RESEARCH IN FOREST MENSURATION, GROWTH AND YIELD

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PREFACE

The theme of the XIX World Congress, held August 5-11, 1990, in Montreal, Canada, was "Science in Forestry: IUFRO'S Second Century." In keeping with the overall Congress theme, Subject Group S4.01 "Mensuration, Growth and Yield" organized three sessions. The first session "Research Frontiers in Forest Mensuration, Growth and Yield" sought to look ahead at the most pressing research needs in the future. A second session on "Research Progress in Forest Mensuration, Growth and Yield" consisted of reports on recent research accomplishments. The third session, organized by S4.01-03 on the topic of "Design, Performance and Evaluation of Experiments," aimed at exploring topics of experimental design and analysis relevant to growth and yield modelling. Papers from these three sessions constitute this volume.

It should be noted that the authors submitted camera-ready copy of their manuscripts and that they are solely responsible for the contents. No attempt was made to obtain review or to edit (other than minor format changes) the material submitted. The papers are arranged in alphabetical order according to the surname of the first-listed author.

Because space in the IUFRO World Congress Proceedings is limited, a special proceedings volume of S4.01 sessions was needed. (Another volume, designated publication FWS-3-90, of sessions of S4.02 "Forest Resource Inventory" and joint sessions of S4.01 and S4.02 has also been compiled.) We have attempted to make full versions of each paper presented at the World Congress in Montreal available. Abstracts of some, but not all, of the papers in this compilation appear in the Division 4 proceedings. In the limited instances where complete papers were published in the Division 4 proceedings, they are not reproduced here. If a paper does not appear in

this volume, it is because it is contained in the Division 4 proceedings or it was not received by the deadline for inclusion in this volume. The Division 4 proceedings can be obtained from:

IUFRO-Secretariat
Seckendorff-Gudent-Weg 8
A-1131 Vienna
AUSTRIA

for a cost of $20 Canadian.

No financial provisions are made for the publication of special proceedings volumes from IUFRO World Congresses. Thus, we are most grateful to various organizations in Canada who generously contributed toward the publication of this volume. Financial aid was received from Forestry Canada in Victoria, British Columbia and in Chalk River, Ontario; the Inventory Branch of the Ministry of Forests of the Government of British Columbia also supported these proceedings, as did MacMillan Bloedel, Ltd, Nanaimo, British Columbia. In addition, the School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, U.S.A. also contributed towards publication and distribution costs.

Finally, I wish to acknowledge the assistance of Klaus Johann, Deputy Leader of S4.01, in the organization of the sessions in Montreal and to thank especially Teja Preuhsler, Leader of S4.01-03, for organizing the session on "Design, Performance and Evaluation of Experiments." We trust that this proceedings volume, and the companion volumes (Division 4 proceedings; publication FWS-3-90), will serve the research community well as IUFRO looks toward its second century.

Harold E. Burkhart
October, 1990

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A COMPREHENSIVE CONCEPT OF TREE AND STAND MEASUREMENTS

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ABSTRACT

The future questions will not be "Permanent plots versus point sampling", or "Why and how critical height sampling?", or "What advantages will Zöhrer's suggestions about sampling with programmed probabilities bring, and how to perform them?", or even the point sampling methods by Hirata and Minowa using vertical angles. All these items are not real "problems" for measurement techniques in the forests, but can be answered by computers alone if we use a comprehensive concept for point sampling, shortly outlined in this article. The basic feature of the proposed system is to develop a special science and technology for getting perfectly adapted taper functions for every sample tree in question. More than ever before, the Tele-Relaskop and the Spiegel-Relaskop (Wide Scale or CP-Scale) with the new micro attachment can be used for the development of taper functions as well as for controlling them occasionally and/or permanently. These two instruments now available can be described as the "corners" for future developments which will allow any enlargement between zero and 8-times. An optimally seeming technique for measuring trees and stands is recommended in this paper, including also indications for "multiple use mensuration" in forestry.

Keywords: Comprehensive Concept, Permanent sample points, Relaskops.

INTRODUCTION

The comprehensive concept of tree and stand measurements presented in this paper rests on the following premises:

1. The still rapidly increasing capability of computer techniques opens unforeseen possibilities for forest mensuration. To take advantage of this progress, the future measurements in the forest should be made on permanent sample points marked by strong capped plastic tubes, sunk in the ground, being used to support an aluminium staff carrying the instrument on its tilting head.

2. The exact horizontal distance from the sample point to every possible tree of a whole sweep, and also the angle between the north direction and the tree axis (turning right), as well as the slope from the base of the point to the base of the tree have to be determined.

3. For each potential sample tree a very close adjusted taper function has to be found. This can be achieved by exact measurements using the Tele-Relaskop and/or the Spiegel-Relaskop following the author's proposals given in his book "The Relascope Idea" (p. 112-115) and in his special programs for the HP-97 calculator.

4. Under these basic conditions, any measurement taken, on any kind of "point sample" can be handled by a big computer, e.g.: angle count sampling at breast height or at other heights using different basal area factors, point sampling with programmed probability by Zöhrer, and even the sampling methods by Hirata or Minowa using vertical angles from one point (see all these methods in the author's book "The Relascope Idea").

5. In the former symposium about "STATE-OF-THE-ART METHODOLOGY OF FOREST INVENTORY" held July 30 - August 5, 1989, in Syracuse, New York, USA, the following topics were discussed:

- Sampling with partial replacement
1) Updating methods for forest inventory
2) Importance sampling
3) Critical height techniques
4) Multi-stage and multi-phase sampling designs
5) Point-model based sampling
6) Integrating multiple value forest surveys into timber surveys
7) Error of inventory biomass or volume estimates
8) Expressing diameter distribution by mathematical functions
9) Estimation of stand ingrowth
10) Estimation of tree and stand mortality
11) Error of volume or biomass regression
- Line intersect sampling
12) Spatial analysis of inventory data
13) Objectives of forest inventory
14) Forest inventory and growth and yield models
15) Permanent plots versus point sampling
- Remote sensing and its use in forest inventory
- New regression concepts
- Geographic information systems

The author is convinced that, if the suggestions presented here are used properly, the topics marked by the numbers 1) to 15) can be answered or demonstrated mainly by computers using the same measurement data.

6. In addition to all these technical points, the proposed measuring system is well suited to classify each sample tree (and/or each sample point) for its contribution to the overall network given by aspects to all possible uses of forest lands (see the author's proposals in his article "Vorschläge zum forstlichen Meßwesen" in "Allgemeine Forstzeitung", September 1961, copy available from the author, and also a translation into English). Many details cannot be offered in this paper, but will be the contents of further research science.

THEORY AND APPLICATION

1. Establishing Permanent Samples

A. Marking locations. As said already in the Introduction this has to be done by strong capped plastic tubes, sunk in the ground (see Fig. 1 and 2).

B. Developing spatial location maps. The grid for permanent sample points should be planned in space in such a manner that the distance between the points can be diminished step wise to a minimum of, let us say, 200 or 100 m. Although the marking of the points is under ground and therefore invisible, the system of the points should be shown on the forest management maps. Square grids should be preferred, although grids of equilateral triangles could also be used.

C. Measuring sample trees. Depending on a minimum dbh and on a maximum of the relative distance to dbh we get a selection of sample trees for which each single tree has to be fixed in the space following point 2 of the Introduction. So we obtain in the first step the polar coordinates of each tree, that can be easily converted to rectangular coordinates by computers. Under such basic requirements structural silvicultural problems can also be handled by computers. The next step is to fit to each single sample tree a suitable taper function. For this, it is not necessary to measure on each single tree one, two or more upper diameters, rather one can take a subsample of a few trees from each species and social class. For the rest of the sample trees only the dbh and the total height is needed to fit good taper functions.

The method works as follows: For the subsample trees two additional diameters between the dbh and the top of the tree must be measured by a Tele-Relaskop, a Wide-Scale or CP-Spiegel-Relaskop (called "Four point method"). These four points should be connected now by a third degree polynomial equation, so that interpolation by computers is possible. As seldom the tree measurements can be taken from the center point marked by the plastic tube in the ground, a separate observation point must be found for many of the trees of the subsample. These secondary points cannot be marked by plastic tubes too and, therefore, in all these cases, measurements have to be taken with the help of tripods. Nevertheless, their location should also be marked for repeated control measurements. This can be done easily, if we install the points in the close neighbourhood (e.g. "2 m NE from tree Nr. 01200") of another sample tree and put a plastic bottle or a plastic or metal stake in the ground. The height of the instrument should also be recorded, as e.g. "195 cm over bottle bottom, or 175 cm over the stake head, or 170 cm over ground". All instrumental measurements are taken using new micro attachments.

The subsample trees are used as patterns for the rest of the sample trees of the same species, similar crown or similar social class. This can be done automatically by computer programs if the height and the size of the two upper diameters of the pattern tree are being reduced (or enlarged) proportional to the change of the total height and of the dbh respectively, and then a new individual taper function is calculated. As an example, in the programs for the small computers HP-97 and HP-41CV, the measuring values of the pattern trees are stored on separate magnetic cards for each tree. For the bigger computers where any amount of storage room is available, many thousands of basic taper functions can be stored and recalled when needed. The same is valid for any other characteristics of the trees, like bent or forked stems, different shapes of crowns, estimated or measured branch volume, biomass etc., and last but not least, any kind of classification of the state of health of trees (e.g. the scales by J. Pollanschütz).

Besides the above-described measurement of sample trees that are reasonably close to the sample point center, there is to be mentioned another kind of tree selection for the special case to enlarge forest mensuration to all possible uses of forest lands (see point 6 of the Introduction). This can be done partly by aerial photographs, partly from permanent sampling points, either giving each sample point itself an adequate classification or attaching a special evaluation to each sample tree in question. The latter case may be illustrated by an example taken from the important value of forests and single trees as protectors against snow avalanches in mountainous countries (e.g. Switzerland or Austria in Europe). Let us assume that above a village there is a small strip of land stocked by 10 old larch trees which protect the settlement against avalanches, and these trees cover an area of 1 hectare. Based on the costs of technical constructions for replacing the protection function of the trees, their protection value amounts 1 million Austrian Schillings (AS). Point sampling concepts can be applied to the problem of evaluating these larch trees by assuming that 1 tree within a sample circle of 1000 m² (radius R_{norm} = 17,84 m) above the village as described has a value of AS 100.000. But it could also happen that we find another tree of which the value is only AS 80.000 (due to species, age, size etc.). In this case we are allowed to evaluate it also by 1 million AS per hectare only if it stands within a circle of 800 m² (radius R_i = 15,96 m). Its probability for selection has to be diminished by 80/100 in order to count it equally for "1 million per hectare".

As we see, all depends on the maximal distance a (in meter) of a tree protecting against avalanches from the sample point for which the following formula is valid:

$$a < R_i = R_{norm} \cdot \sqrt{X_i / X_{norm}}$$

with the following meaning:

- R_i critical radius (e.g. 15,96 m) belonging to the special evaluation X_i
- R_{norm} radius taken as unit (e.g. 17,84)
- X_{norm} evaluation taken as unit (e.g. AS 100.000)

In this example for measuring the value of forest trees for protecting against avalanches the unit radius is probably too small. So, for practical purposes, we would set R_{norm} = 56,42 m that corresponds to a sampling circle of an area of 1 hectare.

For distances of this size and also much more the authors proposal and hope is to use future new devices for acoustics or electric measurements (see his paper in German "Der Ruf nach einer Schallmeßtechnik ist wieder zeitgemäß geworden").

In any case this enlarged kind of tree selection for "multiple-use mensuration" in forestry can be standardized by special relations to permanent sample points installed in a net work over a country or over certain regions of forest lands.

D. Recording data needed to satisfy objectives such as growth prediction.

For computers neither now and in the future it will not matter how many data, measured or estimated, - being even hundreds - will be registered for each kind of trees or sample point qualification. Beside a comprehensive big system of classification data for "multiple-use mensuration" in forestry, data describing the main shape of each single sample tree in question in the close neighbourhood of a sample point will be very important to get information for increment and growth

prediction. Here again, my advise would be to collect and to record all data which are helpful for "critical height sampling", even when not all sample trees are well shaped for this case: we have to find mathematical functions describing the natural shape by an adequate theoretical stem.

Critical height sampling, as suggested by Kim Iles, has the primary advantage that the size of the area sampled grows proportional to the tree cross-sections and to the larger distances between them. The sampled elements are the critical heights h_c. When multiplied by k they represent the volume density in m^3/ha, and have the special advantage that the change in the sampled area is not discontinuous (in leaps and bounds) but will expand smoothly. In addition, by use of the data available, we also get a certain amount of negative critical heights which give us information about the coming ingrowth trees. This means a very important part of future growth can be predicted.

More details of how to collect and to record data will be the results of further research. In general, files of sheets or magnetic devices have to be used.

2. Periodical Remeasurements

A. Relocation of sample points. This can be done in different ways, e.g. using small situation maps showing the sampled trees round a sample point, printed out by computer. Numerous techniques have been developed for this problem.

B. Remeasurement of surviving trees. The first step in the field work is the identification of these trees according to the first or the latest data sheets, or magnetic records, respectively. As a rule only the dbh and the total height will be measured anew, whereas the resulting new form of the stem will be calculated by the computer. Other alterations of condition are to be recorded following the predetermined scheme. Whenever it seems opportune to remeasure upper diameters on single trees the following procedure has to be followed: If such a remeasurement can be taken from the center point marked by the plastic tube the instrument (Tele-Relaskop or Wide-Scale or CP-Spiegel-Relaskop) is mounted on the top of the metal staff screwed in at the bottom of the tube; in this case for all the former and all the following measurements a point in space is fixed from that all inclinations to the height of primarily chosen tree cross sections can be relocated and from that also the directions to the cross section diameters which are measured remain unchanged. If a remeasurement of upper diameters has to be taken from other points (not marked by the plastic tube in the plot center) these secondary points were marked separately by a plastic bottle, or by a plastic or metal nail in the ground. For the relocation of secondary points, from which measurements have to be taken by aid of tripods, their place should be found in the close neighbourhood of other sample trees by a short distance and compass bearing to the tree axis, as already described. In order to use always the same instrumental height we follow the special recording taken from the bottom of a vertical standing plastic bottle (under ground well closed by a screw cap), or from the head of a nail, or from the surface of the soil.

C. Recording trees lost by harvesting or catastrophic events. Records showing the sampled trees round a permanent sample point are helpful for identifying stumps or places of absent trees of former registrations. The cause of loss, whether by cut, elemental damage or natural death, has to be recorded, and also at least the approximate time of that event.

D. Recording new trees. All new trees, which are now in-trees in regard to the predetermined conditions due to their increased dbh, have to be measured according to the instructions of a first recording. Their data sheets or magnetic records have to be filed between the other data sheets (or numbers of magnetic records) in the correct order of the angles from north.

3. Instruments and Tools

The manufacturer of the Relaskops, "FOB, Postfach 12, A-5020 Salzburg, Austria", is a member of the IUFRO and considers himself, not without some pride, to be a "private research institute". During the last few years, FOB has developed the instruments and tools necessary for "permanent comprehensive tree and stand measurements" presented in this paper in close co-operation with the author.

A. For marking locations under ground.

- Plastic tubes. See Fig. 1 and Fig. 2. A hole of about 50 cm depth is dug for the plastic tube, which has thick walls and is 38 cm long including the cap. The one pole metal-shaft ist tightly screwed into the plastic tube, which is protected by its cone-shaped lid against contamination on the inside and stands vertically in the prepared hole. For the firm anchoring in the ground four stakes (about 30 cm long and 1 cm diameter) are driven down through the holes of the flange at the bottom of the plastic tube using a hammer, not too light, and having a handle of about 70 cm. When filling the hole, this hammer is used to consolidate (press in) the soil and to wedge the plastic tube with stones in order to fix it as vertically as possible. This is controlled best by the use of a spirit level which is at disposal in the support for the horizontal base target (see manual of the Tele-Relaskop, part 2, Fig. 8). The support can be screwed on the upper end of the one pole metal-shaft. Having finished this part of the work, the one pole metal-shaft is now unscrewed and taken out of the firmly fixed plastic tube. Care must be taken so that no soil falls into the tube. A ring-shaped cup made for this purpose provides a proper protection against it, also on later occasions when it is dug out again. The plastic tube is closed with an unbreakable cap and soil is put over it.

Some suggestions were already made as to how to procede on stony or rocky ground, when establishing these observation points with the plastic tubes. Generally speaking, one should not hesitate to spend, in some instances, a little more money for our "brother tree" - for example for the transportation of drillers, concrete, etc. even sometimes by helicopters.

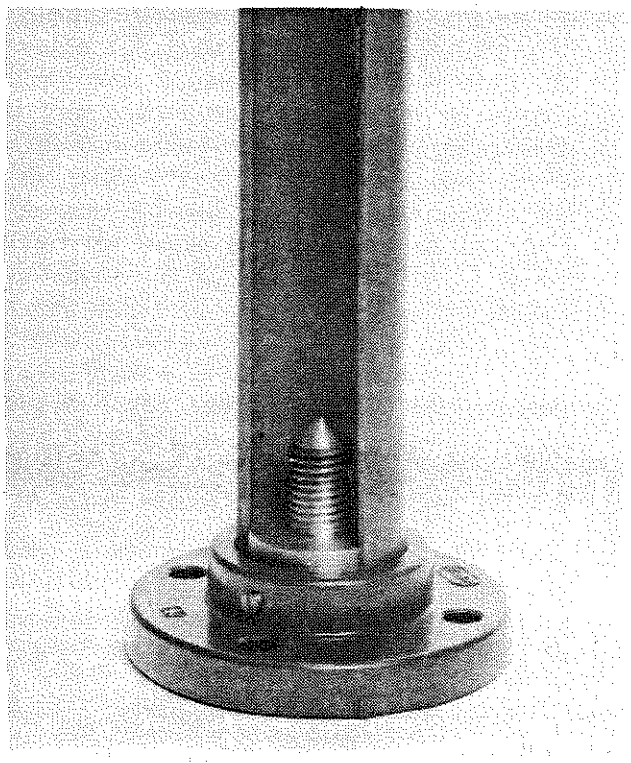

Fig. 1: The lower part of the plastic tube is cut alongside in order to see the metal part with the screw to fix the aluminium staff. On the flange are 4 holes through which long nails as anchors may be driven into the ground

- Plastic bottles. Their necks must be wide enough to introduce a measuring stick to the bottom from where the instrumental height has to be recorded. A screw cap protects against contamination.

- Nails. Plastic or metal, about 30 cm long and 1 cm in diameter.

Fig. 2: The aluminium staff is being introduced into the plastic tube. If screwed in, the conic lid will be pressed against the upper edge of the tube, giving so a strong fixation

B. One pole metal-shaft. It is 170 cm long and, for ease of transportation, it may consist of two or more parts which can be screwed together. It is a tube of 30 mm in diameter and its wall made from Duralumin is 5 mm thick. The lower part has the already mentioned cone-shaped lid rigidly connected and in a distance fitting to the length of the plastic tube. The cone is pressed against the upper edge of the tube, if the one pole metal-shaft is screwed in resulting in an extra strong connection. The lower and the upper part of the one pole metal-shaft has a 10 mm wide cross borehole facilitating the screwing by use of a bolt.

The lower part of the one pole metal-shaft cannot be made from Duralumin but from solid steel if we intend to fix on it a sitting device (see below).

C. Sitting devices. Some different models for that important help to improve observation work with Relaskops from the one pole metal-shaft and also from tripods were already under development. They should be movable in all directions and also to vary in height. One of them consists of a saddle for a bicycle which is mounted on a strong metal pole with a point to be driven in the soil, allowing change for each direction of observation. Another model is fixed on the lower part of the one pole metal-shaft, it can be turned around

to all directions in a full circle, and also its height can easily be changed to each need. This solution of the problem requires a longer plastic tube in the ground (about 70 cm) and a stronger one pole metal-shaft, as mentioned above. More details can not be given in this paper.

D. Tilting head and micro attachment. See Fig. 3, 4 and 5. Experiments have shown that one pole metal-shafts of the described kind, although they may be constructed in a very heavy way, always show small vibrations during work because of the long arm of the lever from the ground to the instrument. It is also known that up to this time there are no tilting heads with micro attachments, simple and suitable for field work in the forest, available on the market. For these reasons a micro attachment for precision measurements has been constructed. This new device is very stable and can be used for the Tele-Relaskop as well as for the Spiegel-Relaskops. The value of the Spiegel-Relaskop of the types "Metric-CP" and "Wide-Scale" was raised remarkably by this micro attachment, especially for single tree measurements.

E. Spiegel-Relaskop illumination device. This new improvement is very helpful in dark forests. It consists of a small lamp to be fixed from outside on the instrument.

F. Future Relaskops. Tele-Relaskop and the Spiegel-Relaskop (Wide Scale or CP-Scale) now available can be regarded as the "corners" for future developments which will allow any enlargement between zero and 8-times, using the so called "zoom-technique". Most probably an enlargement between 2-times and 6-times will be an optimal solution for many practical purposes.

G. Other instruments and tools. Following is a list of other instruments and tools needed to fully implement the comprehensive system of tree and stand measurements described here:

- Compass to be read off inside a monocular or binocular.
- Measuring tapes.
- Base targets for optical distance measurements.
- New devices for non-optical distance measurements.
- Spade and other tools for digging a hole in the ground.
- A long stake with a ring as handle on top for searching former capped tubes under ground.
- Hammer as already described.
- Plastic tube about 50 cm long and 10 cm in diameter to put over the one pole metal-shaft and the smaller plastic tube to protect the cone-shaped lid against the working with the hammer during filling the hole. The protecting tube has to be lifted stepwise in filling up and is taken away after this procedure.
- Two specially made hooks to lift the caps from closed tubes.
- A ring-shaped cap as already described to take care so that no soil would fall into the tube.
- Devices to record measuring data in the field.

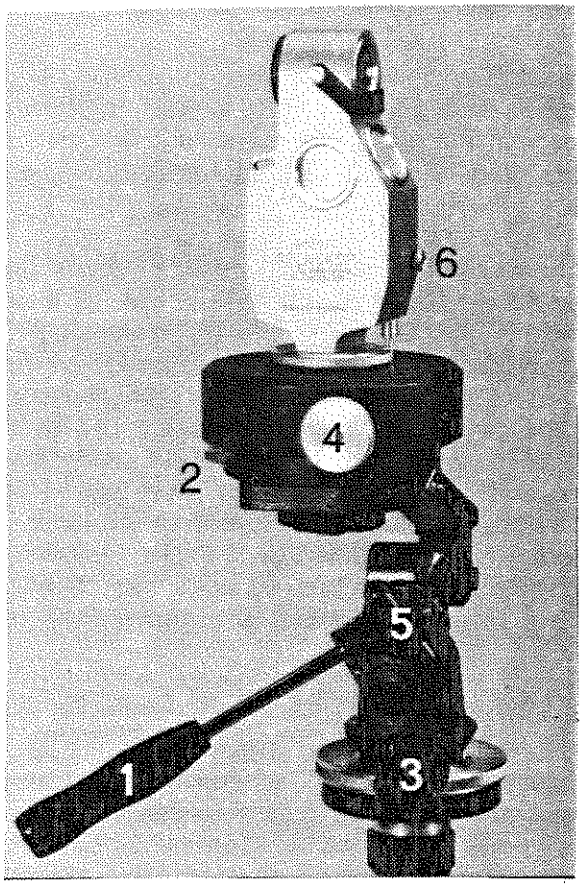

Fig. 3: The Spiegel-Relaskop mounted on a tripod with tilting head and micro attachment for precision measurements

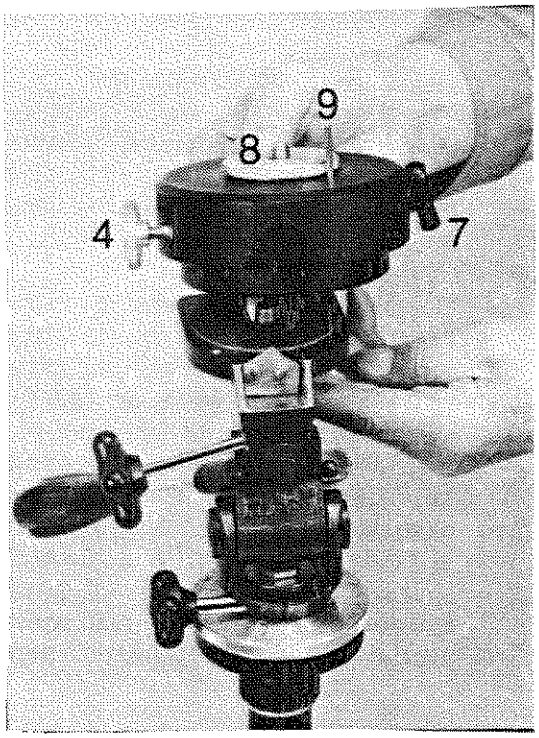

Fig. 4: The micro attachment is mounted on the tilting head

1 long handle of the tilting head
2 knurling knob for vertical adjustment
3 and locking screws for the tilting head
5
4 knurling knob for horizontal adjustment
6 locking break release for the Spiegel-Relaskop
7 lever to fix 8
8 socket plate with screw bolt
9 stop pin
10 stop and device for permanent release of the brake

Fig. 5: Stop and device for the permanent release of the brake

4. Role of Computers

A. In the field. Instead of paper and pencil small pocket computers will serve in the future to collect and to record magnetically data in the forests. Besides that, these small computers will also be used for calculations to be made in the field for certain local decisions.

