROW THINNING. PREVIOUS LOW THINNING.
C
1 F1=CDF(DTHIN1)
XNT=XN*Q/(1.0D0-F1)
IMIN1=DTHIN1+0.51D0
GO TO 3
C
C----- LOW THINNING.
C
2 F1=CDF(DTHIN)
IF(A.LT.DTHIN1) XNT=XN/(1.0D0-CDF(DTHIN1))
IMIN1=DTHIN1+0.51D0
C
C----- LOOP OVER D8H CLASSES.
C
3 CONTINUE
DO 5 I=IMIN1,1MAX
XI=D FLOAT(I)
F2=CDF(XI+0.5D0)
F=XNT*(F2-F1)
IF(111.EQ.111.AND.111.EQ.2) F=F*QTHIN
IF(F.LT.0.0D0) F=0.0D0
F1=F2
XI2=XI*XI
BASAL=0.545415D-2*X12*F
H=DEXP(BH(1)+BH(2)/XI)
D2H=XI2*H
VOB=F*(BOB(1)+BOB(2)*D2H)
VIB=F*(BIB(1)+BIB(2)*D2H)
VOB4=0.0D0
VIB4=0.0D0
IF(1.LT.5) GO TO 4
VOB4=VOB4*(1.0D0+ROB(1)+TOPOB*X1**ROB(3))
VIB4=VIB4*(1.0D0+RIB(1)+TOPB*X1**RIB(3))
5 CONTINUE
C
C----- END OF WTHIN.
C
Appendix 5. Source listing of program WTHN (continued).

4 IF (I.LE.20) CVOB4=CVOB4+VOB4/CF(I)
   DAVG=DAVG+F*X1
   B0=BB+BASAL
   XNRES=XNRES+F
   TV08=TV08+V08
   TV1B=TV1B+V1B
   TV04=TV04+V04
   TV14=TV14+V14
5 WRITE(6,605) I,F,H,BASAL,V08,V1B,V04,V14
605 FORMAT(111,7F11.1)
   DAVG=DAVG/XNRES
   C
   C---- END LOOP.
   C
   IF (I.EQ.2) GO TO 7
   WRITE(6,608) XNRES,BB,TV08,TV1B,TV04,TV14,DAVG,Top,CVOB4,A,B,C
   FORMAT(16X,6(' - '),11X,5(5X,6(' - '))/11X,F11.1,11X,5F11.1)
      /35X,'AVERAGE DBH =',F7.2,2X,'BASED ON 1-INCH DBH CLASSES'
      /27X,'CONE VOLUME TO ',F3.0,' IN =',F7.2
      /31X,'WEIBULL PARAMETERS'
      /45X,'A = ',F7.4
      /45X,'B = ',F7.4
      /45X,'C = ',F7.4
   C1=DABS(BA-BB)
   IER=1
   IF(C(I).LT.0.05) IER=0
   IF(IER.EQ.0) WRITE(6,609)
   FORMAT(/35X,'CONVERGENCE ATTAINED'
   IF(IER.NE.0) WRITE(6,610)
   FORMAT(/25X,'DIFFERENCE IN BASAL AREA > 0.05 SQ. FT./acre')
   GO TO 8
609 WRITE(6,606) XNRES,BB,TV08,TV1B,TV04,TV14,DAVG,Top,CVOB4,A,B,C
   FORMAT(16X,6(' - '),11X,5(5X,6(' - '))/11X,F11.1,11X,5F11.1)
   WRITE(6,611) S1,AGE1,XNRES,BB,DAVG
   GO TO 7
610 FORMAT(/25X,'DIFFERENCE IN BASAL AREA > 0.05 SQ. FT./acre')
   XTHIN=XNRES
   BATHIN=BA-BB
   TVTHIN=TV01-TV08
   TV4T=TV04+TV14
   CV4T=CV04+CV14
   WRITE(6,607) KTYPE,XTHIN,BATHIN,TVTHIN,Top,TV4T,Top,CV4T
   FORMAT(/15X, 'AMOUNT REMOVED IN I, AH, THINNING'
      /15X,6(' - '),1X,7(' - '),1X,5(' - '),1X,3(' - '),1X,8(' - ')
      /31X,'NUMBER OF TREES =',F7.2
      /36X,'BASAL AREA =',F7.2
      /22X,'TOTAL CU. FT. VOLUME O.B. =',F7.2
      /20X,'CU. FT. VOLUME O.B. TO',F3.0,' IN =',F7.2
      /27X,'CONE VOLUME TO ',F3.0,' IN =',F7.2)
   XN=XNRES
   B=BA-BB
   TV01=TV01-TV08
   TV04=TV04+TV08
   CV04=CV04+CV08
   RETURN
8 BA=BB
   TV01=TV01-TV08
   TV04=TV04+TV08
   CV04=CV04+CV08
   RETURN
   END
Appendix 5. Source listing of program WTHIN (continued).

```
SUBROUTINE WTHIN

******************************************************************************
#        #
# SUBROUTINE THIN TAKES CARE OF THE THINNING        #
# OPTIONS AT AGE1.                                  #
#                                                  #
******************************************************************************

IMPLICIT REAL*8 (A-H,O-Z)
COMMON /ONE/ S1,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
COMMON /TWO/ S1,AGE1,XN1,BA1,DTHIN1,AGE2,Q,DTHIN,BRES,BRESR,QTHIN
               INDEX,ITHIN,ILOW,IROW
COMMON /THREE/ TITLE(20),AINV,XNLOG,BLOG,HDLLOG,TVOB1,TVOB41
               CVOB41,IOPT,JJJ
COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA
QTHIN=1.D0
GO TO (1,2,3,2), ITHIN

C-----  ITHIN = 1 = NO THINNING AT AGE1.
C  1 RETURN
C-----  ITHIN = 2 = ROW THINNING AT AGE1. EVERYTHING IS REDUCED
C           BY A FACTOR Q.
C  2 IF(IROW.EQ.2.AND.ITHIN.EQ.2) Q=BRESR/BA
C     IF(IROW.EQ.2.AND.ITHIN.EQ.4) Q=1.D0-Q*(1.D0-BRESR/BA)
C     Q1=100.D0-Q*100.D0
C     WRITE(6,666) (TITLE(11),I1=1,20)
C  666 FORMAT('1'/'10X,20A1)
C     WRITE(6,600) AGE1,Q1
C  600 FORMAT('2'/'32X,'ROW THINNING AT AGE',F4.0)
C           /'F24.2,'% OF TREES IN ALL DIAMETER CLASSES ARE CUT')
C     IF(Q1.GE.100.D0) RETURN
C           CALL OUTPUT(2)
C     IF(IITHIN.EQ.2) GO TO 10
C     ITHIN=3
C-----  ITHIN = 3 = LOW THINNING AT AGE1.
C  3 GO TO (4,5), ILOW
C-----  ILOW = 1 = ALL TREES HAVING DBH LESS THAN DTHIN ARE CUT.
C  4 DTHIN=DTHIN+0.50
C     DTHIN=DFLOAT(DTHIN)-0.500
C     IF(DTHIN.LT.A.OR.DTHIN.LT.DTHIN1) RETURN
C     WRITE(6,666) (TITLE(11),I1=1,20)
C     WRITE(6,601) AGE1,DTHIN
C  601 FORMAT('3'/'32X,'LOW THINNING AT AGE',F4.0)
C           /'F24.2,'ALL TREES UNDER',F5.1,' INCHES DBH ARE CUT')
C           CALL OUTPUT(2)
C     DTHIN=DTHIN1
C     GO TO 10
```