B. For processing and storing measurement data. Here we have the greatest challenge for the future to produce efficient software. Let us consider for now the single sample tree in question: at the one side its stem shape has to be described by a perfectly adapted mathematical function in order to be suited for any kind of special calculation (e.g. "critical height"), on the other side among hundreds of possible different deviations from regularity the individually best description for each sample tree has to be given. Completely new theories have to be involved as for example "Fractal analysis of crown structure" by B.Zeide, or "Profile and thickness distribution of branch and root biomass as specific characteristics of forest biological productivity" by V.V.Demenev and Co-authors. An immense scale of patterns for each tree species has to be available and incorporated into computers following special programs.

5. Permanent Samples for different Objectives

A. Forest inventory. Permanent sample points, as described, allow growth simulation by computers for each sample tree so that forest inventories can be updated at any time, even within the periods of remeasurements. Lost trees, in such a case, can be predicted by computers following former experiences given by statistics, or following certain parameters for catastrophic events, etc. given as inputs to the computer. New ingrowth trees can be calculated also using historical data, and, in the case of critical height sampling, by calculation from negative critical heights. So, permanent sample points can also be regarded as "permanent forest inventories" for all purposes for which they are planned.

B. Forest monitoring. Contrary to forest inventories which give averages for different regions of forests or even for whole countries, forest monitoring is also possible for single stands or single points if we use a close network of permanent sample points as recommended. There is also a certain justification to extrapolate the measuring results round a sample point to its close neighbourhood, or to interpolate such results between neighbouring points if they are not too distant (see "The Relascope Idea", p. 175) within a uniform stand. This kind of evaluation can be done by computer programs as well, and can be used for monitoring purposes.

C. Growth modelling. Provided that we connect an adequate site classification with permanent sample points, or even with the single sample trees in question, we are able to sort out by computers trees of the same species standing on similar sites in order to model growth patterns (see E.Assmann "The principles of Forest Yield Study", pp. 187, 203, 205, and see also as a modern example of what computers can do, the pilot work by H.E.Burkhart and Co-authors "Simulation of individual tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas").

D. Silvicultural prescriptions. Thanks to the very comprehensive measurements round the permanent sample points a good picture of the forest in the close neighbourhood of the points is available: we know something about the mixture of tree species, about diameter and height distributions, and about the social class of the single tree. In uniform stands like plantations we are allowed to extend these point pictures by interpolation to the area between the points, as already mentioned. Computer programs enable foresters to improve the mixture of tree species, telling as examples on the points which trees have to be cut out, or which have to be protected. See also K.Johann's proposal to measure thinning needs in stands of conifers (Österr.Forstzeitung 12/1988). Due to more precisely measured coordinates and dimensions of each sample tree, many structural silvicultural problems will be handled by computers in the future.

E. Multiple-use classification. If a new science will be able to extend permanent samples for multiple-use classification of forests and single trees, computers can give answers to all these items too.

6. Research Needed to Meet Future Demands for Tree and Stand Measurement Data

A warning in advance: Don't try to imitate or to copy our instruments because their price seems high at the moment! Believe me, the market is too small for a real profit. It would be much better to proceed with fundamentally new developments and improvements, such as:

A. Instruments.

- Non-optical distance measurement. This is a most important need for future utilization of permanent sample points. The author is convinced that the electronic technology has already solved this problem but its use is not available for forestry at the moment (see his paper in German "Der Ruf nach einer Schallmeßtechnik ist wieder zeitgemäß geworden!" and others).

- Relaskops. See point F on page 8 !

B. Software for computers. The present Relaskops and other instruments enable us to collect any amount of data for developing relevant computer programs, also for future demands. Besides we have to watch carefully for the fast development in computer techniques in order to coordinate our endeavours in good time. To set priorities in the goals of forest research is an important next step.

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Date: April 1990.

PROJECTING STAND AND STOCK TABLES:

A COMPARISON OF THREE METHODS

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SUMMARY

Often, it is desired to project an existing stand and stock table to a future age in even-aged stands. There are several alternative procedures available to modelers to accomplish this goal. Below, I compare and contrast three such procedures for projecting stand and stock tables for loblolly pine (Pinus taeda L.) plantations grown in the southeastern United States. One procedure is a Weibull parameter recovery model, another is a distribution free percentile-based algorithm, and the third uses an individual tree distance independent basal area projection model.

Each projection procedure was fitted to a large loblolly pine data base. Statistics of fit for the fitted data base and an independent data base were generated for number of trees by diameter class, volume by diameter class, and volume by product class. The procedure based on the individual tree distance independent basal area projection model was superior to the other two modeling procedures for most of the fit statistics.

KEYWORDS: Growth and yield, stand table projection.

INTRODUCTION

Whole stand pine plantation yield prediction and projection models have been designed to 1) predict stand yields explicitly using multiple regression techniques, or 2) to predict stand yields implicitly by predicting a stand's diameter distribution which is then used with individual tree volume equations to obtain the stand yield. The second type of model is generally preferred because more detailed information concerning stand structure can be obtained.

The objective of this talk is to compare three implicit yield projection systems. Although they differ in structure, all three models are capable of projecting yields by diameter class. The first method tested is the widely used probability distribution approach based on the Weibull function (Bailey and Dell, 1973). The second is a percentile-based method (Borders et al., 1987) which does not necessitate the assumption of a particular probability distribution. This method uses a system of percentiles defined across the range of the observed diameter distribution. Stems per acre are proportioned into diameter classes of a specified width assuming a uniform distribution of tree frequency between adjacent percentile classes. This method is very flexible in that reversed J-shaped, mound-shaped, and multi-modal distributions can be reproduced. The third method is a variation of a stand table projection method developed by Clutter and Jones (1980) and subsequently revised by Pienaar and Harrison (1988). Relative size and changes in relative size over time are the basis for this model. Essentially, an individual tree basal area projection equation is fitted to the data that is subsequently used to project surviving individual trees or diameter classes to older ages. The projected stand table is constrained so that trees per acre and basal area per acre are consistent with whole stand estimates. As for the percentile method, this method can reproduce reversed J-shaped, mound-shaped, and multi-modal distributions. This analysis will determine which of the three methods is the most accurate and precise in projecting stand and stock tables for a loblolly pine (*Pinus taeda* L.) data base.

EXPERIMENTAL METHODS

Two separate data sets were used in the analysis. Both data sets were collected by the Plantation Management Research Cooperative at the University of Georgia. One data set was collected during the summers of 1977 and 1981 from lower Coastal Plain loblolly pine plantations in North Carolina and South Carolina. The plantations selected had been established on cutover sites which had been site prepared, but no thinning or fertilization had occurred after establishment. The other data set was collected during the summers of 1981 and 1985 from lower Coastal Plain loblolly pine plantations in Georgia and north Florida. The criteria for selection of these plantations was the same as for the data collected during the summers of 1977 and 1981. The data will hereafter be referred to as the 1977-81 data and the 1981-85 data, respectively. Data collected from the approximate one-tenth acre rectangular plots consisted of plantation age, length and width of plots, diameter at breast height (dbh), crown class, and total height of at least two trees in each diameter class. The 1977-81 data had trees recorded by the total number of trees in each 1-inch dbh class while the 1981-85 data had trees recorded by individual dbh's. Plantation ages at the time of plot installation ranged from 8 to 35 years old, trees per acre ranged from 300 to 900 and site index (base age 25) ranged from 40 to 80.

Models were developed using the three methods presented above. Projected diameter distributions were calculated and compared with observed distributions. Comparison of predicted and observed total and merchantable volumes and pulpwood and sawtimber volumes was also done. The same volume equations and height-diameter equations were used for all three models to prevent confounding results. Observed number of trees per acre, basal area

per acre, and dominant height at the projection age were also used to eliminate unnecessary variation in the comparison of the three projection methods. Each model was fit with the 1981-85 data. The 1977-81 data was then used as an independent test of model performance.

A Weibull parameter recovery method was used that required use of the 0th, 25th, 50th, and 95th diameter percentiles. Thus, a set of linear equations was developed to predict the 0th, 25th, 50th, and 95th percentiles. Projection equations were then developed for each of the percentiles using an algebraic difference approach (Maldonado et al., 1988). Parameters were estimated using linear least squares.

The percentile-based method introduced by Borders et al. (1987) does not necessitate pre-defined mathematical probability distributions. Instead, a set of 12 percentile points is used to characterize the distribution within the stand. The percentiles suggested were the 0th, 5th, 15th, 25th, 35th, 45th, 55th, 65th, 75th, 85th, 95th, and 100th. Based on Borders et al.'s (1987) work, a set of percentile projection equations were derived and parameters for the 1981-85 data were estimated using seemingly unrelated regression (Zellner, 1962).

The basal area growth projection method discussed by Pienaar and Harrison (1988) is based on the concept of relative tree size. Pienaar and Harrison (1988) define relative size of an individual tree as the ratio of the tree's basal to average basal area per tree on a given unit area. The model form they suggested was fitted to the 1981-85 loblolly data.

RESULTS

Stand Table Comparisons

Diameter distributions generated by each of the diameter distribution projection methods were tested against the observed diameter distributions using the Kolmogorov-Smirnoff two-sample test at the .10, .05, and .01 level of significance. As previously stated, observed basal area, trees per acre and dominant height were used in all comparisons. There were no significant differences between the observed and the predicted diameter distributions for any model. Although this is not an exact test, it does serve to indicate that each method reproduces observed stand tables relatively well.

An error index proposed by Reynolds et al. (1988) was used for further comparison. They suggested this error index as a method for selecting and validating models by assessing the error as an alternative to using goodness-of-fit tests. The error index is defined by Reynolds et al. (1988) as a weighted sum of the absolute differences between predicted and observed numbers of trees in each diameter class. Diameter classes may be 1 inch, 2 inch, or by product such as pulpwood and sawtimber. A weight such as volume or dollar value may be assigned to each class. Essentially this error index is a sum of absolute differences between observed and predicted diameter class values, thus an overprediction in one class does not offset an underprediction

in an adjacent class. Reynolds et al. (1988) point out that the error index will be relatively small when all classes are predicted well.

Trees per plot for observed and predicted diameter distributions were compared by 1-inch diameter classes and by diameter groups using all distributions in the data set, unimodal distributions only, and multi-modal distributions only (Table 1). The diameters groups were defined as follows: 1) less than 4.5 inches dbh, 2) 4.5 inches to 9.5 inches dbh, 3) 9.6 inches to 13.5 inches dbh, and 4) greater than 13.5 inches. In all but one case, the basal area method had the minimum error index value (boldface). The percentile-based method always had smaller error index values than the Weibull method.

Table 1 -- Average error index for 1981-85 plots for all distributions, unimodal distributions and multimodal distributions by 1-inch diameter classes and by diameter class groups for the number of trees per diameter class.

	Weibull	Percentile	Basal Area
1-inch classes			
All distributions	13.61	12.52	**11.57**
Unimodal distributions	12.71	12.02	**10.87**
Multimodal distributions	16.50	14.90	**13.82**
Groups			
All distributions	10.73	10.01	**9.82**
Unimodal distributions	10.55	10.09	**9.52**
Multimodal distributions	11.31	**9.78**	10.79

Stock Table Comparisons

Further comparison of the stand table projection methods was done by comparing total and merchantable volumes calculated for the observed and predicted diameter distributions. Total volume of all trees and merchantable volume of trees greater than 4.5" dbh to a 4" top diameter outside bark calculated from the observed and predicted distributions were compared for each of the stand projection methods to determine if there were any significant differences. The statistical criteria used to compare the volumes

were mean residuals, mean absolute residuals, and percent variation explained (PVE) calculated using the following equation:

$$PVE = 1 - \frac{SSE}{SST}$$

where:
SSE = sum of the squared residuals,
SST = sum of the squared deviations adjusted for the mean.

The mean residual was calculated as the average deviation of the predicted value from the observed value and provides an estimate of model bias. Mean absolute residual is the mean or average absolute deviation of the predicted value from the observed value and provides an estimate of precision of mean distance between the observed and predicted values. For the 1981-85 data (Table 2), the basal area method has less bias and has greater precision for reproducing both total and merchantable volumes. The greater PVE values are with the basal area projection method for total volume and the percentile-based method for merchantable volume.

Table 2 -- Statistical criteria for comparison of observed vs. predicted total and merchantable per acre volumes for the stand projection models.

	Weibull	Percentile	Basal area
1981-85 data			
Total volume (cu.ft./ac.)	37.455[1/]	-14.000	-0.102
	61.103	32.294	25.603
	.995615	.998831	.999133
Merch. volume (cu.ft./ac.)	37.397	-20.323	-9.088
	63.750	39.206	34.088
	.995680	.998831	.998777

1/ The first line is the mean of the residuals, the second line is the mean of the absolute residuals, and the last line is the PVE value.

Pulpwood and sawtimber volumes were calculated for further comparisons. Pulpwood volume was defined as the total volume of trees with a dbh greater than 4.5 inches and less than or equal to 9.5 inches and sawtimber volume was defined as trees with a dbh greater than or equal to 9.5 inches to a merchantable top diameter of 6.0 inches.

Pulpwood and sawtimber volumes calculated from predicted distributions were compared with volumes calculated from observed distributions using the mean residual, mean absolute residual, and PVE values (Table 3). The Weibull method has less bias while the percentile-based method has greater precision and the greatest PVE value for pulpwood and sawtimber volumes. Overall PVE values are significantly lower than PVE values in Table 1 which indicates that

Table 3 -- Statistical criteria for comparison of observed vs. predicted pulpwood and sawtimber per acre volumes for the stand projection models.

	Weibull	Percentile	Basal Area
1981-85 data			
Pulpwood volume (cu.ft./ac.)	49.750[1/]	91.191	49.985
	170.603	142.632	151.191
	.887133	.892874	.878301
Sawtimber volume (cu.ft./ac.)	-13.794	-96.573	47.515
	144.088	121.897	134.397
	.960337	.961207	.956855

1/ The first line is the mean of the residuals, the second line is the mean of the absolute residuals, and the last line is the PVE value.

stand tables broken down into pulpwood and sawtimber size classes are not predicted as accurately as the projected diameter distribution taken as a whole. Table 3, like Table 2, does not show great differences between residual values and a very small difference between the PVE values. The error index was also used for comparison of total volume between observed and predicted distributions for 1-inch diameter classes and diameter class groups previously defined for all diameter distributions, unimodal distributions only, and multi-modal distributions only (Tables 4). The basal area model had slightly lower error index values for all distributions and unimodal distributions and the percentile-based method had the lower error index value for multi-modal distributions. For the grouped diameter classes, the percentile-based method had the lower error index value for all three cases.

Table 4 -- Average error index for 1981-85 plots for all distributions, unimodal distributions, and multimodal distributions by 1-inch diameter classes and by diameter class groups for the total volume per diameter class.

	Weibull	Percentile	Basal area
1-inch classes			
All distributions	99.17	88.76	88.19
Unimodal distributions	90.65	84.70	80.32
Multimodal distributions	126.33	101.72	113.12
Groups			
All distributions	94.01	83.92	91.27
Unimodal distributions	85.28	78.32	84.41
Multimodal distributions	121.79	101.76	119.45

Comparisons Using Independent Data

The three stand projection models were each fit to the 1981-85 data. The 1977-81 data was used as independent data to test the models fit to the 1981-85 data. The 1977-81 data can be assumed to be independent since the plots in the two data sets were selected independently of one another.

With the validation data, diameter distributions were predicted using the three procedures under study. These predicted diameter distributions were tested against the observed diameter distributions using the Kolmogorov-Smirnoff two-sample test at the .10, .05, and .01 level to determine if any significant differences existed. No significant differences were detected for any of the projection methods.

Volumes were calculated for predicted diameter distributions and compared with volumes calculated from observed distributions. Comparison of total and merchantable volumes calculated from the observed and predicted diameter distributions is shown in Table 5. Based on the 1977-81 data, the basal area projection method has less bias, shows greater precision, and has the greatest PVE value for both total and merchantable volumes. Overall the difference in PVE values is very small as in Table 2, although the basal area method does appear to be superior. The same statistical criteria are shown in Table 6 for the comparison of pulpwood and sawtimber volumes calculated from the observed and predicted diameter distributions. As in Table 4, the basal area method has less bias, greater precision and the greatest PVE value for both pulpwood and sawtimber volumes.

Table 5 -- Statistical criteria for comparison of observed vs. predicted total and merchantable per acre volumes for the stand projection models using independent data.

	Weibull	Percentile	Basal Area
1977-81 data with models fit to 1981-85 data			
Total volume	20.292[1/]	-38.915	-3.559
(cu.ft./ac.)	52.785	92.238	25.307
	.996815	.992425	.997899
Merch. volume	30.954	-37.092	-3.583
(cu.ft./ac.)	65.292	101.323	30.118
	.996276	.991991	.997808

1/ The first line is the mean of the residuals, the second line is the mean of the absolute residuals, and the last line is the PVE value.

Table 6 -- Statistical criteria for comparison of observed vs. predicted pulpwood and sawtimber per acre volumes for the stand projection models using independent data.

	Weibull	Percentile	Basal Area
1977-81 data with models fit with 1981-85 data			
Pulpwood volume (cu.ft./ac.)	40.192[1/] 166.038 .904684	-12.808 166.546 .922956	6.457 118.850 .949825
Sawtimber volume (cu.ft./ac.)	-18.477 145.200 .965891	-28.954 127.492 .972254	-12.669 111.740 .978202

1/ The first line is the mean of the residuals, the second line is the mean of the absolute residuals, and the last line is the PVE value.

Further comparison of the models with validation data was done using the error index (Tables 7 and 8). Observation shows the basal area method with the lowest error index values for the number of trees per diameter class and the total volume per diameter class in all cases.

Table 7 -- Average error index for 1977-81 plots for all distributions, unimodal distributions, and multimodal distributions by 1-inch diameter classes and by diameter class groups for the number of trees per diameter class.

	Weibull	Percentile	Basal Area
1-inch classes			
All distributions	16.81	17.53	12.50
Unimodal distributions	15.82	16.89	11.93
Multimodal distributions	20.32	19.79	14.51
Groups			
All distributions	3.06	1.88	1.66
Unimodal distributions	2.91	1.92	1.75
Multimodal distributions	3.61	1.74	1.34

Table 8 -- Average error index for 1977-81 plots for all distributions, unimodal distributions, and multimodal distributions by 1-inch diameter classes and by diameter class groups for the total volume per diameter class.

	Weibull	Percentile	Basal Area
1-inch classes			
All distributions	123.37	114.29	91.79
Unimodal distributions	118.88	109.95	87.98
Multimodal distributions	139.22	129.63	105.31
Groups			
All distributions	58.17	33.63	29.15
Unimodal distributions	58.15	34.02	30.38
Multimodal distributions	58.26	32.24	24.81

CONCLUSIONS

Careful observation of Tables 1 through 8 shows the basal area method reproduces the stand and stock tables more accurately and with greater precision for most criteria investigated. Tables 5 through 8, where validation data is used, show the basal area method to have much less bias and have much greater precision and in all cases has lower error index values. The percentile-based method reproduces stock tables with greater precision for total and merchantable volumes and pulpwood and sawtimber volumes more often than the Weibull method. The percent variation explained (PVE) by the percentile-based method is also greater and has lower error index values more often than the Weibull method.

Criteria other than statistics of fit should be taken into consideration when deciding on which model to incorporate into a growth and yield system. The basal area method and the percentile-based method are both able to reproduce multi-modal distributions. As such, these methods are more precise for multi-modal stand tables. Simplicity is desirable when working with mathematical models. All of the models investigated require a survival equation, a basal area projection equation, and a height-diameter equation. The basal area growth projection method requires only one additional equation to be fit. One must remember that there is a special requirement when fitting this model. It is necessary that remeasurement data contains trees that have been identified individually over time so that it is possible to identify survivor trees. There are no special requirements necessary when using the

percentile-based method or the Weibull method. Mathematics involved for the percentile-based method involve fitting twelve equations for projection of each of the twelve percentiles. The Weibull method used in this analysis requires four percentile projection equations to be fit to the data.

To make use of the basal area method, users must have a stand table from which to begin. If stand table information is not available, then either the percentile-based method or the Weibull method would have to be used. The Weibull method should be preferred in this case since it performs almost as well as the percentile-based method and is a more mathematically simple method.

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International Union of Forestry Research Organizations
(IUFRO)

THE RELATION BETWEEN VEGETATION-BASED CLASSIFICATIONS, SYNECOLOGICAL COORDINATES, AND OVERSTORY PRODUCTIVITY: A CASE STUDY

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Summary

Numerous systems for classifying forested landscapes into ecologically homogeneous areas have been developed in the Great Lakes region of North America during the past 10 to 15 years. Although the approaches taken and factors considered differ among the developers, all rely heavily on vegetation to help distinguish different classes. A commonly used tool for classifying this complex, multivariate, floristic data is the computer program TWINSPAN.

Comparisons of overstory productivity among the ecological classes usually show that productivity differs between some classes but not between others. Because overstory productivity is essential information for forest managers but expensive or time-consuming to obtain, a method to simply identify which classes have different productivities is needed. Such a method would reduce the number of classes needing intensive analysis.

Synecological coordinates are semi-quantitative indicators of a species requirements for moisture, nutrients, heat, and light when growing in competition with other species. The average of the coordinates of all species present in a plant community provides an indication of the moisture, nutrients, heat, and light available to that community.

Plant composition is affected by environmental factors, stochastic events, and the temporal sequence of those events. This unknown mixture of environmental and non-environmental factors determines floristic variation, the basis for different TWINSPAN classes. Synecological coordinates may provide a method to identify classes that have different environments and therefore different overstory productivities.

Two long-term studies in north-central Minnesota, USA, provided data to evaluate the proposition that TWINSPAN classes with different moisture or nutrient synecological coordinates also differ in productivity. The .08 ha (.2 ac) circular plots were established about 1950 in an 80-year-old *Pinus resinosa* Ait./*P. banksiana* Lamb. stand of fire origin. A total of 39 plots received the same treatment. Gross basal area growth including ingrowth from about 1960 to 1985 and site index of *P. resinosa* provide estimates of productivity for the area. Floristic data collected from the plots in 1989 provided the basis for developing

TWINSPAN classes and calculating synecological coordinates.

A fixed effects analysis of variance found statistically significant differences in the moisture coordinate ($P = .026$) and nutrient coordinate ($P = .001$) among the four classes indicated by TWINSPAN. The Newman Keuls Studentized range test indicates ($P < .05$) that one of the classes is different from the other three. Those three classes were not different from each other and were combined to form a new class. Site index is significantly different ($P = .002$) between the two classes according to an analysis of variance. Because the initial dbh of the trees differed among the plots, an analysis of covariance with quadratic mean dbh as the covariate was used to test for differences in gross growth including ingrowth between the two classes. A significant difference ($P = .045$) was detected.

Calculating synecological coordinates for members of TWINSPAN classes requires only a list of species present. For the limited data set examined, synecological coordinates differentiate TWINSPAN classes where site index and gross growth including ingrowth are different. Further testing of this approach is needed.

Ecological classification, *Pinus resinosa*, site index, basal area, growth.

YIELD PREDICTION FOR EVEN-AGED BLACK SPRUCE STANDS

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ABSTRACT

In 1989, a mathematical model was created predicting the evolution of dendrometrical variables for stands composed mainly of black spruce. So that this model may apply to all such stands in Québec, both density and site indices have been incorporated in the algorithms. The model applies strictly to pure and even-aged stands. At the taxonomical level, the model can be classified as "whole stand". Furthermore, the model is non-spatial; that is, it does not require information on the exact location of each tree. In American literature these are referred to as "Distance independent Models". The main simulation module of the EPN model consists of multivariate non-linear equations. The model is used to construct yield tables containing the distribution of predicted values by diameter class. Estimates of absolute exploitable age and of maximum mean annual increment can also be obtained.

RÉSUMÉ

Au cours de l'année 1989, un modèle mathématique a été élaboré pour prédire l'évolution des composantes dendrométriques des peuplements composés principalement d'épinette noire. Pour qu'il soit utilisable sur l'ensemble du territoire forestier québécois, nous avons intégré dans l'algorithme de vieillissement des indices de densité et de qualité de station. Au niveau taxonomique, le modèle doit être classé dans la catégorie appelée "Peuplement entier". Il s'applique aux peuplements équiennes et purs. De plus il est non spatial car il ne nécessite aucune information sur la localisation exacte de chacun des arbres; dans la littérature américaine ces modèles sont dits Distance Independant. Le module principal de vieillissement du modèle "EPN" est un ensemble d'équations de régression non linéaires à variables explicatives multiples. Le modèle a comme but principal l'élaboration de tables de production avec ventilation diamétrale des quantités prédites sur une base quinquennale. L'estimation de l'âge d'exploitabilité absolu et de l'accroissement annuel moyen maximum est un sous-produit du modèle.

INTRODUCTION

Well known for its commercial qualities, the Black Spruce is the most commonly used species in the pulp and paper industry. It should be noted that this species is quite abondant in the province of Québec, and that site quality for stands of this species can very greatly. Foresters need mathematical models in order to predict the evolution and growth of stands. The fact that dendrometrical characteristics vary with age and are unique to any given species is a factor in choosing a model.

In Ontario, Plonski (1977) constructed normal yield tables and for stands of optimal density.

Our study uses all the sample plots from a systematically performed sampling. For each plot, both site quality and density indices were calculated and later on, these indices were taken into account in the writing of the prediction software. A model developed in this way can thus allow for a great range of initial conditions.

Payandeh's equation was used to explain the height of black spruce stands. This equation is the result of a nonlinear regression based on Plonski's data and is as follows:

$$H = 6.183 \times SI^{0.616} (1-e^{0.0211 \times AGE})^{5.968 \times SI^{-0.5657}}$$

The prediction software called "EPN" consists of a set of equations relating the various dendrometrical variables.

Simulations performed using the "EPN" software aided in the calculation of absolute exploitable age and of maximum mean annual increment (M.A.I.) using only density and site quality indices as explanatory variables. Knowing the future growth and yield leads, without doubt, to better forest management.

MATERIALS AND METHOD

Sample plot characteristics

The raw data comes from a provincial network of permanent sample plots, a network managed by the Service de l'Inventaire Forestier (S.I.F.) of the Ministère de l'Énergie et des Ressources (M.E.R.).

Figure 1 shows the distribution of black spruce stands in all of Canada while figure 1.1 shows the location of sample plots in the province of Québec. Four hundred and thirty-nine plots were used to develop the prediction model. Table 1 gives various dendrometrical information about the initial sampling. From the table we see that mean age is 84 years with a standard deviation of 32, the youngest plot being 22 years old and the oldest being 196 years old.

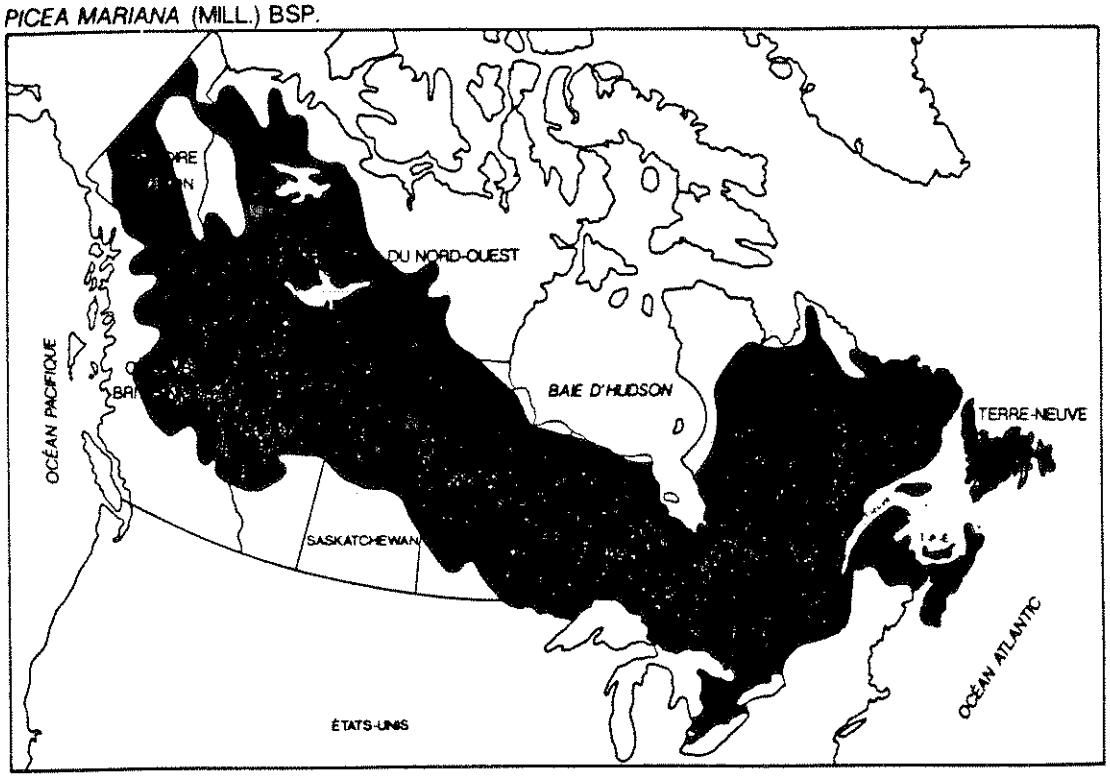


Figure 1 Distribution of black spruce stands in all of Canada

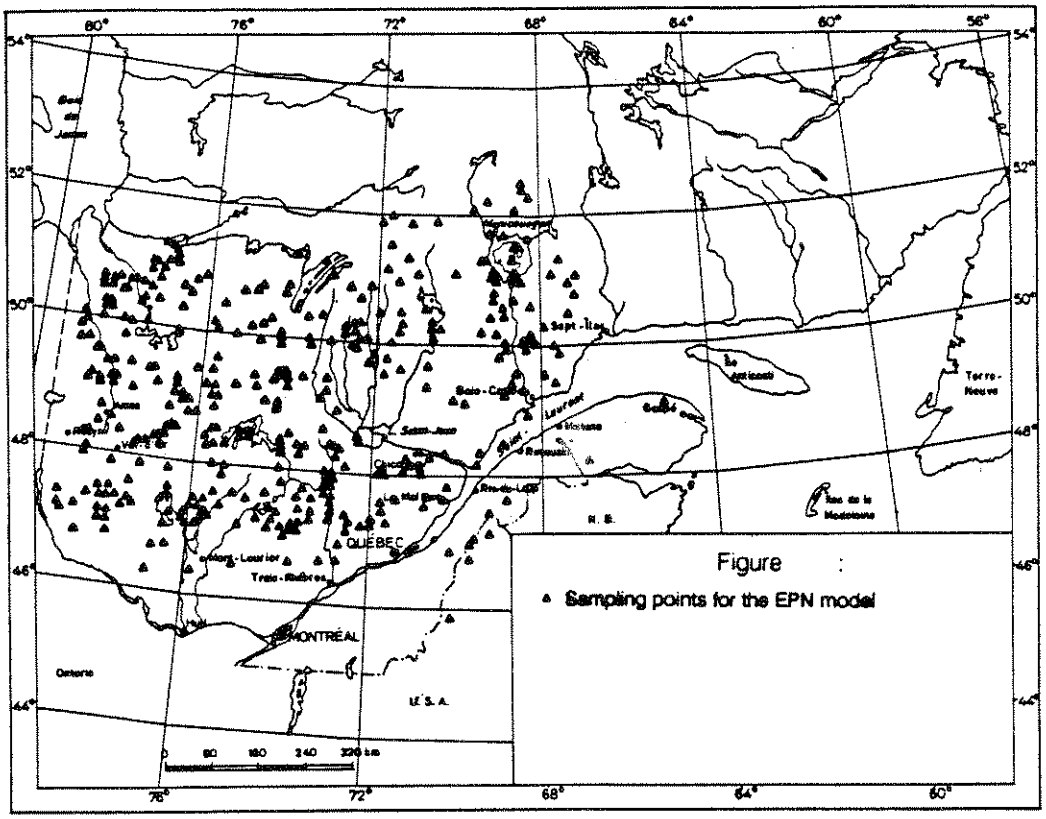


Figure 1.1 Sample points in the province of Québec

Table 1

Main statistics for the black spruce sample plots.

		Mean	Standard deviation	Minimum	Maximum
Age	:	83.7	32.4	21.5	195.8
Mean height	:	12.0	2.9	4.0	20.3
Quality index	:	8.9	3.5	2.6	18.6
Density index	:	1.21	.43	.51	3.12
Percentage of spruce	:	90.5	12.9	50.6	100.0
Total number of stems	:	3503	2188	425	13550
Number of merchantable stems	:	1196	553	25	3000
Total basal area	:	21.80	9.59	2.86	50.57
Total merchantable basal area	:	18.11	10.34	.18	48.29
Mean total dbh	:	10.04	4.14	2.41	25.50
Mean merchantable dbh	:	13.56	2.41	9.70	25.50
Merchantable volume	:	85.85	70.60	.36	382.20

Determining the density index (D.I.)

In order to develop a prediction model for varying density index it was necessary to first develop a model for these plots. The resulting equation is as follows:

$$CNS = bo\ TMDBH^{b_1}\ b_2^{TMDBH}$$

b_0 = 11 111.525949 D.F. = 915

b_1 = -0.138387 E.S.D. = 1941.753

b_2 = 0.889747 R^2 = 0.417

CNS = calculated number of stems
TMDBH = total mean DBH
D.F. = degrees of freedom
E.S.D. = estimated standard deviation
R^2 = determination coefficient

The density index (D.I.) is defined as the total actual number of stems (TANS) divided by the theoretical number of stems CNS calculated in the previous equation.

$$D.I. = \frac{TANS}{CNS}$$

where TANS = total actual number of stems
CNS = calculated number of stems
D.I. = density index

Thus, using this method, a density index was calculated for each plot and all those with an index of less than 0.50 were rejected. The mean density index for the set of accepted plots is 1.21 while the standard deviation is 0.37 and the maximum index is 3.12. The distribution of plots by density class is shown in figure 2.

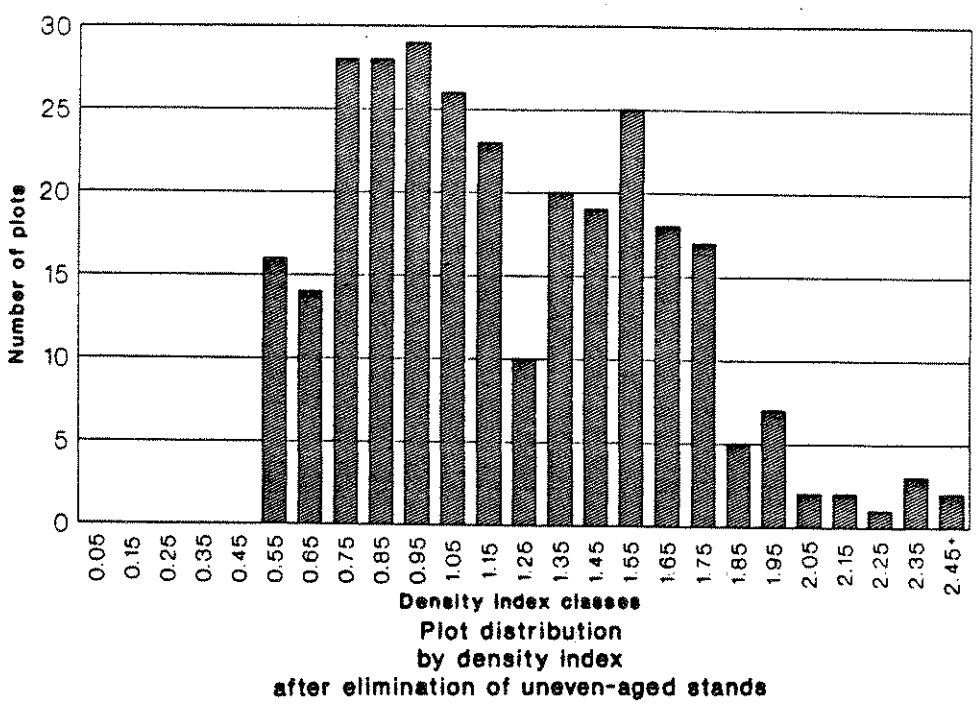


Figure 2 Distribution of black spruce plots with respect to density index.

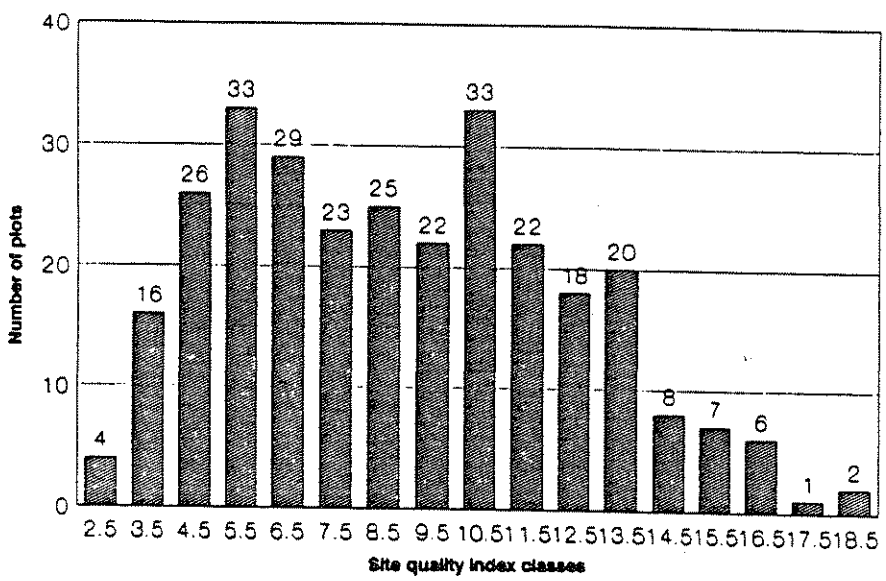
Determining the site quality index (S.I.)

The site quality index allows foresters to quantify the growth potential of a forest stand. This index incorporates many different factors: genetic factors of the given species, pedological factors of the site and microclimatic factors of the growth location. Rather than estimating and quantifying the individual effect of each factor, it was decided that their global effect should be considered instead, using mean age and mean height of dominant and codominant trees. More precisely, the S.I. is defined as the height of the stand at 50 years of age. Since very few plots are exactly 50 years old, an equation estimating the site index is necessary. Payandeh's equation (1978) SI = f(H,age) for prediction is based on Plonski's data and has the following form:

$$SI = b_0 H^{b_1} (1 - e^{b_2 \text{ age}})^{b_3 H^{b_4}}$$

where $b_0 = 0.025$
$b_1 = 1.7509$
$b_2 = -0.00296$
$b_3 = -2.0354$
$b_4 = -0.3212$

and SI = site index in metres
H = mean height of dominant and codominant trees



Plot distribution
by site index
after elimination of uneven-aged stands

Figure 3 Distribution of black spruce plots with respect to site quality index

The development of the prediction model requires the estimation of height as a function of age. Payandeh's relation H = f (SI, Age) was used, it is based on Plonski's data and has the following form:

$$H = a_0 SI^{a_1} (1-e^{a_2 age})^{a_3 SI^{a_4}}$$

where :
$a_0 = 6.183$
$a_1 = 0.515$
$a_2 = 0.0211$
$a_3 = 5.958$
$a_4 = -0.5657$

RESULTS

Prediction of gross merchantable volume

Gross merchantable volume (mv) is a fonction of merchantable basal area (MBA) and mean predicted height.

$$MV = f(H, MBA) = b_0 \, H \, MBA^{b_1}$$

where : $b_0 = 0.299014$ D.F. = 437
$b_1 = 1.068477$ E.S.D = 8.14007
$R^2 = 0.98$

Merchantable basal area is directly related to the total basal area, this relation being expressed by a nonlinear equation. The importance of knowing the SI is made clear in figure 4. Indeed, volume is strongly influenced by the mean height of a stand. It has been noted previously that for any given age, height increases with SI.

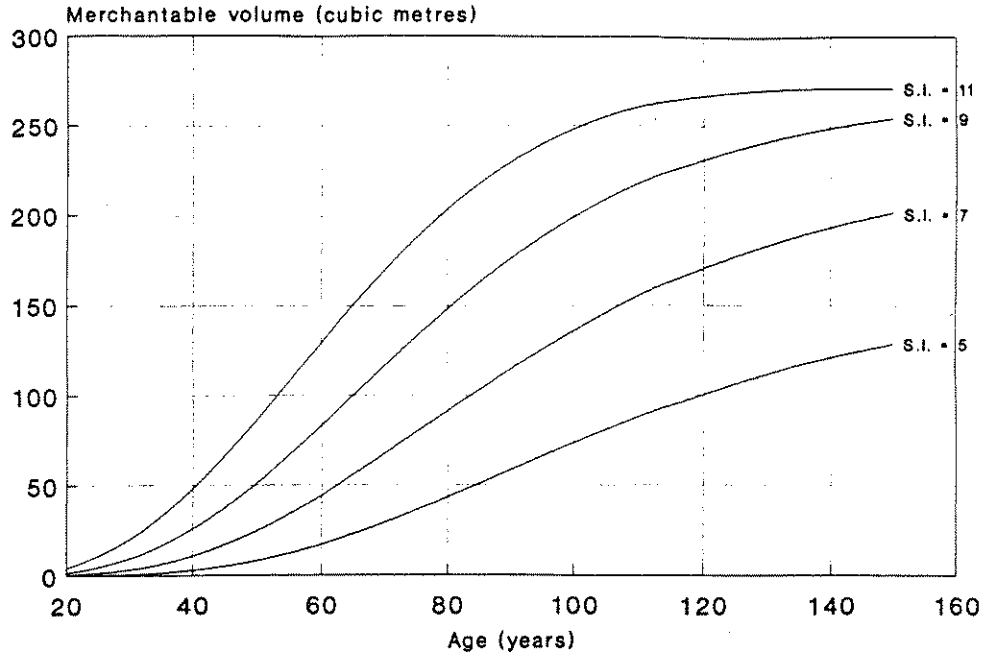


Figure 4 Predicted volume as a function of age for different site quality indices and for a fixed density index of 1.5

Absolute exploitable age

The most common method of determining absolute exploitable age of a stand is to take the point of intersection of 2 curves : the mean annual increment (M.A.I.) and the current annual increment (C.A.I.). The age found at this point also corresponds to the maximum mean annual increment. Figure 5 illustrates absolute exploitable age and the corresponding maximum mean annual increment.

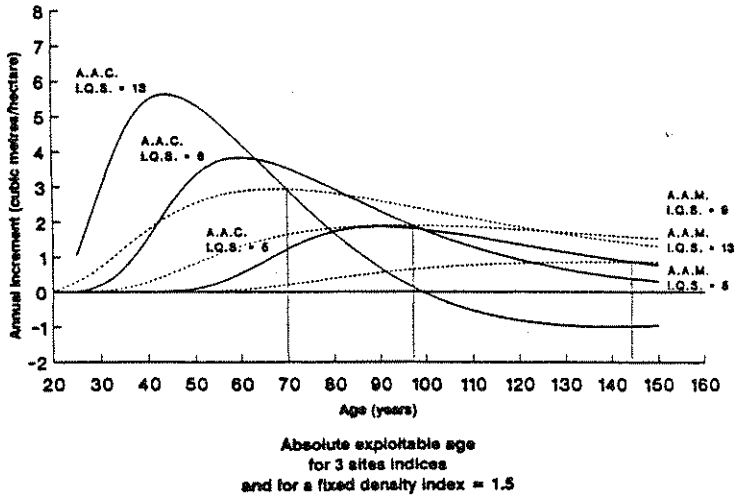


Figure 5 Absolute exploitable age (AEA) for 3 SI's and for a density index fixed at 1.5

From figure 5 one can see that absolute exploitable age (AEA) varies inversely as a fonction of site quality index and that maximum mean annual increment (MMAI) increases with increasing site quality index.

Construction of yield tables

The main purpose behind the prediction model "EPN" is to construct yield tables which predict yield in terms of volume for a period of several years. Such tables can be classified as being either normal or empirical. Indeed, this model which takes into account varying density indices can accept as input a variety of initial conditions with respect to number of stems per diameter class and per acre. This model was also meant to take into account the wide range of SI's that have been observed in black spruce stands found in Québec. The prediction period goes from 20 to 150 years, a period containing 99% of our sample.

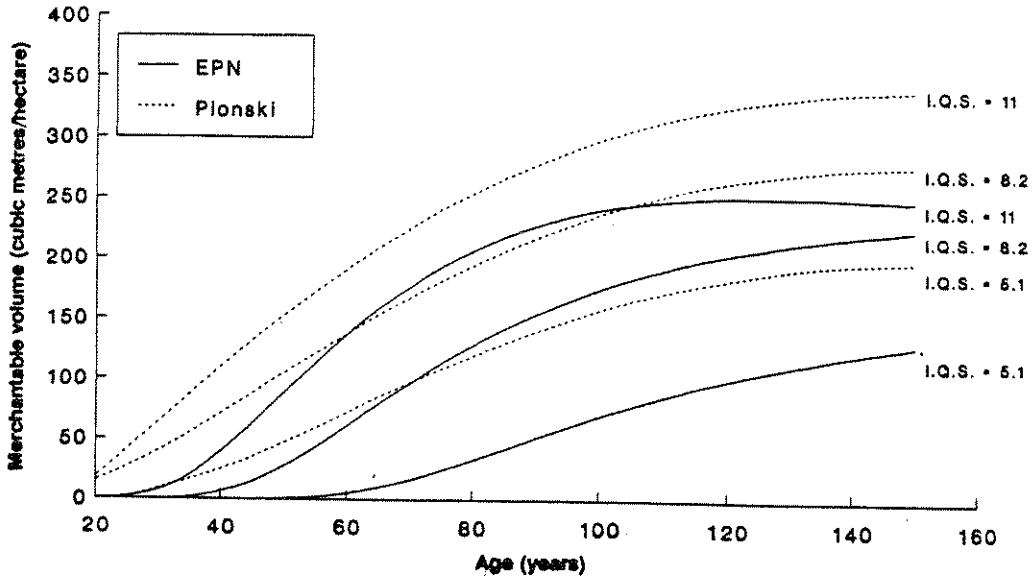
Table 2 - Yield table for varying densities

Age	Height	T.N.S.	T.N.M.S.	T.B.A.	M.B.A.	TDBH	MDBH	Volume	Annual increment Current		Mean	Age
	meters			m^2	m^2	cm	cm	m^3	m^3	m^3	m^3	
20	3.1	12371	0	3.7	0.0	1.9	12.7	0	----	----	0.00	20
25	4.1	10816	5	6.1	0.1	2.7	12.3	0	24.59	0.01	0.00	25
30	5.2	9516	46	9.0	0.5	3.5	11.9	1	23.00	0.12	0.02	30
35	6.3	8411	194	12.0	2.1	4.3	11.7	4	19.79	0.60	0.10	35
40	7.3	7464	480	15.1	5.1	5.1	11.7	11	15.74	1.54	0.28	40
45	8.3	6650	848	18.1	9.1	5.9	11.7	24	11.94	2.59	0.54	45
50	9.2	5948	1215	21.0	13.3	6.7	11.8	41	8.92	3.37	0.82	50
55	10.0	5342	1523	23.5	17.2	7.5	12.0	60	6.71	3.77	1.09	55
60	10.8	4818	1751	25.8	20.6	8.3	12.2	79	5.12	3.86	1.32	60
65	11.6	4364	1901	27.9	23.5	9.0	12.6	98	3.98	3.76	1.51	65
70	12.3	3970	1985	29.6	25.9	9.7	12.9	116	3.15	3.54	1.65	70
75	12.9	3628	2018	31.0	27.9	10.4	13.3	132	2.52	3.25	1.76	75
80	13.5	3331	2015	32.3	29.5	11.1	13.6	147	2.04	2.94	1.83	80
85	14.0	3073	1986	33.2	30.8	11.7	14.0	160	1.67	2.62	1.88	85
90	14.5	2847	1941	34.0	31.8	12.3	14.4	171	1.37	2.31	1.90	90
95	14.9	2650	1886	34.7	32.5	12.9	14.8	181	1.12	2.02	1.91	95
100	15.4	2478	1826	35.2	33.1	13.4	15.2	190	0.93	1.75	1.90	100
105	15.7	2327	1764	35.5	33.6	13.9	15.6	198	0.76	1.50	1.88	105
110	16.1	2195	1703	35.8	33.9	14.4	15.9	204	0.63	1.28	1.86	110
115	16.4	2078	1645	36.0	34.2	14.9	16.3	210	0.52	1.09	1.82	115
120	16.6	1976	1589	36.1	34.3	15.3	16.6	214	0.43	0.92	1.78	120
125	16.9	1885	1537	36.2	34.4	15.6	16.9	218	0.35	0.77	1.74	125
130	17.1	1806	1488	36.3	34.5	16.0	17.2	221	0.29	0.64	1.70	130
135	17.3	1735	1444	36.3	34.5	16.3	17.4	224	0.24	0.53	1.66	135
140	17.5	1673	1403	36.2	34.4	16.6	17.7	226	0.20	0.44	1.61	140
145	17.7	1617	1366	36.2	34.4	16.9	17.9	228	0.16	0.37	1.57	145
150	17.8	1568	1332	36.2	34.3	17.1	18.1	229	0.13	0.30	1.53	150

Importance = 100.0 Density index = 1.50 Quality index = 9.0

Comparison between predictions of the "EPN" model and those of the Plonski tables

It is interesting to compare results obtained in Québec with those published by Plonski (1974). While it is true that, on the whole, natural black spruce stands in Ontario are more productive than those in Québec, it should be noted that only well-stocked sites were sampled in Ontario. Thus, for comparison purposes, we have fixed the density index in "EPN" at 1.0. Figure 6 should be interpreted with care since the volumes were calculated using two different volume tables. Furthermore, the Québec volumes represent only black spruce stems; all other species were omitted.



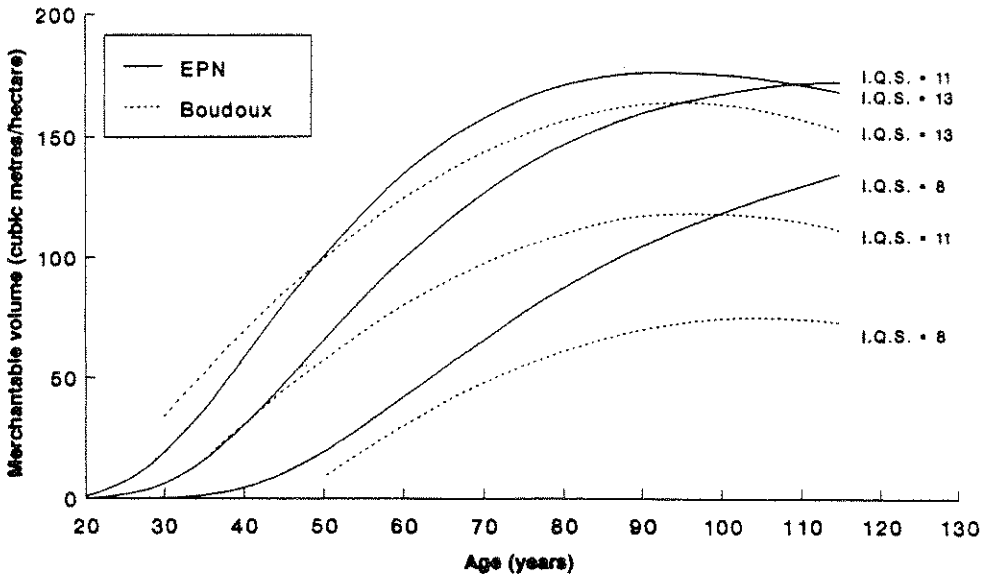
Comparison of Plonski volumes and EPN model predictions for the same site indices

Figure 6 Gross merchantable volume according to EPN and Plonski by site quality index

Comparison between predictions of the "EPN" model and those of Boudoux

The previous section showed a growth template for the black spruce that is different for the province of Québec and for Ontario. Naturally, we could ask if previous studies for the province of Québec on this species gave

the same results. It is for this reason that Boudoux (1978) was considered as a second source of comparison. Figure 7 clearly shows that spruce in Québec is more productive than Boudoux's tables. We attribute the differences to the use of nonlinear models rather than multivariate models and a more exhaustive sampling.