WTH08690
WTH08700
WTH08710
WTH08720
WTH08730
WTH08740
WTH08750
WTH08760
WTH08770
WTH08780
WTH08790
WTH08800
WTH08810
WTH08820
WTH08830
WTH08840
WTH08850
WTH08860
WTH08870
WTH08880
WTH08890
WTH08900
WTH08910
WTH08920
WTH08930
WTH08940
WTH08950
WTH08960
WTH08970
WTH08980
WTH08990
WTH09000
WTH09010
WTH09020
WTH09030
WTH09040
WTH09050
WTH09060
WTH09070
WTH09080
WTH09090
WTH09100
WTH09110
WTH09120
WTH09130
WTH09140
WTH09150
WTH09160
WTH09170
WTH09180
WTH09190
WTH09200
WTH09210
WTH09220
WTH09230
WTH09240
WTH09250
Appendix 5. Source listing of program WITHIN (continued).

C----- ILOW = 2 = THIN TO A SPECIFIED RESIDUAL BASAL AREA (BRES).
C
C 5 BTHIN=BA-BRES
    BB=0.DO
    IF(A.LT.DTHIN1) GO TO 6
    F1=0.DO
    XNT=XN
    IMINI=IMIN
    GO TO 7
C 6 F1=CDF(DTHIN1)
    XNT=XN/(1.DO-F1)
    IMINI=DTHIN1*0.51DO
C
C----- FIND DTHIN CORRESPONDING TO BRES.
C
C 7 DO 8 I=IMINI,IMAX
    XI=DFLOAT(I)
    F2=CDF(XI+0.5DO)
    F=XNT*(F2-F1)
    IF(F.LT.0.DO) F=0.DO
    F1=F2
    BASAL=.545415D-2*F*X1*X1
    BB=BB+BASAL
    IF(BB.GT.BTHIN) GO TO 9
    CONTINUE
C
C----- QTHIN IS THE RESIDUAL PROPORTION (AFTER / BEFORE THINNING)
C
C 9 QTHIN=(BB-BTHIN)/BASAL
    DTHIN=XI-0.5DO
    WRITE(6,666) (ITITLE(11),I1=1,20)
    WRITE(6,602) ACET,BRES
    WRITE(6,602) \n
602 FORMAT(*32X,'LOW THINNING AT AGE',F4.0

    F1(BRES.LT.0.DO) RETURN
    CALL OUTPUT(2)
    DTHIN=DTHIN
    QTHIN=DTHIN
    CONTINUE
C
C 10 XNLOG=DLOG(XN)
    BLOG=DLOG(BA)
    RETURN
    END

C BLOCK DATA

C IMPLICIT REAL*8 (A-H,O-Z)

C COMMON /ONE/ SI,AGE,XN,BA,HD,DMIN,DMED,DMAX,DBAR,IMAX,IMIN
C COMMON /TWO/ SI1,AGE1,XN1,BA1,DTH1,DTH2,G,DTHIN,BRES,BRESR,QTHIN
C COMMON /THREE/ INDEX,ITHIN,ILOW,IPROW
C COMMON /FOUR/ A,B,BMIN,C,CONST,CINV,GAMMA

C DATA AGE/0.DO/,XN/0.DO/,BA/0.DO/,DTHIN/0.DO/,ITHIN/1/
C