Comparison of Boudoux volumes
and EPN model predictions
for the same site indices

Figure 7. Gross merchantable volume according to EPN and Boudoux by site quality index.

CONCLUSIONS

The predictions obtained as a result of running the "EPN" software show that its development was based on sound principles. The methodology of establishing a time series of growths is based on local observations. The development of the software is also based on two widely accepted assumptions : firstly, that the site quality index does not vary during the development of a generation of trees and secondly, that the density index, too, is stable except in cases of extreme. The software does not take into account such occurrences, nor does it take into consideration losses due to such events.

The prediction model is less reliable in situations involving extreme values of site quality, density and age. Care should always be taken when interpreting these extreme cases. In the years to come, there will be more raw data available which will allow a more detailed study of such cases, especially studies of the evolution of old black spruce stands. It is possible that some plots will disappear from the original sample once the proportion of black spruce becomes too low. When this happens, that is when the proportion of other species becomes relatively higher, some of the plots may be rejected. A possible solution could be to consider the proportion of black spruce in plots relative to their age.

ACKNOWLEDGEMENTS

We want to express here our sincere thanks to all the mathematicians who have, at one time or another, supported this project. We would like to mention here Mary Boldrini, Nadine Genest and Lise Grenier.

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MATEMATICAL MODELS FOR THE INTERDEPENDENT GROWTH OF TWO TREES

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SUMMARY

The present paper is concerned with a class of deterministic mathematical models for the simultaneous interdependent growth of pairs of even-aged pines. Numerical examples based on data from an even-aged non-thinned pine stand shows that with certain of these models a better fitting is achieved than by using the classical "one-tree" approch.

Keywords: growth analysis, interdependent growth

INTRODUCTION

In order to provide precise forecasts of forest growth, several attempts have been made towards a mathematical description of the tree growth (c.f. the references in Chuanzhong Li (1988) or Saborowski (1982)).

Often deterministic growth functions have been used, e. g. the so-called GOMPERTZ function

$$y(t) = B \exp(C \exp(kt)), \qquad t \geqq t_0 \qquad (1)$$

where $y(t)$ denotes the growing quantity of interest at time t, and B, C and k are certain parameters.

As the individual tree growth clearly is a random phenomenon, the use of deterministic models can be justified from at least two points of view: First, they are intended to describe the "mean" beheviour of the whole stand. And second, they are considered as ideal individual tree growth functions which cannot be immediately observed due to random fluctuations. In both cases, deterministic models have received a certain practical justification.

However, from the second point of view it seems to be reasonable to make the models more precise by simultaneously modelling the interdependent growth of several tree individuals. For example, whereas the GOMPERTZ function can be described by an one-dimensional ordinary differential equation:

$$y(t) = k\, y(t)\, (\ln B - \ln y(t)) \qquad (2)$$

with initial condition $y(t_0) = y_0$, now it seems reasonable to consider several such equations simultaneously (each of them representing another fixed individual) and to disturb each of these equations by a certain expression which models the inhibition raised by the competitive neighbours of each individual.

In order to investigate the possible advantages of such an approch, in the present paper we restrict attention to pairs of trees, where each tree is the nearest neighbour of the other. That is, only the influence of the current nearest neighbour is taken into account. Further it is assumed that inhibition acts additively in the differential equations.

In the section "A class of models for two-tree growth" several specific models of the above type are introduced. They have been fitted to numerical data from an even-agged non-thinned pine stand. The section "Fitting of the models to the data" gives an explanation of this procedure which can be

found in greater detail in the research report from Renz and Vorpahl (1989). In the section "Some numerical results" we give an outline of the results. The main result is that certain of these models allow a substantially better fitting than by the classical "one-tree" approch.

A CLASS OF MODELS FOR TWO-TREE GROWTH

Assume that an adequate model for an ideal one-tree growth is given by a GOMPERTZ function generated by the differential equation:

$$y(t) = k\, y(t)\, (\ln B - \ln y(t)) \qquad (2)$$

with the initial condition $y(t_0) = y_0$ and the parameters B and k. Now we declare a denotation, when the tree number i is considered then the nearest neighbour is the tree 1-i. A fairly general structure of a related two-tree problem with an additive inhibition is

$$y_i(t) = k_i\, y_i(t)\, (\ln B_i - \ln y_i(t)) - h\,(y_i(t),\, y_{1-i}(t),\, \theta_i) \qquad (3)$$

for $t \geq t_0$, with initial conditions $y_i(t_0) = y_{i0}$ and specifying parameters B_i, k_i and θ_i $(i = 0,1)$.

Here the function h should be chosen suitably to represent the inhibition acting on the i-th tree which is caused by the competing (1 -i)-th tree.

The restriction to an additive influence of the inhibition (and not, e.g., to a multiplicative one) can be understood as modelling an a b s o l u t e loss of resources (water, light, nutrients) for the i-th tree raised by its competitor.

There is no general answer yet to the question how to choose the function h optimally. When we cast a look on suitable h's, however, it is reasonable to require that h be an increasing function with respect to y_{1-i} (the bigger the competitor, the stronger the inhibition) and a decreasing function with respect to y_i (by an analogue reason). Further it could be desirable to involve also the influence of the distance of the trees in the model. In fact, in (3) this distance can be taken into account by a suitable choice of the (vector) parameter θ_i.

The purpose of the present investigations was first of all to study a class of models (3) involving comperatively simple functions h. In particular, $h = h\ (y_i,\ y_{i-1},\ \theta_i)$ was chosen as

$$h_1 = c_1\ y_{i-1} + e_i \qquad \theta_i = (c_i,\ e_i)$$

$$h_2 = c_1\ \frac{\sqrt{y_{i-1}}}{y_i} \qquad \theta_i = c_i$$

$$h_3 = c\ \sqrt{y_{i-1}} \qquad \text{(that is, } \theta_i = \theta_{i-1} = c)$$

$$h_4 = c_1\ \frac{\sqrt{y_{i-1}}}{y_i} + e_i \qquad \theta_i = (c_i,\ e_i)$$

$$h_5 = c\ \frac{\sqrt{y_{i-1}}}{y_i} \qquad \text{(that is, } \theta_i = \theta_{i-1} = c).$$

The computations within model (3) and with inhibition functions h_1 and h_3 refer to the stem diameter, whereas those with inhibition functions h_2, h_4 and h_5 refer to the stem area as the growing quantity of interest. For the functions h_1 to h_4, the initial values have been considered as free parameters as well.

FITTING OF THE MODELS TO THE DATA

For the investigation of the mathematical model, one trial plot was chosen. This trial plot was a non-thinned pine stand in the forest district Finowtal. It was established in 1941 and the breast high diameters of all stems were measured at ages 22, 25.75, 29, 33, 41 and 46.5 years. The extent of this area is 30 x 30 m and it contains after 22 years 1298 stems. Moreover the co-ordinates of every tree as to a local co-ordinate system were recorded. From this trial plot, 18 pairs of surviving stems were picked such that each stem of a pair was the nearest neighbour of the other; moreover, the influence of all other stems from the neighbourhood of the pairs was apperently neglectable. The

system of differential equations (3) with the inhibition functions h_1 to h_5 is a nonlinear system, which cannot be solved by analytic procedures. Therefore the integration must be performed numerically, for example with the RUNGE-KUTTA-Method. This method is only available when all parameters are known or can be estimated. The statistical estimation of the parameters is possible with the least squares method, which raises a regression problem. This regression problem is nonlinear with nonanalytic approximation functions. For solving this regression problem we can use the MARQUARDT algorithm. Details of these procedure, in particular the applied software, instructions for users, and all numerical results are extensively described in Grüning and Belitz (1989).

S O M E N U M E R I C A L R E S U L T S

In the following we can confine ourselves to rather few numerical examples as alredy these exhibit some main effects. (Some of them are exhibited graphically in the Appendix.)

Consider two pairs of trees:

P_1: trees number 325 and 404 (distance 1.34 m)

P_2: trees number 367 and 368 (distance 0.40 m)

The corresponding measurements are given in the following table

	age	22	25.75	29	33	41	46.5	[y]
P_1	d_{325}	70	82	87	98	121	136	[mm]
	d_{404}	80	90	96	107	137	152	[mm]
P_2	d_{367}	66	70	78	80	81	82	[mm]
	d_{368}	63	71	78	84	87	89	[mm]

Now compare the results of an individual curve fitting of the GOMPERTZ function (1) to each of these 4 trees with those of the fitting of (3) to the two pairs.

Let SAQ_i and SQ_l denote the sum of squared deviations in individual and pair-wise curve fitting applied to the i-th tree and l-th pair, respectively. Fitting results in the following table:

	h_1	h_2	h_3	h_4	h_5	$\widetilde{SQ}_1$
SQ_1	11.79	27.40	20.61	33.08	34.08	24.45
SQ_2	10.13	13.12	25.48*	12.10	14.10	13.70

* case of no convergence Note that $\widetilde{SQ}_1 = SAQ_{325} + SAQ_{404}$ and $\widetilde{SQ}_2 = SAQ_{367} + SAQ_{368}$ refer to the stem diameter and cannot be compared with SQ-values corresponding to h_2, h_4 and h_5.

From these results it follows that:

1) by using type (3) models a substantially better individual curve fitting than by using (1) can be achieved,

2) it need not be possible to find one function h among (h_1, h_3) or (h_2, h_4, h_5) which gives an optimal approximation for each pair of trees.

C O N C L U S I O N S

The results of this problem can be generalised. We have also tried to solve a "three-tree problem" succesfully. Now the forester has the possibility to decide objectively which of two stems he will remove from the stand. The results of this work are also a foundation for the computer aided forecast of growth and the individual management of the stand.

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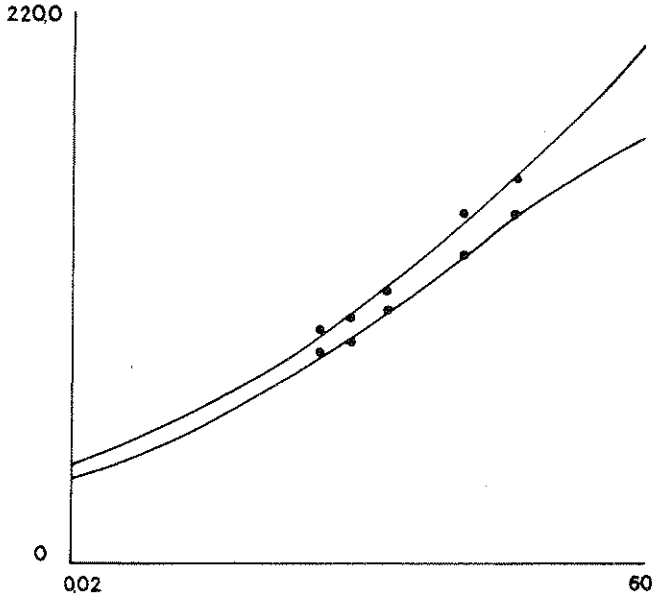

Figure 1:
Trajectory and realization of a stochastic process of tree pair 1 with the inhibition function h_3

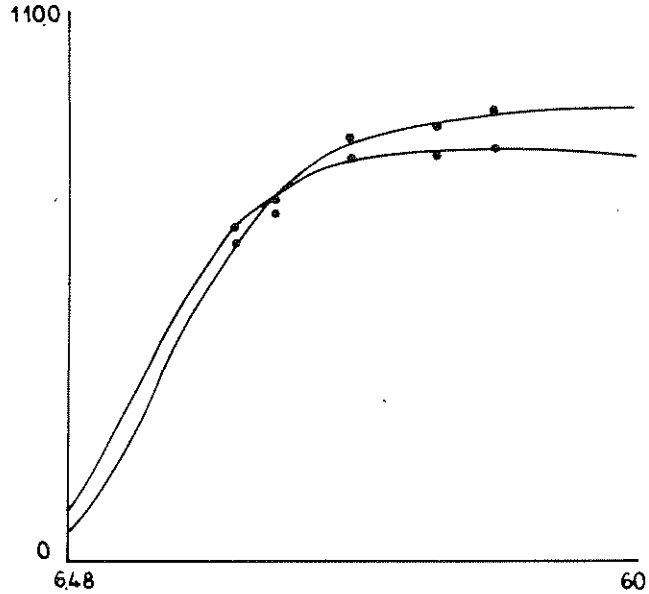

Figure 2:
Trajectory and realization of a stochastic process of tree pair 2 with the inhibition function h_2

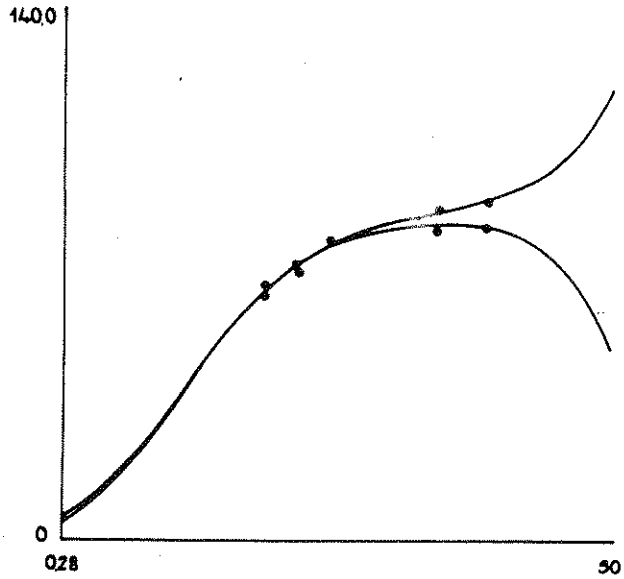

Figure 3:
Trajectory and realization of a stochastic process of tree pair 2 with the inhibition function h_3. A case with an unsuitable result.

GROWTH AND YIELD PREDICTORS FOR NATURALLY REGENERATED, EVEN-AGED SOUTHERN PINE STANDS: RECENT RESULTS AND FUTURE RESEARCH PROSPECTS

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SUMMARY

Systems to predict the growth and yield of thinned, natural, even-aged pine stands are of considerable utility in the South due to the extent of the resource and its probable future importance. Past prediction efforts are reviewed, and examples of prediction systems are given along with some current modeling activities. A recently developed, d.b.h.-distribution system for thinned stands utilizing parameter-free, d.b.h.-distribution recovery procedures and a developing individual-tree-based system are discussed. Adequate data bases are the most important need for the future. Keywords: growth, yield, natural, even-aged, pine.

INTRODUCTION

Naturally regenerated pine stands are a very important forest resource in the South. Approximately 35 percent of the 184 million acres of commercial forest in the South is occupied by stands of essentially pine or combinations of pine and hardwood of natural origin. The seven State Mid-South area, served by the Southern Forest Experiment Station, has 35 percent of its 99 million acres of commercial forest land in pine and oak/pine forest types, mostly of natural origin. Natural stands account for 72 percent of the pine and 86 percent of the oak/pine area. Furthermore, 92 percent of the area in these stands is occupied by types involving the four major southern pines: loblolly (*Pinus taeda* L.), longleaf (*P. palustris* Mill.), shortleaf (*P. echinata* Mill.), and slash (*P. elliottii* Engelm.). About half of the area

occupied by these major species contains even-aged stands[1].

Thus, naturally regenerated, even-aged stands of southern pines constitute a considerable resource now and will likely remain so in the future. Natural regeneration methods are more economical than artificial methods, and increasing future energy, equipment, chemical, and labor costs may make natural stand management even more attractive, particularly to some landowners. These include many public agencies and some industries. Management of natural stands should be most attractive to the nonindustrial, private owners who possess some two-thirds of the forest land in the South and tend to be quite conservative regarding management investments. Stands regenerated under natural systems are likely to be less degraded environmentally and may also retain more biodiversity than stands artificially established on prepared sites. There is a growing interest by some owners in uneven-aged systems, but probably most natural pine stands will be managed as even-aged ones due to the simplicity of even-aged systems and their ease of application.

Successful, even-aged management requires a capability to forecast the growth and yield resulting from various stand conditions and management treatments. Our objective is to present a background on past efforts, to discuss current activities, and to outline future needs in research aimed at developing prediction systems for managed, natural, even-aged pine stands. We will not provide an exhaustive literature review for each area but will cite illustrative examples.

B A C K G R O U N D

During the last 30 years, stand growth and yield research in the South has concentrated on stand-level, lump-sum predictors; stand-level, d.b.h.-distribution predictors; individual-tree-based predictors; and tree-volume-defining functions. Essentially, the predictors have been developed for pure stands of a single pine species.

L u m p - s u m p r e d i c t o r s

The major innovation in growth and yield models for managed, even-aged, natural southern pine stands was the compatible system first developed by

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1 Area information supplied by Forest Inventory and Analysis (FIA) Research Work Unit, U. S. Forest Service, Starkville, MS, 1990.

Clutter (1963) and later improved by Sullivan and Clutter (1972) into a simultaneous growth and yield system. These systems are based on the fundamental and logical premise that compatibility should exist between growth and yield. Growth should be the derivative of yield, and yield should be the integral of growth. The functions are easy to fit, do not demand complicated data, and are easy to apply. They have been developed for natural stands of the four major southern pines (Brender and Clutter 1970; Farrar 1979, 1985b; Murphy 1982, 1983a; Murphy and Beltz 1981; Bennett 1970, 1980) and serve the needs of many who require relatively simple predictions. These systems are usually developed from data bases where plots have been thinned to various levels of residual density, usually by low thinning. Their major disadvantage is that they do not provide product-volume predictions by d.b.h. class or tree size and cannot simulate detailed merchandising of trees and stands.

These models typically operate in the following manner. Current stand volumes are estimated by stand-volume functions in terms of stand age, site index, and basal area (total or merchantable). The volume functions may predict total cubic feet, merchantable cubic feet, and board feet by some log rule. Future stand volumes are predicted by the same functions but use future age, site index, and future basal area. A basal area projection function (or "driver") estimates future basal area by current and future age, current basal area, and sometimes site index. Thus, both net basal area growth and net volume growth can be estimated by subtracting current from future stand values or their estimates. Thinning can be simulated by predicting the volume for a before-cut level of basal area, predicting the volume for an after-cut level, and subtracting the after-cut level from the before-cut level to obtain the cut. Growth can be estimated for a single period, or a thinning regime can be simulated over an entire rotation.

Recently, these simultaneous type predictors have been enhanced by providing two basal area drivers (Murphy 1983b). One driver predicts the future total or merchantable basal area, and a compatible, second driver predicts the future sawtimber basal area as a subset of the total or merchantable basal area. Thus, the future sawtimber basal area is predicted by current and future age, current and future total or merchantable basal area, current sawtimber basal area, and, sometimes, site index. Sawtimber volumes are predicted using stand volume functions in terms of age, site index, and sawtimber basal area. This refinement permits more flexibility and versatility because both the total or merchantable basal area and the sawtimber basal area can be manipulated in thinning simulations and limited merchandising into merchantable, sawtimber, and pulpwood volumes is permitted. These systems are implemented by simple computer programs or electronic spreadsheets on a microcomputer.

D.b.h.-distribution predictors

D.b.h.-distribution predictors also predict at the stand level but volume is disaggregated into d.b.h. classes. Therefore, they provide more detailed information than lump-sum models and enable simulated merchandising of stands

into multiple products. Most activity in this area has been for unthinned pine plantations, but some work has been done for unthinned natural stands (Burk and Burkhart 1984, Farrar 1985a, Schreuder and others 1979). However, until recently, no d.b.h.-distribution models had been developed for thinned, even-aged, natural stands of southern pines.

The simultaneous type, stand-level predictors accommodate thinning simulation easily; and, historically, even-aged natural stand data have come from thinned stands so development work here gravitated toward this model type. Parallel research in plantations dealt mainly with unthinned stands which are more amenable to modeling by d.b.h.-distribution predictors. Generally, unthinned-plantation models employed probability density functions (pdf), most commonly the Weibull pdf, fitted to diameter-frequency data. The parameters of these fitted pdf's were either predicted directly from stand-level variables such as age, site index, and trees per acre (Dell and others 1979) or were recovered from their relationship to predicted stand-level variables (Matney and Sullivan 1982) such as arithmetic mean d.b.h., quadratic mean d.b.h., or percentiles. These techniques work relatively well for the generally smooth unimodal distributions of unthinned stands and have been developed for thinned plantations (Bailey and others 1981, Baldwin and Feduccia 1987) as well as row- or broadcast-seeded stands (Baldwin 1989). However, they are difficult to employ in thinned or unthinned stands with d.b.h. distributions that are irregular, truncated, or multimodal; conditions which unfortunately commonly occur in the field. Therefore, in recent years, there has been considerable activity in plantation modeling in projecting stand tables without assuming any underlying pdf (Borders and others 1987, Pienaar and Harrison, 1988).

Individual-tree-based predictors

Individual-tree-based predictors are of two basic types: distance dependent and distance independent. These predictors are the most versatile and provide the most detailed production information of any models. But they also require detailed data, are time consuming to develop, may be inaccurate due to accumulating errors, and may be expensive to use on computers, particularly if they are distance-dependent. Distance-dependent, individual-tree models utilize information on tree location to help predict the growth of the individual trees; therefore, they require stem-map data for development. Distance-independent, individual-tree predictors do not use tree location data and are, therefore, less difficult to develop and require less computer time.

There are two such systems for managed, even-aged, natural stands of southern pines. One is a distance-independent system based on TWIGS (Miner and others 1988) in which the coefficients for the various functions have been fitted by Bolton and Meldahl (1990) with FIA data from the 1980-83 survey of Georgia. TWIGS is essentially a microcomputer version of the STEMS (U.S.D.A., Forest Service 1979) system designed to run on mainframe and minicomputers.

The other, a distance-dependent system, has been partially developed for seeded stands (including natural) of loblolly pine by Daniels and others (1979). Development of the initial stand generation stage was successful, but the subsequent tree survival and growth were not well described by the available plantation relationships. Completion awaits the availability of suitable seeded-stand data.

Volume-defining functions

Stem-profile functions that describe stem diameter change with height and that can be integrated over height to obtain cubic foot volume to various heights or top diameters (Bennett and others 1978) have been the most significant recent advance in volume-defining functions for southern pines. These functions provide estimates of the dimensions of veneer bolts or logs within a tree and,thereby,permit estimates of tree veneer volume or board foot volume. When coupled with d.b.h.-distribution or individual-tree-based growth prediction systems, they permit versatile and valuable predictions of various product volumes in trees, diameter classes, and stands.

These functions are usually expressed in terms of tree d.b.h. and total height. The addition of an expression of stem form, such as crown ratio percentage, has been found useful (Dell 1979, Valenti and Cao 1986). Three stem-profile functions for natural southern pines that employ crown ratio have been developed (Farrar 1987, Farrar and Murphy 1987, 1988). Recently Parresol and Thomas (1989) have combined a stem-profile function with a tree specific gravity function. Doubly integrating the functions provides estimates of stem dry weight. Certain other tree biomass components can be estimated from existing prediction equations (Clark 1986).

CURRENT ACTIVITIES

Currently, most thinned, even-aged, natural stand models for southern pine are the relatively simple, stand-level, lump-sum simultaneous type employing dual basal area drivers. They are available for loblolly, longleaf, and shortleaf pines in pure stands (Farrar and others 1986). These systems are quite useful due to simplicity and ease of application but are quite limited in stand conditions and the variety of stand and volume output permitted. We think that this technology has been developed to the point of diminishing returns. We are now concentrating our efforts in the more promising areas of d.b.h.-distribution and individual-tree-based predictors.

To be truly versatile and most useful, a prediction system should be able to accept almost any current tree list or tree frequencies by d.b.h. class from an inventory, account for the growth and mortality of these trees or classes for a period of time, and return an accurately-estimated future tree list or d.b.h. distribution. Companion tree-height-defining and stem-profile functions permit estimation of tree heights and desired volumes or weights. Individual-tree-based predictors have this potential, and some developing d.b.h.-distribution techniques have similar capabilities. We are participating in these arenas of emerging efforts in natural, even-aged, pine growth prediction, and they are illustrated by the following two examples.

For 25 years, the Southern Forest Experiment Station has cooperatively maintained a comprehensive stand-density study in thinned, natural, longleaf pine stands. It now consists of some 275 permanent one-fifth-acre plots covering an array of stand ages, site indices, and stand densities maintained by low thinning. Stand ages range from 15 to over 100 years; site indices, from 45 to over 90 feet at age 50; and basal areas, from 30 to 150 square feet per acre. The plots have been inventoried every 5 years and periodically thinned to assigned densities. There are three comparable time replications 10 years apart installed in the youngest age class. These time replications will be maintained through a rotation of perhaps 120 years. All trees are positively identified, mapped, and measured for d.b.h., crown class and for utility pole class and length. Systematic subsamples are made for total height, height to crown base, and age of dominants or codominants. This data base has supported development of volume-defining functions, site-index functions, and stand-level, volume-growth predictors and is currently supporting development of both d.b.h.-distribution and individual-tree-based predictors.

The d.b.h.-distribution system is an adaptation of one devised for thinned, loblolly pine plantations by Matney and Farrar (1990) and is being applied to natural, longleaf pine stands. The system consists of two parts: (1) a three-parameter, Weibull-recovery system to simulate stand development up to first thinning and (2) a parameter-free, d.b.h.-distribution recovery system to simulate stand development after thinning.

The system was developed in two phases. First, stand-level regression equations were calculated to estimate and project the arithmetic and quadratic mean d.b.h.'s and to estimate minimum stand d.b.h., number of surviving trees per acre, site index and mean height of dominants and codominants, total height/d.b.h. relationship, and number of trees following thinning from below. Differences between thinned and unthinned stands required development of separate equations for survival and arithmetic and quadratic mean d.b.h.'s for conditions before and after thinning. Poor predictability prevented development of an equation to estimate minimum d.b.h. of thinned stands. In the second phase, models were developed to obtain d.b.h.-distribution projections from the stand-level relationships determined in the first phase.

Before first thinning, the arithmetic mean, quadratic mean, and minimum d.b.h.'s are estimated at projection age. The Weibull "a" parameter is set to one-half of the minimum stand d.b.h.. The Weibull "b" and "c" parameters are recovered so the resulting d.b.h. distribution has expected mean d.b.h.'s

equal to the projected mean d.b.h.'s. An ordered tree d.b.h. list is used in simulating either lower d.b.h.-limit thinning, low thinning, or both. After thinning, the residual stand is grown to the next projection age. Future arithmetic and quadratic mean d.b.h.'s and surviving trees per acre are predicted from stand-level equations. To allocate stand-level growth to individual trees, mortality trees are removed from the list, and a weighted, constrained, least squares procedure is used to derive linear, tree-growth, projection equations. These are used to grow the residual stand tree list into a future tree d.b.h. list that has the same arithmetic and quadratic mean d.b.h.'s as predicted. An integrated stem-profile function, tree total height and crown base height equations, and the predicted stand tables are used to estimate current and future product volumes.

The system requires a personal computer with FORTRAN capability, but it offers considerable versatility and output detail. The nonparametric, d.b.h.-distribution recovery technique seems to handle nonsmooth distributions, such as those sometimes left after thinning, rather well.

Our second major effort with the natural, thinned, longleaf pine data base is the construction of an individual-tree-based prediction system at Auburn University. It is in the early stages of construction, and specifics regarding it are few. Generally, it is expected to follow the conceptual format of the distance-independent system of STEMS. It is thus expected to operate from a tree d.b.h. list, use an "expected maximum" function to set the upper limit on tree growth, and estimate the growth of individual trees as a departure from the maximum as dictated by the characteristics of the tree and the surrounding stand. Product volumes will be estimated for stand and stock tables via a stem-profile function incorporating crown ratio as a surrogate for stem form. The distribution of trees by utility pole class and length will also be considered.

FUTURE NEEDS

It sounds trite but we simply need to keep pursuing prediction systems that will operate on any given tree list of d.b.h.'s and species in a thinned or unthinned stand and accurately project the d.b.h.'s and heights of these trees into the future. To provide the most information, these systems will likely be individual tree based and employ stem-profile functions to estimate tree and stand volumes and weights although d.b.h.-distribution systems may provide sufficient accuracy and detail for certain purposes (Jayaraman and Bailey 1988). Concurrently, we will need to improve our survival functions to incorporate mortality unrelated to suppression and expand growth functions to account for climatic and other environmental effects or changes. The problem of negative/positive feedback inherent in projection equations causing underestimates/overestimates of stand-level parameters needs to be addressed. Also, we will need to better define the geographic area of applicability,

i.e., where a system is suitable and where a variant or new system is needed.

However, we are really not concerned about the future of such modeling per se. Plenty of bright modelers have new and innovative ideas. Our universities are graduating very competent individuals, so there will be no lack of technological advances. Instead, the most pressing need is to insure that the data bases needed to develop future systems are available and sufficient. However, we do not think this need is being met in any organized and effective way. We need to install and maintain the necessary systems of permanent field plots to produce the data on tree and stand development under current and anticipated future management practices that will be required to support development of future models.

Sufficient, dependable funding must be available to do the field inventory, treatment, and maintenance work. It is expensive. In current US dollars, it may take $80,000 to $100,000 per year in direct costs for 5 years to locate, install, inventory, and treat a set of some 200 permanent plots and manage the data base. An established system of such plots and its data base may take $40,000 to $50,000 per year to inventory and maintain. To obtain the data on managed tree and stand development, these plot systems need to be continued for a rotation. If this work is as important as we and others think it is, there should be some mechanisms to insure that stable and adequate funding is available each year to carry on the work. If such mechanisms exist, we are not aware of them. We are painfully aware of the problem but have not yet found the solution. We submit, however, that some national, nonprofit foundation, or regional foundations, with sufficient capital contributed by both public and private concerns is needed to finance the work. A capital base of $10,000,000 (a fraction of the cost of one F-14 "Tomcat" aircraft) would probably provide perpetual annual support for permanent-plot systems in 8 to 20 timber types. Such an effort should be scientifically directed and removed from political and administrative jurisdiction.

We need to broaden the scope and include other interests to improve support for this work. We should account for development of the entire ecosystem both spatially and temporally. We need to start investigating development in the regeneration and premerchantable stages and carry through to senescence. Any discipline of resource management concerned with the development of forest ecosystems and their impact on the resource of interest is a potential supporter and contributor. These interests or disciplines include wildlife, endangered species, watershed, fisheries, and land aesthetics; research associated with these activities; and research associated with environmental impacts on these activities. Traditional timber management interests, although remaining very important, may not be the major users of tree and stand development prediction systems in the future.

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FIRST RESULTS OF SINGLE TREE THINNING EXPERIMENTS

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SUMMARY

Thinning experiments are normally controlled by basal area or stem number of the remaining stand. The object of these experiments is the stand, not the single tree. Even if a large number of trees is measured, the degrees of freedom for statistical tests frequently are low. The protocols of these experiments normally do not include replications of treatment with regard to single trees. Thus the reaction of single trees to specific changes of their growing space can be assessed only in a limited range of conditions.

In 1983 the German Association of Forest Research Stations issued "Recommendations for yield science experiments for the observation of the reactions of single trees to different types of thinning". In these recommendations the type, intensity and time of treatment (removal of trees) are defined in units of tree dimensions (dbh, height) and their mutual distances. Neighbouring trees have to be removed if the distance to the subject tree is shorter than a distance limit (DL) calculated by a formula. The formula is:

$$DL = H/A * d/D,$$

where H = height of the central tree,
D = dbh of the central tree,
d = dbh of the neighbour tree,
A = coefficient governing the intensity of thinning, fixed by the experimental protocol.

Two thinning experiments have been installed in Austria according to these recommendations. The coefficient A in these experiments was 4, 5 and 6 (three treatments). Five years after the first treatment significant differences in diameter-increment were found dependent on intensity of thinning in one of the experiments.

The results of this early evaluation lead to the conclusion that information about growth reactions to predefined neighbourhood conditions can be found by this method at quite low cost. The calculation of thinning intensity by a formula guarantees reproducible neighbourhood conditions by means of an objective rule. The paper discusses an international protocol of single tree thinning experiments.

THE PROBLEM

Thinning strategies are normally defined by the type of thinning method (e.g. high, low, selective, etc.), thinning grade (light, heavy, etc.), age of the stand at first thinning and frequency of thinning. Special rules are applied to the thinning of forest stands to define which trees and how many trees should be removed at certain development stages of the stand. The individuals of a stand can be grouped into different classes (e. g. social, numerical, economic). The thinning rules can determine special tree classes either to remain or to be removed. Grade of thinning can be expressed in stem number or basal area per unit area. The object of these experiments is the stand, not the single tree. Even if a large number of trees is measured, the degrees of freedom for statistical tests frequently are low. The protocols of these experiments normally do not include replications of treatment with regard to single trees. Thus the reaction of single trees to specific changes of their growing space can be assessed only in a limited range of conditions.
It is commonly suggested that a specific thinning treatment applied to similar stands will result in similar growth and yield responses. The theory of long term sampling uses the hypothesis that thinning treatments as well as growth reactions are repeatable. In my opinion, this theory can be applied to stands, but not to single trees, unless the treatment standards are adapted to single tree sampling.

Furthermore, forest stand modelling is based on the assumption that thinning treatments can be expressed by appropiate mathematical functions. In order to derive these functions a suitable data base is required. Long term permanent sample plots, temporary plots, inventory data and data from stem analysis and/or combinations can serve as data bases. Single tree modelling requires sufficient information on competition by neighbouring trees with the subject tree. This information has to be derived frequently from data bases (e. g., permanent sample plots), which have been installed for other goals than providing data for modelling.

SINGLE TREE THINNING BY MEANS OF A FORMULA

Since 1983 in Austria two permanent sample plots have been installed according to the "recommendations for yield science experiments for the observation of the reactions of single trees to different types of thinning", issued by the German Association of Forest Research Stations. In these experiments the removal of neighbouring trees depends on a special competition coefficient derived by a formula. Neighbouring trees have to be removed if the distance to the subject tree is equal to or shorter than a distance limit (DL) calculated by a formula. The formula is:

$$DL = H/A * d/D,$$

where
- H = height of the subject tree,
- D = dbh of the subject tree,
- d = dbh of the neighbouring tree,
- A = coefficient fixed by the experimental protocol.

The coefficient A determines the level at which the competition between subject and neighbouring trees is reduced. Since it is not possible to decrease continuously the amount of competition to which a single tree is subjected (e. g. by removing only half a tree), this formula provides the possibility of reducing competition down to a predefined maximum level. In detail this means that the distances between the subject tree and its neighbouring trees are compared with the calculated distance limit; neighbours within a shorter distance than the limit are removed. On the other hand, the amount of competition due to the remaining trees is a result of the random distribution of those trees and cannot be predicted. The effect of this fluctuation can be explained by an example (figure 1):
Let us assume that a subject tree has 3 neighbouring trees of identical dbh and let the distances from the neigbouring to the subject tree be 2, 3, and 4m respectively. If the height of the subject tree (H) is 15 m and A=5 or A=4, the distance limit is 3m and 3.75m respectively, and two of the neighbouring trees have to be removed. The remaining tree will be 4m distant. It so happens that, in this example, regardless of whether A= 4 or 5, the treatment of the subject tree is the same. But if height of the subject tree is 16m and A=4, the three neighbouring trees would be removed, whereas if A = 5 only two would be removed.
As an effect of this fluctuation the differences in growth response after only a short period of sampling may be of low significance, because of high variances within and low variances between treatments. An increased duration of the experiment should eliminate most of the fluctuations, and variances between groups should increase.

Other approaches of distance-dependent competition measures (e.g. TOME',M. and BURKHART,H.,E, 1989) calculate competition measures from existing experiments. The formula presented above is a tool for using the competition measure in a thinning experiment. There is some similarity to the HART (1928)-BECKING (1952) stand density index, if this is expressed as

$$DA = H/A',$$

where
- DA = average spacing between trees in a stand
- H = average height of 100 tallest trees per hectare
- A' = proportion of H determining the grade of thinning.

HART-BECKING's stand density index was found to be useful in governing thinning experiments on a stand basis in Great Britain, Netherlands and Java (HUMMEL,1954). Contrary to the HART-BECKING index, the formula presented above was derived especially for single tree thinning.

EARLY RESULTS

The two Austrian single-tree thinning experiments were both applied to spruce trees in pure spruce stands. The dominant height of the stands was about 12m (Weitra) and about 16m (Loelling) respectively. In 1983 (Loelling) and 1985 (Weitra) neighbouring trees of selected subject trees were removed using A=4, 5 or 6 respectively in the formula. The number of subject trees (number of replications) was about 100 per treatment in Loelling and 25 in Weitra.
Five years later the subject trees were remeasured. Some of the early results are presented here. In figures 2 and 3 diameter increment is plotted against dbh. These figures, as well as regression analysis, demonstrate that there is no correlation between dbh and diameter increment.

Analysis of variance was employed to test for significant differences between mean annual diameter increments of the subject trees due to treatment. In the Loelling experiment the differences in diameter growth between the treatments are significant at a probability of 99.9%. However, they are not different in the case of the Weitra experiment. These contradictory results may be explained by the fluctuation mentioned above. Since subject trees were treated using this formula once only, the fluctuation happens to be high. In the case of Weitra, the number of replications (N = 25) is low in comparison to that of Loelling where 'N' equals 100. Furthermore, the average height of the subject trees differs between the experiments. In the Loelling experiment the average height was 16m, but in Weitra only 12m. The relationship between distance limit and subject tree height is inversely propotional. Therefore, since the subject trees in the Loelling experiment have a larger average height than those in Weitra, the resulting distance limit is greater and therefore also the potential for increment is higher in the more abundant growing space.

DISCUSSION

The single tree thinning experiment was governed by a formula derived from tree dimensions and intertree distances. A significant increase in diameter growth dependent on treatment was found after only an initial thinning and five years of observation. However, in another case of single tree thinning no difference between treatments in diameter increments could be determined. Treatment and observation of both experimental stands will be continued in the future. It is presumed that after a longer duration in experimentation, growth responses will become more clearly visible. The advantage of this experimental method is that any additional trees to be removed will be determined according to an objective standard. In my opinion, this standard could be applied to a great number of tree species and also possibly to mixed forests. It is conceivable that this standard could be successfully applied to experimental stands in which the composition differs so greatly that a repetitive application method, as

in that of classic plot experiments, is impossible. I also believe that other possibilities of application are within stands of high variation in age composition or that are too small for plot experiments. I would like to point out clearly that this proposed method can be applied only to the control of the experiment, not however, to the explanation of the growth responses. In order to explain these growth responses, special competition parameters can and must be calculated, where competition as a factor interference is demonstrated. On the other hand the location-dependent competition parameters in which the entire competition situation is measured with regard to a single subject tree is, in my opinion, unsuited for the control of an experiment with single trees as experimental subjects.

I would like the Subject Group "Design, performance and evaluation of experiments" to consider the need for single-tree experiments. If the group agrees, I would suggest procedures for carrying out such experiments.

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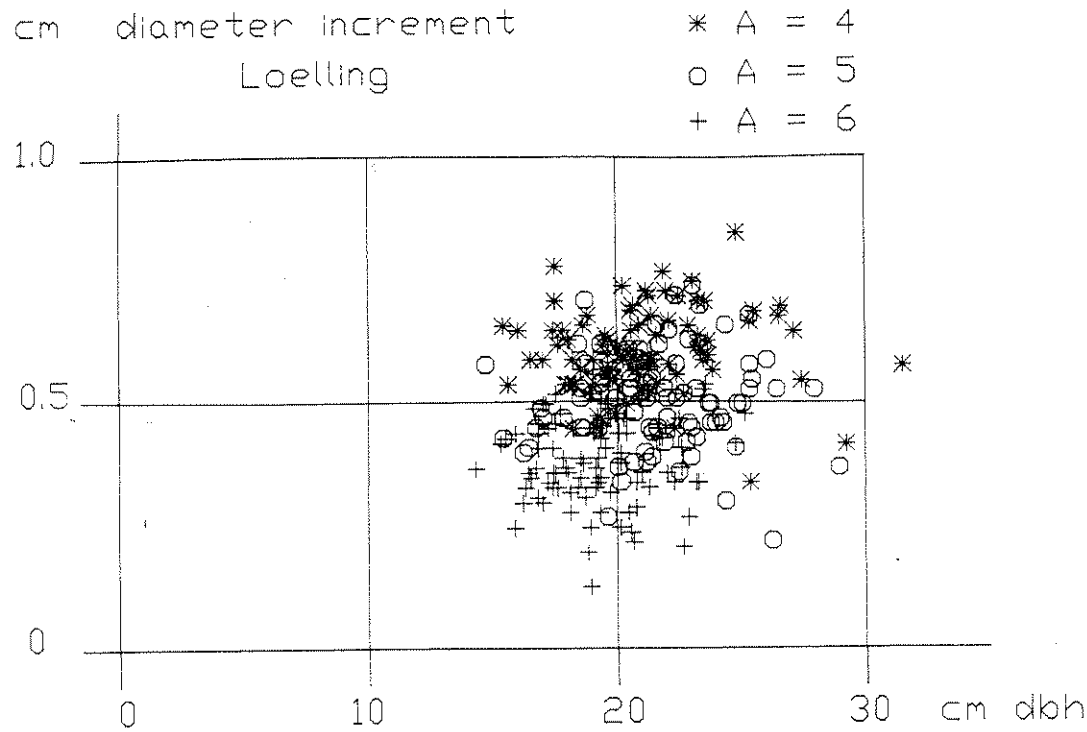

Figure 2: There is no correlation between annual diameter increment and dbh (Loelling, N per treatment = 100)

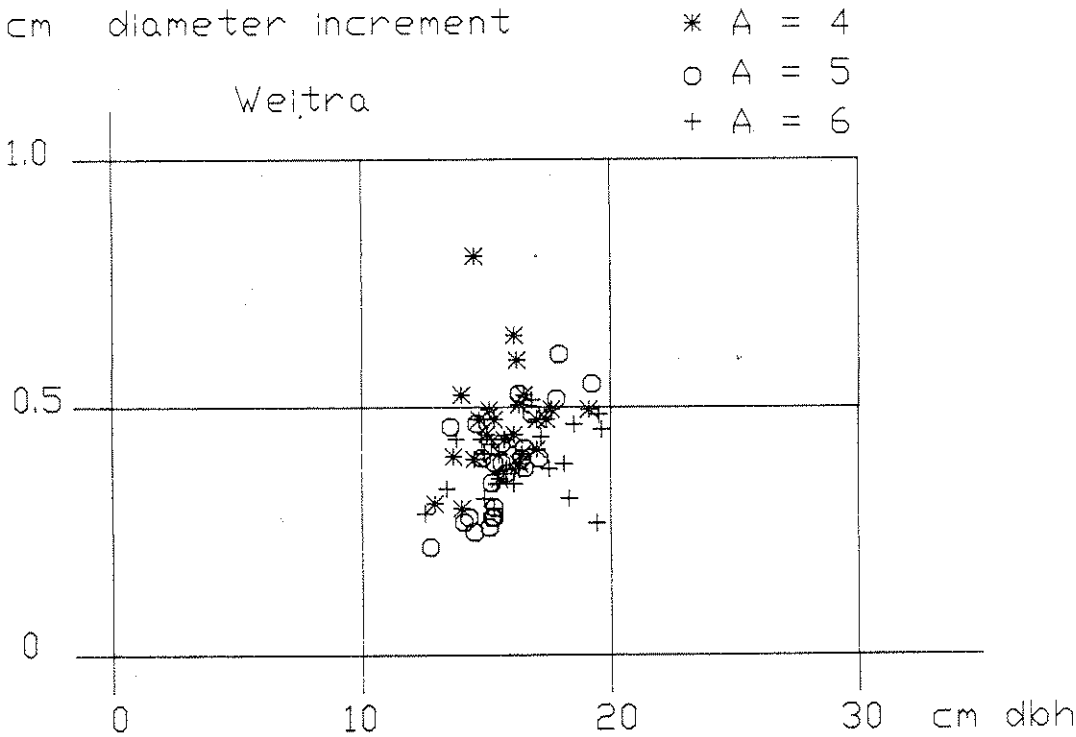

Figure 3: There is no correlation between annual diameter increment and dbh (Weitra, N per treatment = 25).

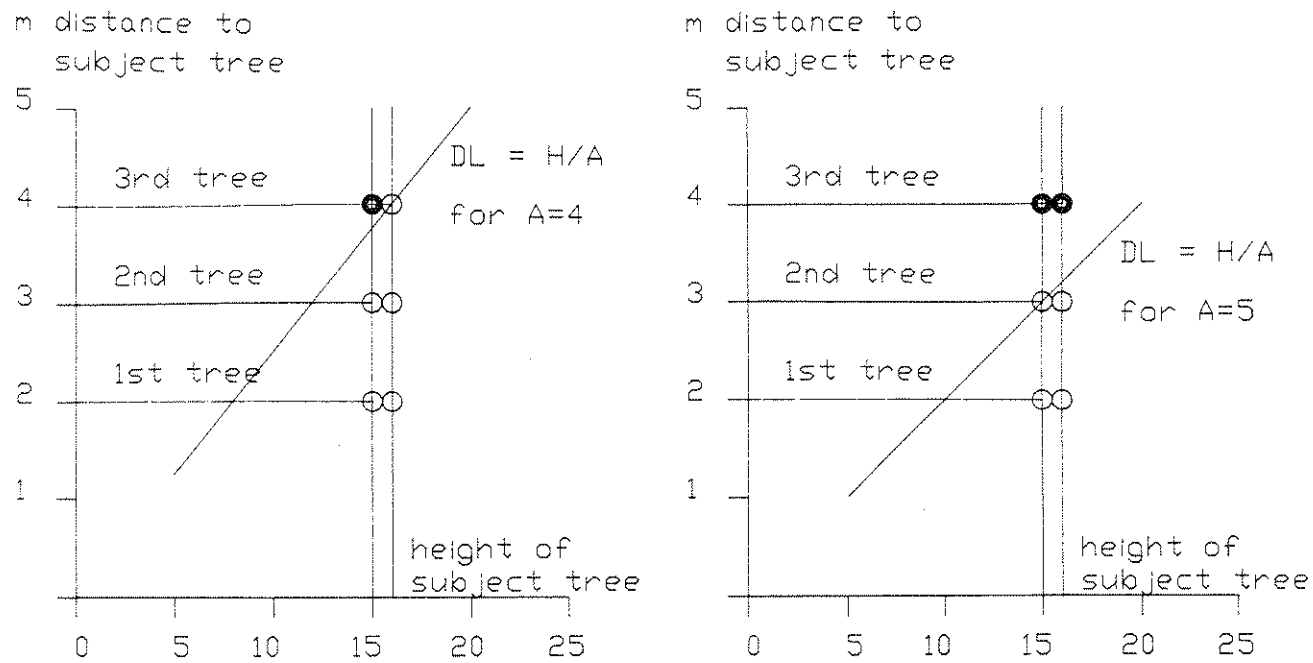

Figure 1: If the height of the subject tree is 15 m and A = 4 or A = 5 two trees have to be removed.
If height is 16 m and A = 4, three neighboring trees have to be removed, for A = 5 only two.

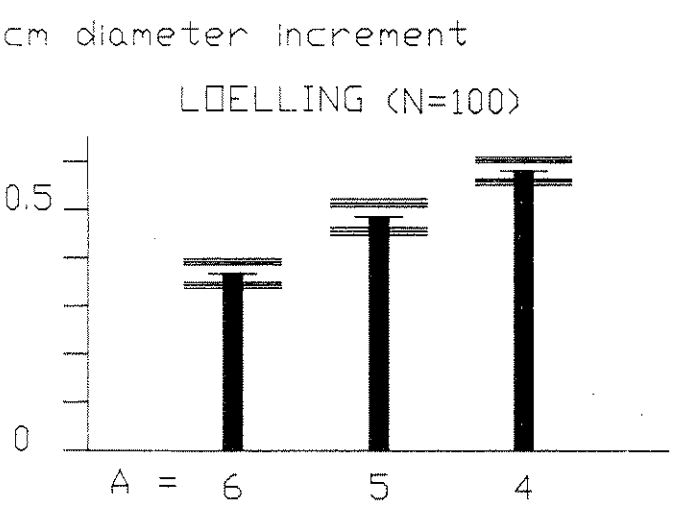

Figure 4: Loelling

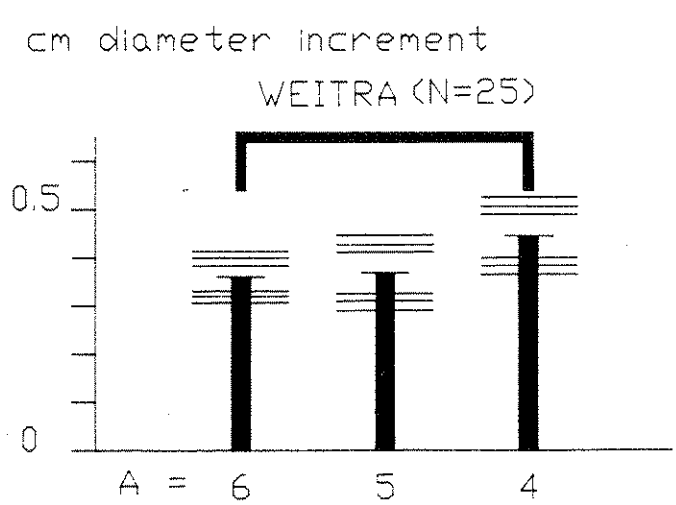

Figure 5: Weitra

Average annual diameter increment and confidence intervals for p = 0.05, 0.01 and 0.001.

THE MAIN PRINCIPLES OF MODELLING OF MAXIMALLY PRODUCTIVE STANDS AND TARGET PROGRAMS OF THINNING CUTTINGS*

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Keywords: thinning, Modelling, stand, ecosystem

SUMMARY

On the basis of long-term (over 30 years) investigation data (more than 400 permanent and 80 temporary experimental plots), the scientific background for modelling of maximally productive stands and target thinning programs have been created. It was determined that the maximal stand productivity is described by an increased quantity of absorbed solar energy, by its effective utilization by trees and storeys, by the tree quality and their greater productivity as well as optimal density. Based on this, original methods for the determination of optimal stand density were suggested.

The elaborated methods of density optimization are based on a search for the optimal crown parameters and optimal space rates ensuring maximal increment of the whole stands. On the basis of the revealed new phenomenon, so called stress effect during the process of the ecosystem (biocenoses) creation, it was established that the criteria of optimal stand density in different phases of stand development are different. In young stands the optimal density is that density which eliminates the mutual influence (competition) of the trees and ensures maximum height increment for a possibly greater number of trees. Following the ecosystem creation, in middle age and maturing stands, the optimal density is such that provides the maximum current increment of the growing stock and maximal total stand productivity. This is achieved in the case when the crown closure is maximal with an optimal rate of mutual overlap and the stand is formed from maximally productive trees distributed over an optimal distance from one another.

Based on the above, the model of maximally productive forest stands have been worked out. The standard programs have been created and have implemented in exploitive forests in the Lithuania as well as in western and north-western regions of the USSR. Specialized forest growth undertaken in accordance with set standards have already resulted in a productivity increase of up to 20-15 %. The utilization of wood from an area unit is significantly augmented and the cutting cycle of stands is noted to be much shorter.

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*Paper for presentation at the 19th IUFRO World Congress to be held in Montreal, Canada (August 5-11, 1990)

I. INTRODUCTION

Thinning cuttings is one of the most effective means to ensure qualitative stands are obtained. However, it must be taken into account that positive results are achieved when scientifically sound efficient and qualitative thinning is applied. Thus, a forester must know which, and how, stands must be formed according to species composition, structure and productivity, i.e., he must have a simulation model which provide an insight into the probable stand development. In practice, he must deal with prototypes or standards of a maximally productive forest and have concrete programs for forming such a forest. Such standards and programs must be differentiated according to species composition of stands, ecological and geographical regions as well as to site conditions.

We have, therefore, studied the biological grounds of forming a maximally productive forest in more detail elucidating the regularities of natural and artificial stand formation. Our task was to:

- develop the principles and methods for constructing simulation models of maximally productive stands;
- elaborate standards for growing of stands of maximal productivity;
- elaborate purposeful programs for thinning cuttings.

2. EXPERIMENTAL AND THEORETICAL BACKGROUND FOR THE MODEL CONSTRUCTION

Data from more than 400 permanent experimental plots of 5-30 years duration were used (Kairiukstis, 1977). The trees were mapped, measured and calculated. Intermediate cutting on permanent experimental plots was applied 2-6 times. Data from 80 temporary experimental plots were also used. The stands included different species composition, structure and age. To ascertain the correlation between separate assessment indices, more than 300 different equations were derived.

It was found that natural stands do not attain maximally possible productivity because 40-60 % of the trees are of lower or low productivity. The processes of self-regulation in stands occur through tense interrelations between the individual trees. This results in wasted energy and, consequently, in retardation of forest growth. Thus, artificial regulation of stand density and structure at an optimal level is the essential factor ensuring increase in stand productivity. The investigations revealed that stand productivity on comparatively fertile soils is determined by the following conditions:

- First: quantity and quality of solar energy received in the stand. This depends very much upon the cover surface and its depth being regulated through the thinning systems. The more deeply stepped the surface of the canopy, the lesser the albedo, and the more solar energy is received in the stands.

- Second: effectiveness of the solar energy utilization by trees and storeys and by the tree quality and productivity itself. The efficiency of the use of solar radiation obtained by the crowns of variously developed trees is not equal. The comparative coefficient of profitable solar energy use for both the Physiologically Active Radiation (PhAR) and Summary Radiation (SR) is assumed for class A* trees = I.0, for trees of class B = 0.8-0.7 and for trees of class C = 0.7-0.5 respectively. The stand formation by thinning cuttings enable one to change the composition of tree classes and to increase the percentage of more productive trees and to produce a much more and better quality wood.
- Finally: stand productivity is determined by the optimal number trees per area unit.

We have verified a vast number of methods to determine the optimal stand density and have noted that most of them are far from perfect for the construction of dynamic models of maximally productive stands. We carried out silvicultural/physiological investigations on the productivity of stands and class structure of trees in storeys (Kairiukstis, I973). We find that the stand density reflects tree growth conditions better than the stock density. We arrived at the conclusion that density optimization must be based on crown parameters of maximally productive trees (class A), on the optimum dynamic space for each tree in each time span. In such optimal space the highest current increment of stock growth can be ensured. With regard to the crown, the following must be mentioned. First, the crown is a sensitive index simultaneously reflecting the development and productivity level of a tree and its state in space (canopy). Second, as previously determined, the correlation of the magnitude of current tree increment with horizontal crown projection area is considerably closer (r = 0.75) than with nutrition area (r = 0.47). Therefore, the crown was selected as the main criterion of stand density optimization.

Thus, we can infer that the search of the optimal crown parameters and optimal space rates for each age class will enable the optimal stand density to be determined for the entire period of stand growth.

We have investigated growth peculiarities and formation of stands in various phases of their ontogenic development. We discovered a new phenomenon, the so-called stress effect which occurs during the process of creation of forest ecosystems (Kairiukstis and Juodvalkis, I975). We also discovered fixed limits of the critical approach of crowns at which trees enter into an intra-specific competitive interrelationship, resulting in a high increment decrease irrespective of soil or climatic conditions.

Taking into consideration the above phenomenon, the optimal density criterion cannot be identical during the whole period of stand growth even when the aim is the same-to grow

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*The classification of trees was worked out by L.Kairiukstis (I969).

maximally productive stands. In young stands, therefore, the optimal density must be considered such that it eliminates intraspecific competition, ensures maximal height and diameter increment for the greatest possible number of trees. Following the effect of stress and ecosystem creation, the optimal density in middle-age forest stands is that which provides maximal current increment of the growing stock and maximum total stand productivity. Such density also provides the greatest timber volume by the age of final felling.

Our investigations revealed that stands meet such requirements when the following three conditions are combined in them:

a) crown cover is maximized;

b) the stand consists of maximally productive trees (class A according to the authors, or class I.8-II.2 according to Kraft);

c) maximally productive trees are distributed at an optimal distance from one another.

Conforming with the first condition, maximal stand productivity is obtained in the case where the forest ecosystem utilizes the space maximally, i.e., in the case where the area of the crown cover is maximal. We have determined that the maximally possible crown cover area depending upon species composition and age comprises 7500-9000 m^2/ha, whilst the area of inevitable openings is IO-25 %.

Under the second condition, maximal stand productivity is achieved only when possible crown cover area is chiefly covered by crowns of maximally productive trees, i.e., trees with the most productive crowns-maximal current increment of stem wood per I m^2 of crown area.

The investigations and theoretical calculations have indicated that when the stand consists of well-developed trees, its productivity depending upon species and age is by 5-25 % higher than the current increment of natural stands of the same density.

According to the third condition, maximal stand productivity is achieved when the crowns are situated at an optimal distance from one another with optimal mutual overlap. A search for the best index reflecting the distance among trees showed that the correlation between the magnitude of the current tree increment and the distance among trees is highest when the distance is expressed through crown parameters. Therefore, in the proposed method of the optimal stand density determination, the indices of the distance among trees may be substituted by those of crown diameter and the extent of the optimal crown overlap. The optimal crown overlap is established according to the extremum of the curve of the dependence of the current stand increment magnitude on the extent of crown overlap. For practical purposes, the indices of the optimal crown overlap may be replaced by those of the mean crown overlap of well-developed trees (class A).

Consequently, the optimal density of middle-age and those of maturing stands is ascertained with the help of the follo-

wing formula:

$$N_{opt} = \frac{Q_{max}}{S_{opt}\left(I - \frac{P_{opt}}{ICO}\right)},$$

where

N_{opt} = optimal number of trees, trees/ha;

Q_{max} = maximum possible crown cover area; m^2/ha;

S_{opt} = optimal area of horizontal crown projection of well-developed trees, m^2;

P_{opt} = extent of optimal crown overlap, %.

While establishing the optimal density and constructing the models of maximally productive one-storied stands, great attention was paid to the external and internal relations between single structural elements of a tree and a stand. For two-storied stands, the optimal growth conditions for trees of the most valuable species was also ensured. The total productivity of mixed stands was considered to be negligible

The stand growth and formation regularities and correlation between the single assessment indices of a tree and a stand as well as the method of stand density determination enable the construction of models of maximally productive stands. The models include pure and mixed stands formed from the main species of the Lithuanian SSR according to predominating forest types.

3. THE PRINCIPLE OF THE MODEL CALCULATION

The model algorithm consists in determining the maximum density of the layer, calculating the area of the horizontal projection of the optimal crown, determining the optimal overlapping percent of the optimal crown, and calculating the number of trees at a certain age. To determine the maximum density of the layer, one should use experiment plots in the most dense stands in the whole range indicated in the constraints and to draw the plan of tree locations and crown projections. The whole area of crown projection and area projection except when overlapping, are presented separately for each tree. These measurements are necessary to determine the area of inevitable glades in the stand and to calculate the optimal crown overlap. If the stand contains "windows" (or squares), the area which is greater than that of an average tree crown, a certain quantity of trees is artificially included in them and their crown are included into the initial information. The maximum density of the layer in I ha area is found by the computer. This is the sum of the crown projection areas without overlapping.

In order to determine the optimal crown parameters, the cor-

relation between increment of a stand basal area of a separate tree and crown projection area of an individual tree at a particular age is being sought. The crown producting the maximum wood increment in the area unit is considered optimal. Having the optimal crown parameters at a certain age interval; we find its relation with age in all the age ranges.

When the maximum layer density and the optimal crown area are know; the optimal number of trees in the area unit can be found. For this purpose, however, the optimal distance between the trees must be found. In the model, this distance is established in terms of the optimal crown parameters and the percent of optimal crown parameters and the percent of optimal crown overlap. Therefore as a result of the calculations, the optimal number of trees, the optimal stand base area and volume affecting the maximum wood increment are obtained for the given type of forests and tree species at any age.

4. STANDARDS OF MAXIMALLY PRODUCTIVE STANDS AND THINNING PROGRAMS

For elaboration of thinning programs according to standards of maximum productive stands we used the data from more than 300 permanent experimental plots in which intermediate cutting of different density was applied 2-6 times. It resulted in determining the following:

- the extent and duration of tree response to thinning for each species (Fig. I);

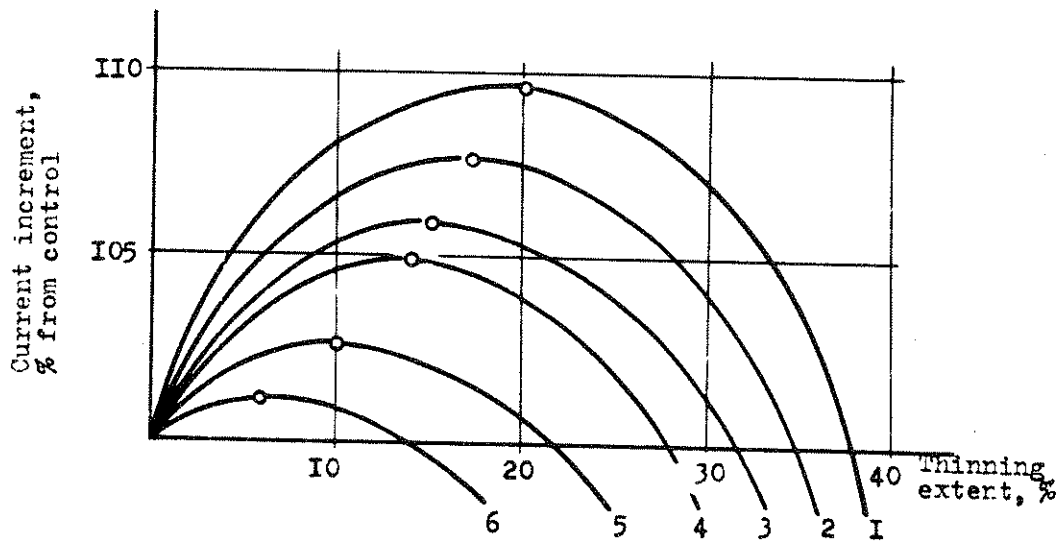

Figure I: Dependence of current increment on thinning extent and the age of pine stands: I - 20 years, 2 - 30 years, 3 - 10 years, 4 - 40 years, 5 - 50 years, 6 - 60 years. o - optimal extent of thinning.

- the dependence of the size of the current timber volume increment on the extent of thinning, the optimal and critical extent of thinning;
- dependence duration of rethinning on the extent of thinning

and age of stand (Fig. 2).

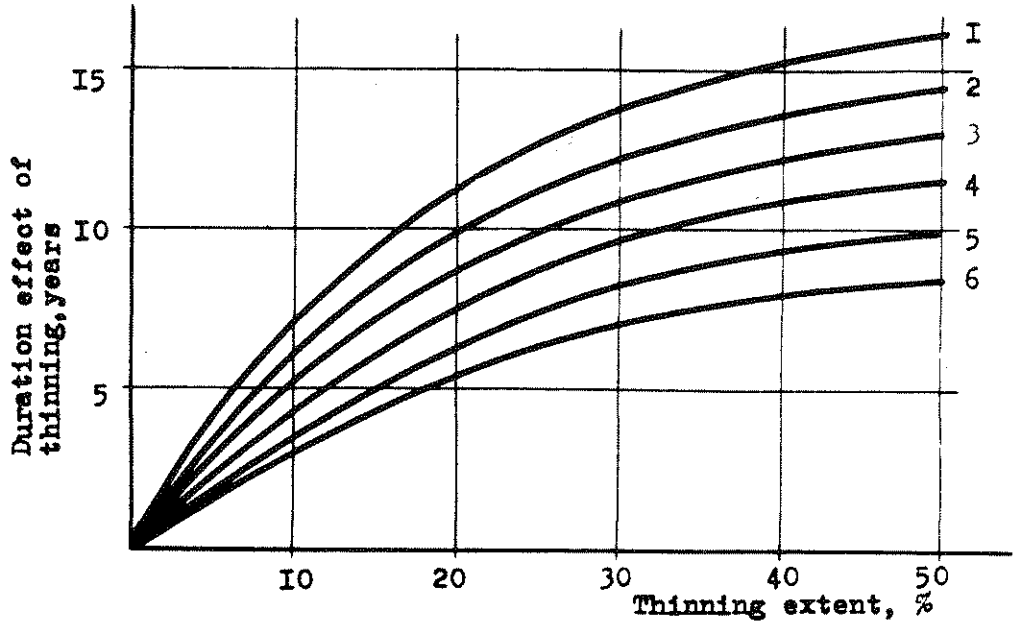

Figure 2: Dependence of Duration of thinning effect on the extent of thinning and the age of pine stands: I - 60 years, 2 - 50 years, 3 - 40 years etc.

The optimal terms of repetition and the optimal regime of intermediate cutting in stands of different species composition, structure and age was established (Kairiukstis and Juodvalkis, I985). An example for spruce (Picea abies, Kaisten) is given in Figure 3.

Hence, the programs of thinning according to standards of maximally productive stands were elaborated. As one of example such programs is given in table I. The essence of the standards lies in the fact that for a regular repetition of intermediate cutting the standards indicate the number of well-developed (class A) trees, stand basal area, and timber volume that must be left after thinning. The number of trees is ascertained according to the conformity of the stand with the optimal density in the middle of the period between the two applications of intermediate cutting.

The standards presented are assigned not only for carrying out intermediate cutting but also in projecting the intermediate forest use and forest management by computer. The main assessment index in carrying out intermediate cutting is the number of trees that must be left while projecting the intermediate cutting - the volume of cutting-timber that is to be left after felling.

In order to simplify the standards for practical use, the assessment indices of trees that must be left are expressed depending upon the mean height of well-developed trees. A generalized standard for forest types of similar productivity can thus be applied.

In the process of standard forest formation in practice, the assessed indices of the specific stand at a certain height

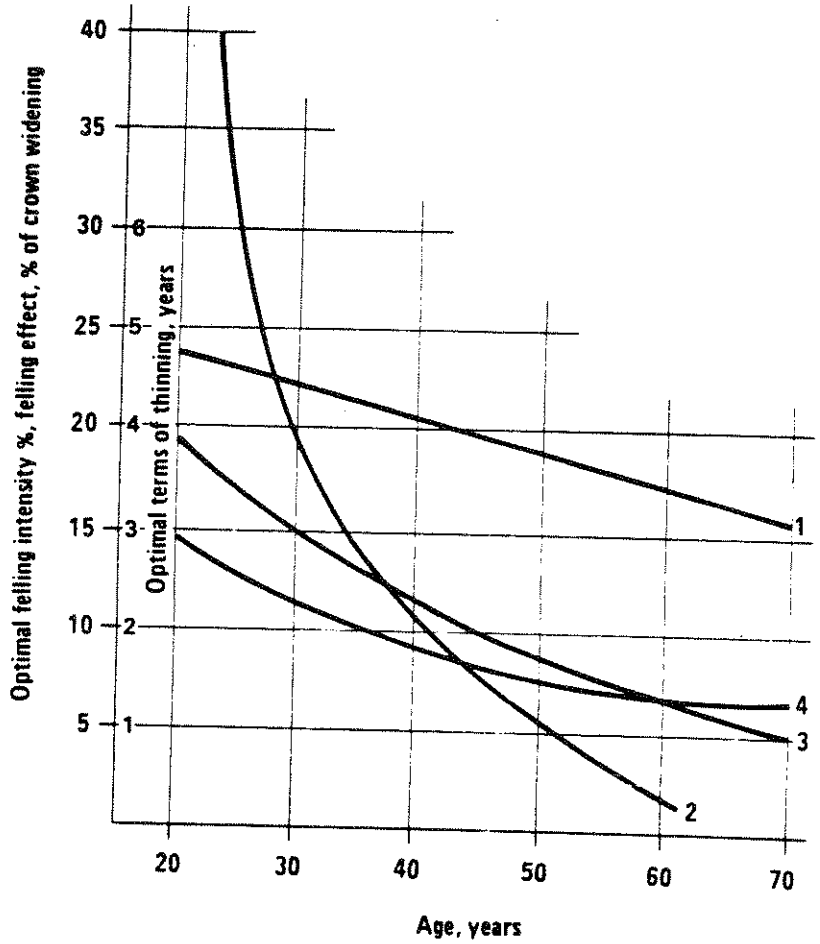

Figure 3: Optimum time span of re-thinnings (I), maximum thinning effect (2) optimal intensity of thinning (3) and percent of crown widening (4) in Oxadilosum spruce stand depending upon age.

are compared with the corresponding indices presented in the program (Table I). In the planning of intermediate cuttings, the volume or stand basal area in the forest is compared with the corresponding indices presented in the program for such calculations.

Table I. Number of trees (trees/ha) that must be left after felling in pure stands of different species depending upon the mean height of well-developed trees when opening of stands and clearings are repeated every five years while thinning and intermediate cutting every ten years.

Height of well-developed trees	Oak (Quercus robur stands	Ash (Fraxinus excelsior) stands	Spruce (Picea abies Kar) stands	Aspen (Populus tremula) stands	Birch (Betula verrucosa) stands
3			2400		
4			2390		
5	5380	5360	2370	59I0	5540
6	4250	4280	2320	5240	4900
7	3460	3530	2270	4640	4330
8	2890	2970	22I0	4II0	3830
9	2450	2540	2I40	3650	3390
I0	2I00	2200	2070	3230	2990
II	I830	I920	I980	2860	2650
I2	I600	I690	I890	2540	2340
I3	I4I0	I500	I800	2250	2070
I4	I250	I340	I700	I990	I830
I5	III0	I200	I590	I770	I620
I6	990	I070	I490	I560	I430
I7	890	960	I380	I390	I270
I8	800	870	I280	I230	II20
I9	7I0	780	II80	I090	990
20	640	7I0	I080	960	880
2I	580	640	980	860	770
22	520	580	890	760	680
23	470	520	800	670	600
24	420	470	720	600	540
25	370	420	650	530	470
26	330	380	590		420

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GROWTH FEATURES OF BLUE PINE (PINUS WALLICHIANA) IN NORTHERN PAKISTAN

by

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S U M M A R Y

Based on recently developed new yield tables for Blue Pine (Pinus wallichiana) the growth performance of this important commercial species in the hill areas of Pakistan is described. For the purpose of detailed analysis of the new tables individual growth parameters are compared with those from Blue Pine yield tables prepared in 1929 on the basis of data from a separate region located in India. This comparison reveals that both yield tables differ substantially concerning methodology used for their preparation and concerning the way the stands were treated.
For the fact that both growth regions show different production classes some preliminary results could be found.
Additional keywords: yield tables, site index, stemnumber regime, production class.

I N T R O D U C T I O N

Pakistan's main timber resource is located in the north of the country, in the hill areas between the mountain ranges of the Himalaya and Hindu Kush. The climate is subtropical in the lower parts of the valleys and becomes temperate to subalpine until it reaches the timberline in an altitude of about 3300m.

Chir Pine (Pinus roxburghii) is the main species in the subtropical area whereas Blue Pine (Pinus wallichiana), Deodar (Cedrus deodara), Fir (Abies pindrow) and Spruce (Picea smithiana), beside several broadleaved species, dominate the temperate and subalpine zone. Among all the conifers Blue Pine covers the largest range and grows in an altitude between 1700 and 3300m.

Intensive management of these high hill conifer forests was started about 10 years ago. Until that time only a very conservative type of utilization was in practice. Mainly the single tree selection system without proper measures to enhance natural regeneration, the lack of systematic stand management and traditional felling and extraction methods (scantlings) led to undesirable conditions of these forests. These are characterized by:

- overmaturity of the stands
- poor stocking and therefore low productivity
- considerable loss of valuable timber in terms of quantity and quality.

In order to improve the situation and to intensify the management multifold research activities in various fields like Forest Engineering, Ergonomics, Silviculture and Forest Management
are required and have been started some years ago.

At present one of these research projects is being implemented by the Mensuration Branch of the Pakistan Forest Institute/Peshawar aiming at the preparation of yield tables for all important commercial tree species of the high hill forests of Pakistan.
The first of these yield tables, dealing with Blue Pine, was completed recently (CHEEMA et al,1990).

This paper describes the main growth features of Blue Pine in Pakistan and compares individual growth parameters with those of an old Indian yield table, which was prepared in 1929 by the Silviculture Branch of the Forest Research Institute, Dehra Dun (CHAMPION et al,1929).

M A T E R I A L A N D M E T H O D S

The preparation of both, the Indian and Pakistani yield

tables was done with totally different methods. Detailed descriptions of the techniques applied can be found in each of the respective yield table publications. The Indian researchers used mainly graphic methods to find the yield table parameters whereas in Pakistan a comprehensive equation system was developed using linear and nonlinear regression models.

Data used in this paper were taken directly from the yield tables. The comparison concentrates on graphic presentation and shows the main growth parameters of both yield tables in combined diagrams.

R E S U L T S

G r o w t h F e a t u r e s o f B l u e P i n e

Heigth Growth

The new Pakistani yield tables are based on a site index system which uses dominant height and age for determination of the site quality. A modification of the growth model, created by BERTALLANFY in 1959 (STERBA et al, 1988) was fitted to stem analysis data in order to establish a relation between the two parameters. The site index is defined as the dominant height at age 100 years and given in the tables in 1m steps ranging from 26 as lowest to 42 as highest site index.

Due to the fact that the Indian yield tables are based on the mean height and the age for the determination of its three site quality classes (I,II,III) a comparison of the height growth of both the tables is only possible using the mean height as shown in Figure 1.

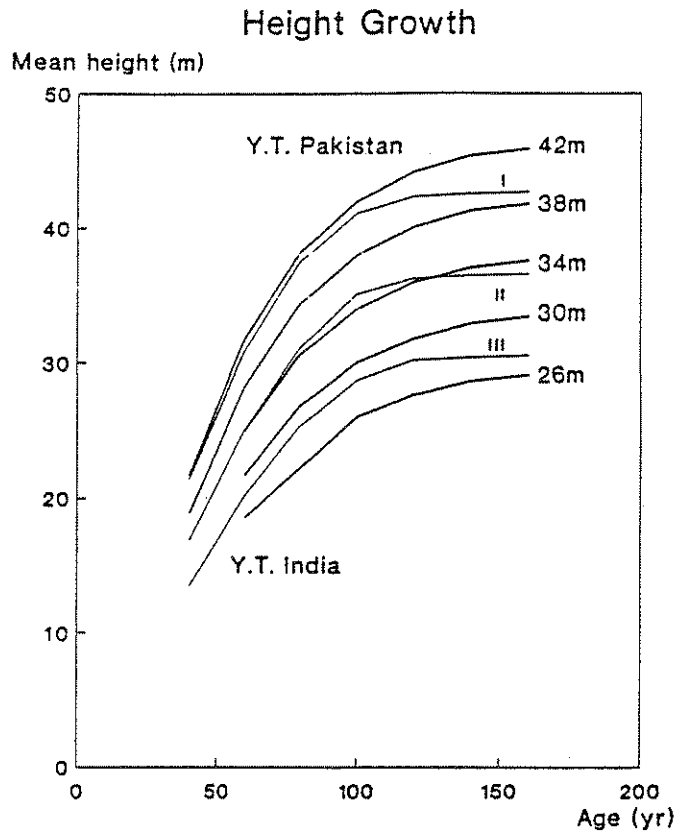

Figure 1: Height growth curves of the Pakistani and Indian yield tables

As far as the range of site qualities is concerned both the curve systems are quite similar.But the growth pattern differs considerably in higher ages. MONSERUD (1985), who studied site index curve systems calculated from stem analysis data and those derived graphically by a so called harmonized guide-curve method, found substantial differences in the growth pattern applying both methods to the same data. Considering this it is very likely, that above mentioned differences in the growth pattern of the Pakistan and Indian yield tables is caused by the type of data and the methodology used for preparing the site index system. The Indian one was derived by the guide curve method which is very sensible to the number of plots in different age-classes. The lack of plots belonging to better site qualities of higher ages could have caused the differences in the curvature.This factor does not effect the height growth pattern derived from stem analysis data used in the new Pakistani yield tables.

Stemnumber Regime and Mean Diameter

As given in Figure 2 and 3 both yield tables show completely different stemnumbers and mean diameters. Since the Pakistani tables have got less stemnumbers especially in ages higher than 70 years logically the corresponding mean diameters are much bigger.

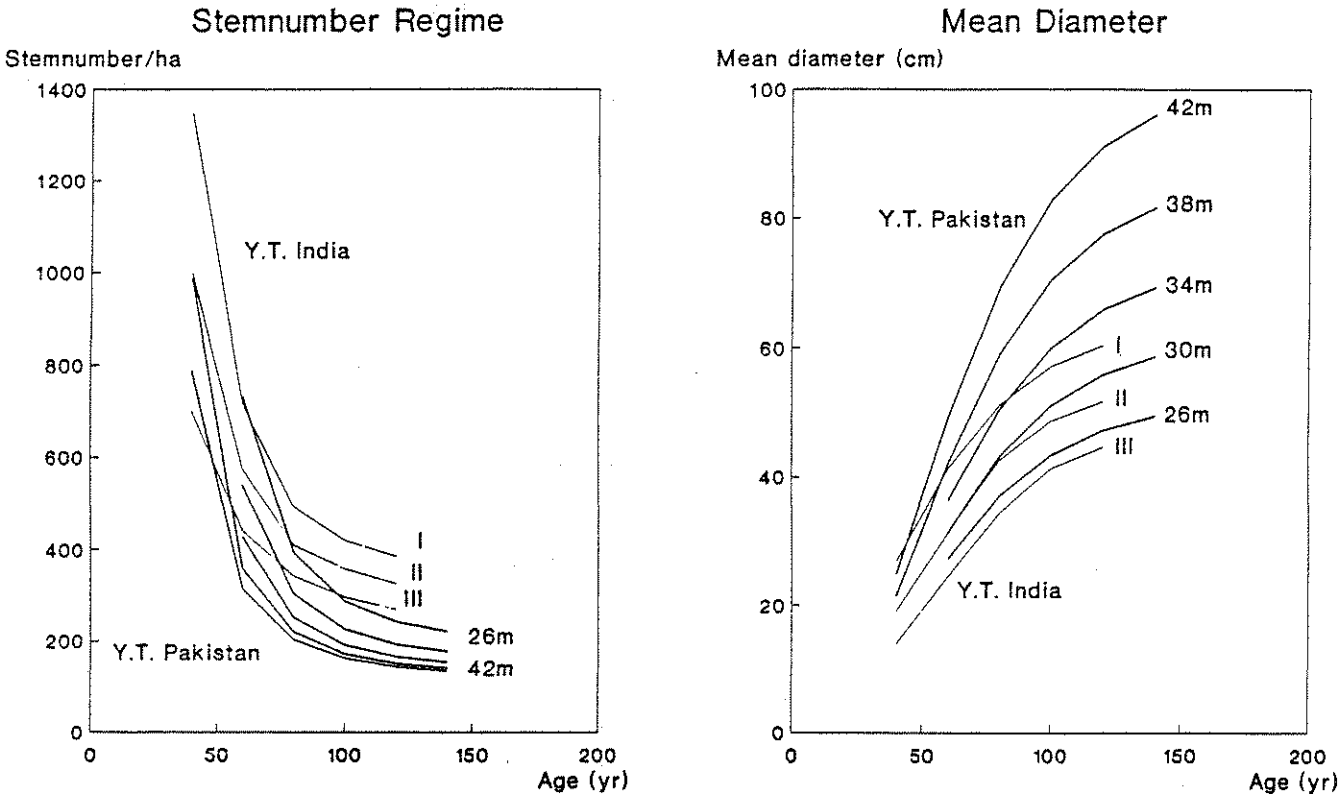

Figure 2 and 3: Stemnumber regime and mean diameter curves of the Pakistani and Indian yield tables

These differences in stemnumber regimes can be caused by:

- site factors which allow different potential densities

Since the data on which each of the yield tables are based represent separated regions of the Blue Pine distribution it could be possible that they belong to different production classes provided the same treatment was given to the stands in the past.

Beside the site index above mentioned production class is a second dimension of the site quality and can be characterized by the potential density which is defined as the capacity of a site to support trees (STERBA,1987). An evaluation of the potential density could be done either by using data of completely unthinned stands or by calculating REINECKE'S maximum stand density index with the help of existing plot data. For Blue Pine this type of research has not been carried out so far.

- different type and intensity of treatments

Sample plots for growth and yield studies were laid out in the past mostly in unmanaged stands and an initial C grade thinning reducing the density to 0.8 -1.0 were given to them (CHAMPION et al,1929). The Indian yield tables are based exclusively on data from such permanent plots whereas for the Pakistani tables also temporary plots covering a wide range of managed stands were used. Lower stemnumbers especially in higher age-classes in the Pakistani yield tables are caused by both single tree as well as shelterwood system aiming at the promotion of natural regeneration through heavy thinnings in old stands. Results of field checks show that the parameters given in the Pakistani yield tables coincide quite well with most of the actual stand conditions. The use of the Indian yield tables as a stand management guideline in Pakistan would possibly lead to undesirable stand conditions.

Total Yield

As discussed before both yield tables have different stemnumber regimes which means that the stands have undergone a different type of treatment. Due to the fact that the increment decreases to a comparatively low extent with decreasing basal area in a nonlinear relationship the effect by a certain treatment on the total yield is very small. This holds true if the stemnumbers are kept at a level which is above or equal to the critical basal area (ASSMANN,1970).

Assuming this for both yield tables the curves (Figure 4), representing the relationship between total yield over mean height for different site qualities can be considered as indicators for the production class. The Pakistani yield tables show with increasing site index also an increase in the production class. Here the site index and the production class are positively correlated whereas the contrary can be observed in case of the Indian yield tables where the worst site quality (III) shows the best production class.

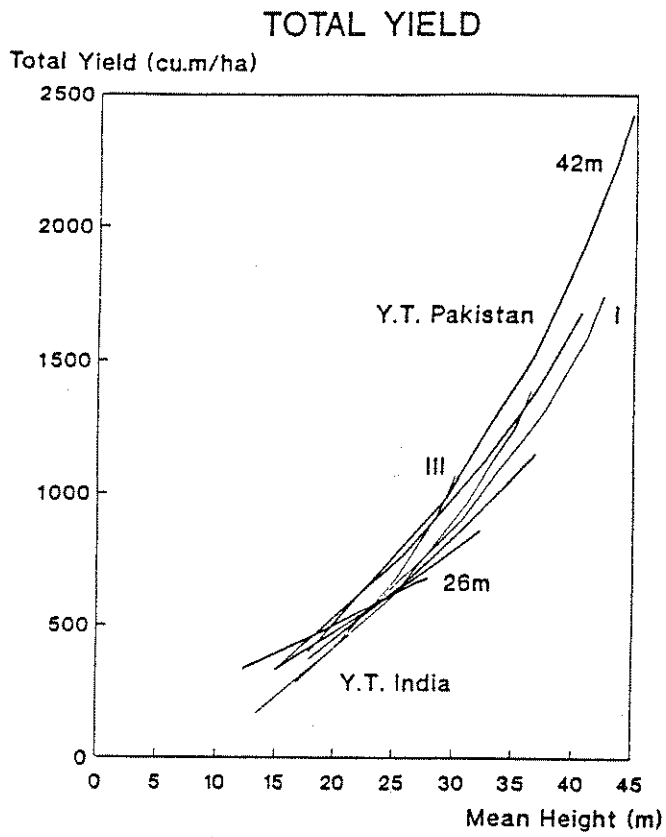

Figure 4: Relationship between total yield and mean height of the Pakistani and Indian yield tables

However, in order to confirm or revise these preliminary results further research on potential density along with improvement of the available data basis (longterm thinning experiments) is necessary.

C O N C L U S I O N

The results of above described comparison of the two Blue Pine yield tables can be summarized as follows:

Methodology

For the preparation of the Indian and Pakistani yield tables two completely different methods viz. graphic harmonized curve

technique and regression analysis respectively were used. This led to site index systems with significant differences in the shape of the curves. Applying the Indian yield tables in the Blue Pine area of Pakistan would cause wrong site quality determinations.

Stand treatment

The comparison of the stemnumber regimes revealed that the stands in both growth regions (India and Pakistan) have been treated in different ways. Much smaller stemnumbers but bigger mean diameters given in the Pakistani yield tables reflect the past management of the stands in Pakistan in a better way than the Indian tables would do.

Production class

Indicators were found for the fact that the production class in both growth regions is not the same particularly concerning the correlation between site index and production class. Due to the provisional nature of this result further research on this topic is necessary.

Finally one can conclude that there is justification for the use of the new yield tables in the growth region of Blue Pine in Pakistan. Despite the fact that various growth parameters are still not fully explained and correctly interpreted such yield tables are definitely a valuable tool for proper stand treatment and contribute to an intensification of the management.

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SIMULATING BRANCH DIAMETER AND BRANCH DISTRIBUTION IN YOUNG DOUGLAS-FIR

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Summary - Branching pattern and branch dynamics relate closely to the ecophysiological processes of growth in young plantations, and influence the quality of wood recovered from the butt log of crop trees. A system of equations was constructed to simulate the number, diameter, and distribution of primary branches along the stem of young Douglas-fir trees prior to crown closure. Three to eleven annual shoots forming the lower main stem were sampled from each of 189 Douglas-fir trees; these annual shoots served as the basic modeling unit. Within each annual shoot, relative branch diameter was expressed as a function of relative depth in the shoot, maximum diameter was predicted as a function of depth into crown, branch number was expressed as a function of shoot length, and branch distribution was reconstructed from the average cumulative branch distribution over relative shoot depth. Simulated branch diameter and branch distribution on 21 trees from a validation data set highlighted the strengths and weaknesses of the described prediction system. **Key Words: branch diameter, branch distribution, nodal branch, internodal branch, annual shoot.**

The dynamics of crown structure and related ecophysiological processes influence the structure of bolewood and, hence, determine the quality of timber as a raw material (Brazier 1977, 1980). On a relatively coarse scale, crown size is commonly recognized to influence stem form, taper, and bole volume (Larson 1963, Hann et al. 1987). On the individual log scale, knot size, knot frequency, and proportion of juvenile wood exert primary control over lumber grade recovery (Fight et al. 1989). Likewise, knot size has been shown to exert a major influence on machine stress ratings in lumber (Whiteside 1974, Tustin and Wilcox 1974, Whiteside et al. 1977). These wood quality factors relate ultimately to crown development resulting from a given silvicultural regime (given the genetic disposition of the tree and stand), since crown size dictates both branching patterns (James and Revell 1974, Kenk 1981, Carter et al. 1986) and size of the juvenile wood core (Cown 1974).

As the effects of branch size, branch distribution, and juvenile wood core become more clearly defined in second-growth Douglas-fir (*Pseudotsuga menziesii*), concurrent advancements in silvicultural technology to control these attributes will facilitate production of logs with desired properties (for example, Whiteside 1962, Whiteside et al. 1977). An integral part of this technology will be the link between growth and yield models and milling simulators.

Some growth and yield simulators have been adapted to predict tree and log characteristics that feed directly into product recovery equations (for example, Maguire et al. 1987, Fight et al. 1989). The latter equations usually use an index of maximum knot or branch diameter per log as a predictor variable. Although they can define the empirical relationships between log characteristics and the relative volume of lumber and veneer by grade, simulated milling of "glass logs" avoids the limitations imposed by product dimension, sawing pattern, and sawyer skill in product recovery studies. As a result, numerous sawing simulators have been developed to assess lumber quality under a range in mill specifications. These simulators can utilize information on locations of all branches, yielding simulated dimension lumber with

very specific knot characteristics (Mitchell 1987, Todoroki 1988, Väisänen et al. 1989). A capacity to simulate branch diameter and location on the tree is therefore an important part of the link between growth models and milling simulators. Information on branch structure and dynamics have the added potential benefit of facilitating a link to ecophysiological studies and models.

Various types of branching models have been constructed for *Pinus sylvestris* (Kellomäki and Tuimala 1981, Jokinen and Kellomäki 1982), *Pinus radiata* (van Laar 1973), *Chamaecyparis obtusa* (Yamakura et al. 1973), and *Pseudotsuga menziesii* (Mitchell 1975, 1987). The model characteristics vary due to differences among species in branching structure, primarily the prominence of nodal vs. internodal branches. The objective of the present study was to model the initial size and distribution of nodal and internodal branches on stems of young, plantation-grown Douglas-fir. Sample trees and plots will be monitored over time to facilitate expansion of the present static model to dynamic models of branch growth and mortality, especially as they respond to differences in silvicultural regime.

Table 1. Stand attributes for 21 plots from which trees were sampled.

Installation (elevation, slope)	Plot	Trees per hectare	Breast height age	Ave. height (m)	Mean diam. (cm)	Relative density*
703 Longbell	16	400	4	4.05	5.3	0.99
(305 m	18	751	4	3.84	5.1	1.72
10-25%)	22	761	4	3.60	5.3	1.61
	24	1195	4	4.27	5.8	3.44
705 East Twin	42	820	7	7.59	8.9	4.32
(823 m	43	721	7	6.68	8.1	3.29
20-40%)	44	1507	7	6.80	8.1	7.02
	45	380	7	7.41	8.9	1.97
709 Mill Creek	1	776	6	9.11	12.2	6.66
(594 m	4	395	6	9.91	13.7	4.02
20%)	7	178	6	10.33	14.7	2.01
711 Kitten Kn	1	1270	4	5.33	7.4	4.94
(152 m	2	553	4	5.03	6.6	1.83
0-10%)	5	1294	4	5.64	7.1	4.79
	7	1210	4	5.27	7.1	1.17
713 Sauk Mtn	2	331	4	4.51	5.1	2.74
(305 m	6	701	4	6.00	8.4	3.40
10-20%)	7	1522	4	6.04	8.4	6.99
723 Formader	2	204	5	6.83	10.5	1.81
(274-488 m	4	408	5	5.91	8.8	2.39
20%)	6	815	5	6.66	9.8	7.03

*RD = plot basal area (m^2/ha)/$\sqrt{}$quadratic mean dbh (cm) (Curtis 1982)

METHODS

Data Collection

Trees were sampled from permanent plots established by the Pacific Northwest Stand Management Cooperative (Chappell et al. 1987). At least three plots from each of two plantations were sampled during three consecutive summers (1987, 1988, and 1989). Characteristics of the sample plots are summarized in Table 1. All plantations were sampled before crown closure and, hence, base of live crown was essentially at ground level.

In each sample plot, a stratified random sample of 10 trees was chosen across the range in dbh (diameter outside bark at 1.37 m). Height and diameter of all primary branches (those attached to the main stem) >0.6 m in height and >3 mm in diameter were recorded to the nearest 2 cm and 1 mm, respectively. Branch diameters were measured outside bark by caliper at a distance from the bole that was approximately equal to one branch diameter. If horizontal branch diameter differed by 1 mm or more from the vertical branch diameter, both were recorded and the geometric mean was used in model construction. Finally, annual shoots of the main stem, as identified by bud scale scars, were numbered consecutively starting with one at the top shoot (ignoring the current season's leader) and ending with the last discernible annual shoot at the base of the tree. In total, 17438 branches from 1031 annual shoots on 210 trees were sampled. Sample tree, shoot, and branch characteristics are summarized in Table 2.

Table 2. Summary of sample tree, shoot, and branch attributes.

Attribute	Min	Mean (sd)	Max
Tree:			
Total height (m)	2.15	6.10 (2.14)	12.31
Number of sample branches	30	93 (25)	219
Number of sample shoots	3	5 (1)	11
Annual Shoot:			
Height at top (m)	0.72	2.91 (1.43)	10.90
Length (m)	0.06	0.71 (0.29)	1.69
Depth into crown (m)	0.08	3.42 (2.23)	11.41
Maximum branch diameter (cm)	0.4	1.7 (0.6)	3.9
Number of sample branches	1	17 (6)	44
Branch:			
Height (m)	0.60	2.89 (1.36)	10.88
Diameter (cm)	0.3	1.0 (0.6)	3.9

Data Analysis

Modeling overview

On these young plantation Douglas-fir, primary branches are concentrated, both in size and density, at the top of the annual shoot (Fig. 1). Nodal branches are generally larger and more numerous than internodal branches. This pattern is repeated within successive annual shoots, although there is less differentiation in branch size from top to bottom in younger annual shoots. Maximum branch diameter within trees occurs near the bottom one-quarter of the tree, although this varies with the height of competing vegetation.

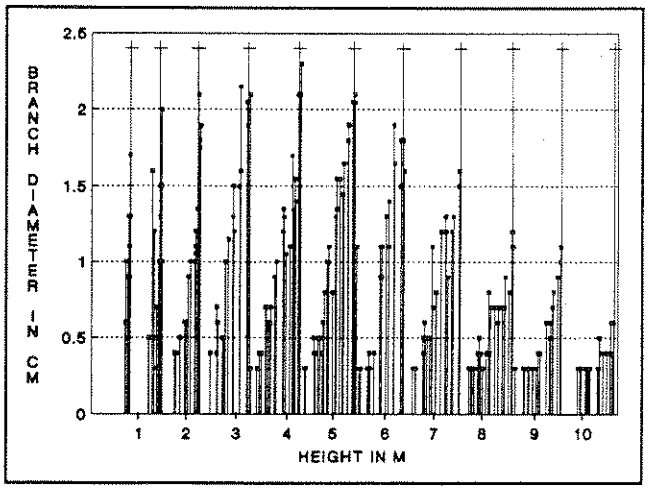

Figure 1. Diameter and height of primary branches along the main stem of a sample Douglas-fir tree. Vertical lines ending in a cross represent bud scale scar heights.

Analysis

Four separate components were developed to form a model system by which branch diameter and branch location within an annual shoot could be predicted: 1) a model predicting the total number of primary branches originating from an annual shoot, 2) the empirical frequency distribution for the number

of branches by relative depth in shoot, 3) a model expressing relative diameter as a function of relative height in the annual shoot, and 4) a model predicting the maximum branch diameter attainable within an annual shoot.

Alternative model forms for each model were compared by a modified likelihood criterion which was equal to the root mean squared error from unweighted and untransformed models, and to a rescaled root mean squared error from weighted and transformed models (Furnival 1961). For the relative branch diameter analysis, candidate models were fit to the observed data separately for each plantation. The other three analyses were performed across all six plantations. Twenty-one trees (one randomly selected tree per plot) were set aside for later validation of prediction equations. The data to which models were fit are referred to as the "modeling" data, vs. the set aside "validation" data. The following dependent and independent variables appeared in the final models:

Dependent variables

BD_{max} Maximum branch diameter, the geometric mean diameter of the largest branch within the annual shoot (cm).

BD_{rel} Relative branch diameter, the ratio of branch diameter to maximum branch diameter within the annual shoot.

NBR Number of branches within the annual shoot.

Independent variables

NODE Annual shoot identifier, in ascending order from 1 at top of tree.

RELH Relative distance from top of annual shoot, computed as [(node height - branch height)/annual shoot length].

DINC Depth into crown, computed as the difference in height between node 1 and the midpoint of the shoot (m).

NLEN Annual shoot length (m).

Number of branches

Various models were fit to predict number of branches (NBR) from shoot length (NLEN), both with and without various combinations of shoot and tree variables.

Relative frequency of branches by relative height

Relative frequency of branches by depth into annual shoot was plotted by installation, by shoot order from top of the tree, and for all shoots and installations combined. Several parametric models were fit to the cumulative data by maximum likelihood, and similarities of empirical distributions among installations and whorl orders were tested with Kolmogorov-Smirnov tests (Daniel 1978).

Relative branch diameter

Relative branch diameter (BD_{rel}) was explored as a function of relative position in annual shoot (RELH). Several model forms appropriate for describing nonlinear functions defined on the range 0 to 1 were compared. Modifications of a cumulative Weibull density function (for example, Somers et al. 1980), a generalized Chapman-Richards function (for example, Pienaar and Turnbull 1973), and a Johnson Schumacher function (for example, Grosenbaugh 1965) were fit and compared by overall mean squared error, visual comparison of the fitted curves to the observations, and resulting residual patterns. For each of these functions, one minus the cumulative density yields an expression for relative branch diameter

as a function of relative distance from top of the annual shoot. Several log-linear transformations were also examined. For each model form, each parameter alone and in combination with other parameters was allowed to vary by depth of shoot into crown (either NODE or DINC). This modification improved overall model fit by accounting for variability associated with increasing diameter differences between nodal and internodal branches in successively older annual shoots.

Maximum branch diameter

Nodal branches in Douglas-fir exhibit increasing diameters with increasing depth into the crown, as would be expected given the corresponding increase in branch age (Kershaw et al. 1990). As has previously been found in Douglas-fir, depth into crown accounts for most of the variation in maximum branch diameter at a given node (Maguire et al. 1987). Depending on the nature of competing vegetation, however, the largest branches occasionally occur slightly above the base of the live crown. Hence, some of the model forms tested allowed for a peak in maximum branch diameter within the live crown.

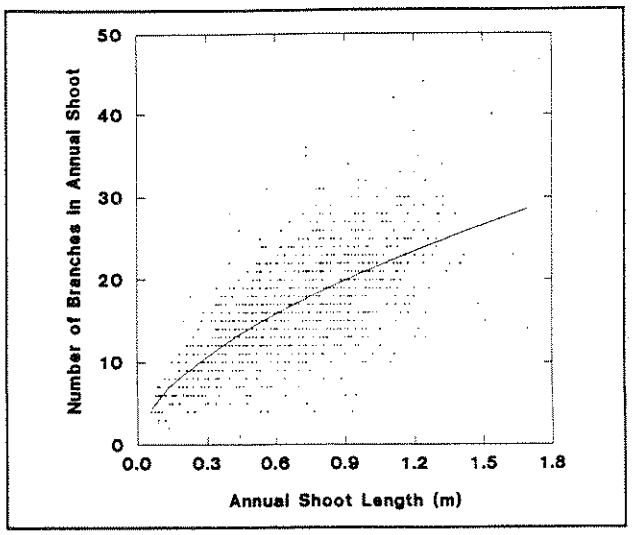

Figure 2. Relationship between number of primary branches and length of annual shoot.

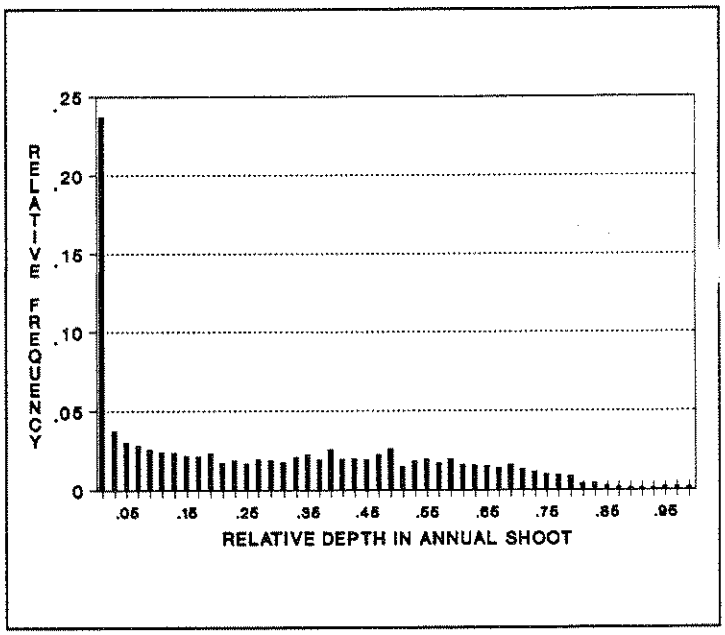

Figure 3. Relative frequency of branches across relative depth into annual shoot.

RESULTS

Individual Models

Number of branches

The number of branches per annual shoot (NBR) was largely a function of shoot length (NLEN), with relatively little systematic variation among shoots, trees and plots (Fig. 2). The best form and final parameter estimates for the equation predicting total number of branches in an annual shoot were

[1] $NBR = a_0 NLEN^{a1}$

a_0 = 21.16266 (appr. sd = 0.24142)
a_1 = 0.57111 (appr. sd = 0.01736)

A weighting factor of $NLEN^{-1}$ provided the best index of fit (Furnival 1961).

Relative frequency of branches by relative height

Parametric models fit to the the empirical distribution of branches by depth in annual shoot were not sufficiently flexible to produce the rapid drop in relative branch frequency away from the node, nor the slight but consistent increase in relative frequency near the middle of the shoot. Because the K-S tests revealed relatively little variation in general shape of this distribution among installations and shoot order, the overall empirical distribution was applied to estimate the relative frequencies of branches by relative depth into annual shoot (Fig. 3).

Relative branch diameter

The Weibull model proved to be more versatile for describing the relative diameter-relative position relationship. The Weibull cumulative probability function takes the form

$$BD_{rel} = \exp[-((RELH - a)/b)^c]$$

with location parameter *a*, scale parameter *b*, and shape parameter *c*. Allowing the scale parameter to vary by node number produced a family of curves with a different zero-intercept and steepness of descent for successive annual shoots (shown for plantation [713] in Fig. 4).

Table 3. Parameter estimates and their approximate standard deviations for the relative branch diameter model [2].

Installation (pseudo-R^2)	Parameter	Parameter Estimate	Standard deviation
703	b_1	-0.047603	0.008911
	b_2	0.862885	0.024288
(.94)	b_3	-0.049708	0.006305
	b_4	0.779730	0.038619
705	b_1	-0.017416	0.002417
	b_2	0.511637	0.013949
(.95)	b_3	-0.011788	0.002663
	b_4	0.736521	0.019298
709	b_1	-0.025392	0.003234
	b_2	0.530277	0.018028
(.94)	b_3	-0.026109	0.002286
	b_4	0.728085	0.024191
711	b_1	-0.055080	0.009578
	b_2	0.876356	0.023076
(.94)	b_3	-0.083076	0.006273
	b_4	0.834321	0.043450
713	b_1	-0.018524	0.002416
	b_2	0.571256	0.013079
(.95)	b_3	-0.040418	0.002356
	b_4	0.724422	0.020921
723	b_1	-0.034188	0.005447
	b_2	0.804092	0.017951
(.94)	b_3	-0.064835	0.003784
	b_4	0.822618	0.030302

For all plantations, the modified Weibull function accounted for at least 93% of the total variation in relative branch diameter (Table 3):

$$[2] \quad BD_{rel} = \exp\{-[(RELH-b_1)/(b_2+b_3NODE)]^{b4}\}$$

There is very slight bias (mean residual is not zero) in the predicted values for these models. This bias is consistently positive for the current year's annual shoot (NODE=1), but inconsistent in sign for all other annual shoots. Overall there is a very small positive bias in predicted relative diameter, but this bias does not seem serious relative to the observed data; the mean residual is less than 0.2% of the range of observed values for every plantation.

Validation of the relationship between relative diameter and relative position was performed

separately for the six plantations. Relative diameter is slightly underpredicted for four of six plantations, and overpredicted for two plantations. There are no consistent directions in sign of the mean residual for particular annual shoots, nor is bias consistently related to branch size.

Maximum branch diameter

A logarithmic transformation of maximum branch diameter produced a substantially better index of fit than corresponding weighted nonlinear models (Fig. 5). The chosen model was therefore

$$[3] \quad \ln(BD_{max}) = c_0 + c_1 \ln(DINC) + c_2 DINC$$

$$c_0 = 0.099070 \quad (sd = 0.012109)$$
$$c_1 = 0.493171 \quad (sd = 0.015972)$$
$$c_2 = -0.032204 \quad (sd = 0.006142)$$

The maximum branch diameter model peaks at 15.3 m, yielding a maximum branch diameter of 2.6 cm. Although this behavior is commensurate with the range in tree size and age in the modeling database, peak location in reality would vary across a wider range in tree size.

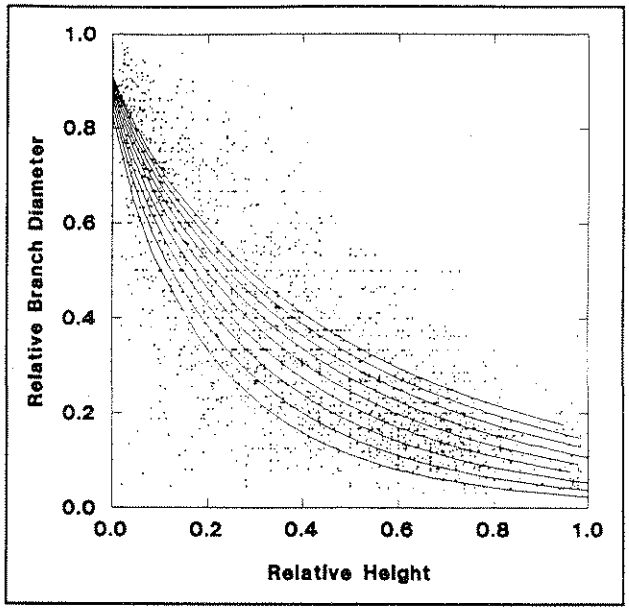

Figure 4. Relationship between relative branch diameter and relative depth into annual shoot. Successively higher curves represent successively younger shoots.

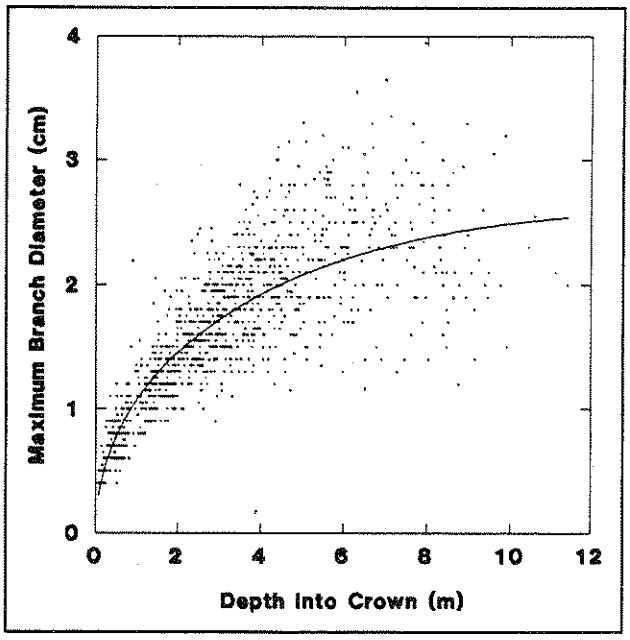

Figure 5. Relationship between maximum branch diameter within shoot (BDmax) and depth into crown.

Simulation Package

Simulation of branch diameter and branch distribution started with input of heights to bud scale scars, or annual cumulative height growth. Number of branches in each annual shoot was then estimated from shoot length by equation [1]. The relative height of each branch was assigned by the empirical distribution illustrated in Figure 3, and was subsequently converted into an absolute height above ground. Given the assigned relative depth into annual shoot, relative diameter for the branch was predicted from equation [2], then multiplied by the maximum branch diameter prediction from equation [3]. Output from the simulation program included installation number, tree number, branch height, and branch diameter.

All 21 trees in the validation dataset were simulated by the described procedure. Both actual and simulated branch diameters were then plotted on branch height to provide a rough assessment of prediction system performance. Figures 6-7 illustrate results for two trees from the validation dataset.

DISCUSSION

Model Evaluation

Branch model performance can be evaluated at two different levels: 1) performance of the four separate system components; and 2) performance of the prediction system as a whole. Because the validation data represented only a stratified random sample from the full database, performance of individual equations was expected to be comparable to their ability to fit the modeling data; in fact, this was borne out. Validation statistics (overall R^2) and qualitative comparison indicated good correspondence between predicted and observed NBR, BD_{rel}, and BD_{max} on trees from the validation sample.

Greater uncertainty was associated with performance of the prediction equations as a system. Ultimately, four dimensions corresponding to the four prediction components form the validation space of interest. The third and fourth components can be combined into predictions of individual branch diameters. The second component can be translated into predicted branch heights and, combined with diameter predictions, yield Figures such as 6 and 7. Thus the general predictive ability in regard to location and size can be assessed qualitatively. The tree represented in Figure 7 exhibits a problem with estimates of maximum branch diameter for a given node. Conversely, the tree in Figure 6 is described relatively well, except near crown base. Finally, the prediction for number of branches also appears satisfactory in these diagrams.

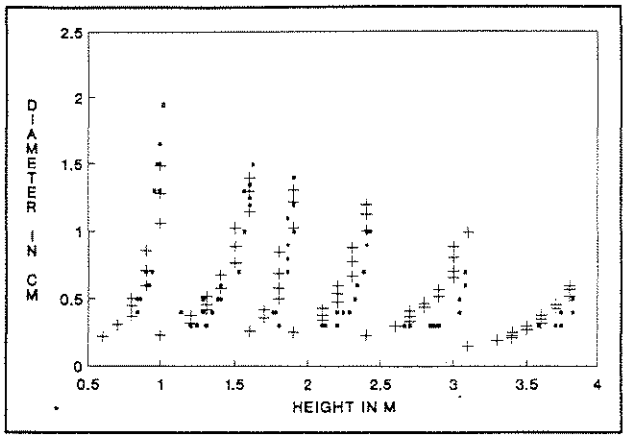

Figure 6. Actual (▪) and simulated (+) branch diameters and branch heights for Tree 713-2-255 in the validation dataset.

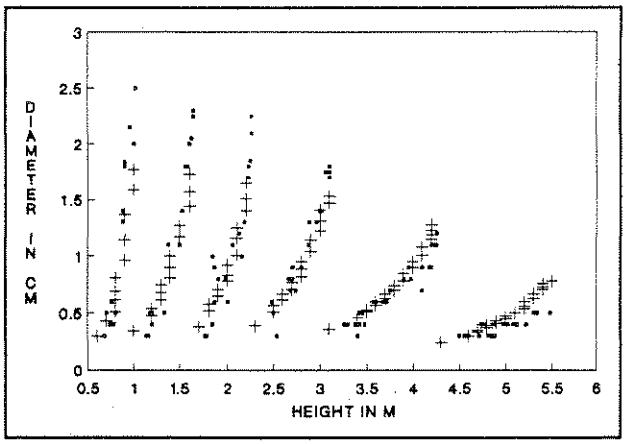

Figure 7. Actual (▪) and simulated (+) branch diameters and branch heights for Tree 723-6-692 in the validation dataset.

Comparison to Existing Models

The branch models developed for *Pinus sylvestris* differed from the described Douglas-fir model primarily because *P. sylvestris* does not form internodal branches (Kellomäki and Tuimala 1981, Jokinen and Kellomäki 1982). Similarly, other branch models for Douglas-fir disregarded internodal branches because they are substantially smaller and more evenly dispersed, producing a much smaller impact on log and lumber quality (Mitchell 1975, 1987, Maguire et al. 1987). As Jensen and Long (1983) emphasized, however, internodal branches may assume greater importance in an ecophysiological context.

Future Work

Several foci for future refinement include prediction of nodal branch diameters, assignment of relative height within annual shoots, and application of parameter estimation techniques that recognize correlations among equations and observations. The mean diameter of nodal branches is probably of lesser significance to wood quality than the maximum, suggesting the desirability of building deviations around the equations into predictions. This will become even more important in future dynamic models, since "growth is a deviation-amplifying process" (Stage 1987).

Other approaches to assigning relative height of branches within a shoot have not been fully explored. Models that incorporate spatial autocorrelation or that represent branch occurrence as a Poisson process are currently under consideration.

Finally, the error structure of the prediction system is considerably more complex than is implied by application of ordinary least squares. Hence, parameter estimation techniques that recognize the actual error structure will be pursued.

In summary, the described models represent a first approximation and an initial approach to a prediction system for simulating the primary branching structure of young Douglas-fir stems. The trees and plots from which the described data were collected will be monitored as part of the Pacific Northwest Stand Management Cooperative research program. Future crown models will therefore take advantage of both remeasurement data and innovations in analysis and modeling techniques.

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PARAMETER-FREE DIAMETER-DISTRIBUTION RECOVERY

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SUMMARY

Constrained weighted least squares methods are discussed and illustrated for allocating mortality, thinning removals, and diameter growth to a tree diameter list. In the procedure, allocation functions are derived to recover the arithmetic and quadratic mean diameter of a residual stand following thinning, an initial stand, and a projected stand after mortality trees have been removed. Because the procedure operates from a tree diameter list, instead of assuming a parametric probability model, it provides a highly flexible framework in which to model stands having diameter distributions poorly approximated by probability functions.

INTRODUCTION

Forest planning models depend on accurate and computationally efficient methods of estimating projected product yields from stand inventory data. The modeling technique implemented for this purpose must be amendable for modeling widely different stand types, flexible enough to accommodate modeling mixed species stands, and robust enough to handle irregular initial diameter distributions.

Diameter distribution models, either direct parameter prediction or indirect parameter recovery types, lend themselves well to modeling single species stands having smooth diameter distributions closely approximated by the assumed underlying probability function. Any attempt to extend these models to the mixed species stands and/or stands of irregular diameter distributions injects a level of complexity that virtually negates their use for this purpose.

Individual-tree models present the best framework for modeling mixed species conditions and complex diameter distributions because they manipulate the individual instead of the distribution. For this simplicity in modeling a

price is paid in terms of reduced precision and accuracy on the derived aggregated lump-sum stand level response. When tree growth and mortality are allocated to the individual our statistical expectation in general is that the lump-sum estimates of growth will have a larger variance and bias than those obtained through the use of stand-level growth equations. Consequently, the positive or negative feedback due to the use of predicted variables as independents tends to accumulate bias faster for the individual tree models than for stand-level models--the net result being that individual-tree models have a higher propensity to produce biased lump-sum estimates than stand-level projection equations over several projection intervals.

To allow stand-level projection equations to be used in conjunction with a tree list, a technique was sought that would allocate individual-tree response to the elements of a tree list from estimated stand-level response. These investigations led to two constrained least-squares allocation procedures, one for allocating mortality to a tree list and one for allocating diameter growth to a tree list. A tree list is a list of tree diameters and the associated numbers of trees represented per unit area by each diameter in the list.

In this paper we present each model and list the computer program (FORTRAN 77) prepared to implement the allocation procedures. A detailed document is available from the authors describing and illustrating the algorithms used in the FORTRAN programs.

The least-squares mortality model minimizes the weighted sum of squared difference between tree-list elements before mortality and after mortality numbers per unit area subject to the constraints on the adjusted tree list that:

1. all trees per unit area represented equals the predicted total number of trees per unit area surviving, and

2. the diameter moments of the adjusted list equal the predicted diameter moments of trees at time 0 that will survive to time 1.

The least-squares growth allocation model calculates the diameter growth of a tree-list element by minimizing the weighted sum of squared differences between tree-list elements at time 0 and time 1 subject to the constraints that the adjusted tree list has mean and quadratic mean diameters equal to projected.

In both models a weight variable is used to control the magnitude of the allocation to each tree list element. Elements of a list given higher weights relative to other elements receive relatively smaller adjustments. By controlling the shape of the weighting function we also control the shape of the allocation function, and we may derive mortality and growth allocation functions approximating a desired shape.

MORTALITY/THINNED TREE ALLOCATION MODEL

To derive the least-squares procedure to allocate mortality trees to a tree list and introduce the required rotation we denote by

n the number of trees in the tree list before mortality allocation;

d_i the diameter-breast-high (d.b.h.) of the ith tree;

S_i the number of trees per unit area represented by the ith tree before mortality;

S_i' the number of trees per unit area represented by the ith tree after adjustment for mortality occurring between t_0 and $t_0+\Delta t$;

TS_0 the total number of trees alive per unit area at time t_0 (i.e. $TS_0=\sum_{i=1}^{n} S_i$);

TS_1 the total number of trees per unit area that will survive to time $t_0+\Delta t$ (i.e. $TS_1=\sum_{i=1}^{n} S_i'$);

$\bar{d}$ the arithmetic mean d.b.h. of all trees at time t_0 that will survive to time $t_0+\Delta t$;

$\bar{q}$ the quadratic mean d.b.h. of all trees at time t_0 that will survive to time $t_0+\Delta t$.

Given that regression equations have been estimated from the data to estimate TS_1, $\bar{d}$, and $\bar{q}$, the objective of a moment recovery procedure supported on arithmetic and quadratic diameter is to determine the $\hat{S}_i'$'s so that

a. $\hat{TS}_1=\sum_{i=1}^{n} \hat{S}_i'$,

b. $\hat{\bar{d}}=[\sum_{i=1}^{n} \hat{S}_i' d_i]/[\sum_{i=1}^{n} \hat{S}_i']=[\sum_{i=1}^{n} \hat{S}_i' d_i]/\hat{TS}_1$, and

c. $\hat{\bar{q}}^2=[\sum_{i=1}^{n} \hat{S}_i' d_i^2]/[\sum_{i=1}^{n} \hat{S}_i']=[\sum_{i=1}^{n} \hat{S}_i' d_i^2]/\hat{TS}_1$

where the caret (^) indicates the estimate of the quantity. One convenient method for determining the $\hat{S}_{i,s}$ satisfying the above three constraints is to use a constrained least-squares procedure that minimizes the sum of the weighted squared difference between S_i and $\hat{S}_i$ where the weighting function is used to control the form of the allocation function. Formally stated using Lagrangian multipliers (Whittle 1971), this least-squares recovery model is

$$\underset{\hat{S}_{i,s}}{\mathrm{Min}} L(\hat{S}_{i,s}\lambda,\delta,\gamma)=\sum_{i=1}^{n} W_i(\hat{S}_i-S_i)^2+2\lambda(\Sigma\hat{S}_i-TS_1)+$$

$$2\delta[\sum_{i=1}^{n}\hat{S}_i d_i-(TS_i)\hat{\bar{d}}]+2\gamma[\sum_{i=1}^{n}\hat{S}_i d_i^2-(TS_1)\hat{\bar{q}}^2] \tag{1}$$

where W_i is the weight and λ, δ, and γ are Lagrangian multipliers. The values of $\hat{S}_i$ that minimize the above equation are found by taking the partial derivative of the equation with respect to $\hat{S}_i$, equating it to 0, and solving the resulting equation for $\hat{S}_i$. That is

$$\frac{\delta L}{\delta \hat{S}_i}=2W_i(\hat{S}_i-S_i)+2d_i+2d_i^2=0 \tag{2}$$

and thus

$$\hat{S}_i=S_i-(\lambda+\delta d_i+\gamma d_i^2)W_i^{-1} \tag{3}$$

The constant Lagrangian multipliers λ, δ, and γ are determined from equation 3 by imposing each of the required constraints on the equation. This results in the following simultaneous system of three linear equations solvable for λ, δ, and γ.

$$[\sum_{i=1}^{n} W_i^{-1}]\lambda+[\sum_{i=1}^{n} d_i W_i^{-1}]\delta+[\sum_{i=1}^{n} d_i^2 W_i^{-1}]\gamma=TS_0-T\hat{S}_1 \tag{4a}$$

$$[\sum_{i=1}^{n} d_i W_i^{-1}]\lambda+[\sum_{i=1}^{n} d_i^2 W_i^{-1}]\delta+[\sum_{i=1}^{n} d_i^3 W_i^{-1}]\gamma=\sum_{i=1}^{n} S_i d_i-(T\hat{S}_1)\hat{\bar{d}} \tag{4b}$$

$$[\sum_{i=1}^{n} d_i^2 W_i^{-1}]\lambda+[\sum_{i=1}^{n} d_i^3 W_i^{-1}]\delta+[\sum_{i=1}^{n} d_i^4 W_i^{-1}]\gamma=\sum_{i=1}^{n} S_i d_i^2-(T\hat{S}_1)\hat{\bar{q}}^2 \tag{4c}$$

Appendix program A applies standard matrix inversion procedures to solve the above system of equations. The program takes an initial tree list and returns an adjusted list for any specified set of moment constraints.

How much mortality is allocated to a tree list element can be controlled by its assigned weight W_i. From equation 3, if an element is assigned a large weight relative to other elements it will receive a relatively small mortality allocation (i.e. mortality = $(\lambda+\delta d_i+\gamma d_i^2)W_i^{-1}$. Consequently, for competition mortality, we want the assigned weight to decrease with increasing diameter so proportionally more mortality will be allocated to the small diameter classes.

In applying these models to construct growth and yield models for planted loblolly pine (Pinus taeda (Matney and Farrar unpublished) and natural longleaf pine (Pinus palustris) (Farrar and Matney unpublished), setting $W_i=d_i^{-4}$ produced the best results.

Stands thinned from below behave similarly to stands having heavy competition mortality. Because of this, the mortality model can be readily modified for use by letting TS_1 equal the number of trees remaining after thinning, $\bar{d}$ equal the residual stand arithmetic mean diameter, and $\bar{q}$ equal the residual stand quadratic mean diameter.

The shape of the weighting function for thinning is more concave than for mortality because more trees are removed from the upper diameter classes in thinning than occur due to mortality. For the two data sets studied, satisfactory results for thinning were obtained by setting $W_i=d_i^{-2}$.

Growth Allocation

A weighted least-squares tree-list projection recovery model supported on predicted future stand arithmetic and quadratic mean diameter is

$$\underset{\hat{D}_{i,s}}{\mathrm{Min}}\, L(\hat{D}_{i,s};\lambda,\delta)=\sum_{i=1}^{n} W_i S_i'(\hat{D}_i-d_i)^2+2\lambda[\sum_{i=1}^{n} S_i'\hat{D}_i-(\hat{TS}_1)\hat{\bar{D}}]+$$

$$\delta[\sum_{i=1}^{n} S_i'\hat{D}_i^2-(\hat{TS}_1)\hat{\bar{Q}}^2] \qquad (5)$$

where

$\hat{D}_i$ is the estimated future d.b.h. of the ith tree list element,

$\hat{\bar{D}}$ is the predicted future arithmetic mean stand d.b.h.,

$\hat{\overline{Q}}$ is the predicted future quadratic mean stand d.b.h., and

λ and δ are Lagrangian multiplier constants to be determined so that the estimated list has

a. arithmetic mean stand d.b.h. equal to predicted

$$(\text{i.e. } \sum_{i=1}^{n} S_i'\hat{D}_i=(\hat{T}S_1)\hat{\overline{D}}) \text{ and}$$

b. quadratic mean stand d.b.h. equal to predicted

$$(\text{i.e. } \sum_{i=1}^{n} S_i'\hat{D}_i^2=(\hat{T}S_1)\hat{\overline{Q}}^2).$$

The $\hat{D}_1$ satisfying the least-squares model are determined by taking the partial derivative of equation 5 with respect to $\hat{D}_i$, setting the result equal to 0 and solving for $\hat{D}_i$.

$$\frac{\delta L}{\delta \hat{D}_i}=2W_iS_i'(\hat{D}_i-d_i)+2\lambda S_i'+2\delta S_i'\hat{D}_i=0 \text{ and}$$

thus

$$\hat{D}_i=(W_id_i-\lambda)/(\delta+W_i) \tag{6}$$

To solve for the Lagrangian multipliers, the two constraints are imposed on equation 6 to produce the following two nonlinear equations solvable for λ and δ.

$$\sum_{i=1}^{n} S_i'[(W_id_i-\lambda)/(\delta+W_i)]=(\hat{T}S_1)\hat{\overline{D}} \tag{7a}$$

$$\sum_{i=1}^{n} S_i'[W_id_i-\lambda)/(\delta+W_i)]^2=(\hat{T}S_1)\hat{\overline{Q}}^2 \tag{7b}$$

Appendix program B uses Newton's iteration (Burden, Faires, and Reynolds 1981) procedure for determining λ and δ from equations 7a and 7b.

Since the regression of future diameter on initial diameter for short projection intervals (10 or less years) is usually well approximated by a straight line model, a logical choice for the weight, W_i is 1. With $W_i=1$, equation 6 becomes

$$\hat{D}_i=a+bd_i$$

where

a is the intercept is $-\lambda/(\delta+1)$, and
b is the slope is $1/(\delta+1)$.

DISCUSSION

These models have been successfully applied to construct thinned and unthinned growth and yield models for planted loblolly pine on cutover site-prepared land (Matney and Farrar unpublished) and repeatedly-thinned natural even-aged longleaf pine stands (Farrar and Matney unpublished). The models are not founded on any biological basis but are rather artificial constructions designed to mimic the biological process observed and to achieve a diameter-distribution recovery-type system not dependent on an assumed probability density function--hence the title of this paper.

Data from the Mississippi State University growth and yield cooperative were utilized to compare observed and predicted distributions with simple Chi-square tests. The data set afforded 503 growth model comparisons (thinned and unthinned), 503 mortality model comparisons, and 198 thinning model comparisons. The rejection percentages at an alpha level of 0.05 for the growth, mortality, and thinning models were 1.9, 3.5, and 2.1, respectively.

These models, as in most mathematical procedures, can result in illogical results under certain circumstances. The mortality/thinning allocation function can either cause a list element to have negative numbers of trees or to have more trees than before adjustment. In testing the procedure we found that the problem could be eliminated by repeatedly applying the procedure until a logical list is obtained to a new list formed by setting negative list elements to zero and positively adjusted list elements to their original values.

The least-squares diameter-projection procedure can produce negative tree growth. Because these negative growths were negligible for the data sets used for testing, they were ignored.

How well these models handle the projection of stands with irregular diameter distribution and mixed species stands is presently being evaluated by a major southern forest products industry. All preliminary results indicate that the method will prove to be a very robust means of handling stands even with extremely irregular diameter distributions and mixed-species stands. This robustness is due to the diameter moment constraints and to the principle of moments (Kendal and Stuart 1977) that states that most distributions can be closely approximated if a sufficient number of its moments are known.

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A P P E N D I X

```
C -------------------------- Program A
C
C
C SUBROUTINE LSQ_MORT
C
C Weighted least squares procedure for allocating mortality
C to a size class distribution.
C
C SUBROUTINE LSQ_MORT INPUT ARGUMENTS
C
C     X_LIST = REAL*4(N) array of size classes.
C     X_SCL  = REAL*4(N) array containing the number
C              of individuals in each size class of X_LIST.
C               X_SCL must not have any elements less then
C              or equal to 0.
C     N      = INTEGER*2 scalar containing the number of
C              elements in arrays X_LIST, and X_SCL.
C     MK_VEC = REAL*8(M) vector of containing the moments
C              of size of all individuals at time 0 that
C              will survive to time 1.
C              MK_VEC(1) must be set to 1.
C     OR_VEC = INTEGER*2(M) array containing the orders of
C              the moments stored in the array MK_VEC.
C              OR_VEC(1) must be set to 0.
C     M      = INTEGER*2 scalar containing the number of
C              moments in the vector MK_VEC and the number
C              of orders in OR_VEC.
C     SURV   = REAL*8 scalar containing the number of
C              indivduals surviving to time 1.
C     WT_FUN = REAL*8 function for evaluating weight
C              function.
C     WT_PAR = REAL*8(10) array of parameters to be passed
C              to WT_FUN.
C
C
C SUBROUTINE LSQ_MORT OUTPUT ARGUMENTS
C
C     Y_SCL  = REAL*4(N) array of containing the number
C              individuals represented by each size class
C              of X_LIST after mortality.
C              The moments of (X_LIST,Y_SCL) equal those
C              given in MK_VEC.
C    PAR_OUT= REAL*8(M) array containing the values of the
C             Lagrangian multipliers satisfying the moment
C              constraints.
C     ERR    = INTEGER*2 scalar error code. If ERR = 0 the
C              allocation was successful. Otherwise if ERR
C              = 1 the allocation is in error
C
C SUBROUTINE LSQ_MORT SOURCE CODE
C
      SUBROUTINE LSQ_MORT
     &           (N,X_LIST,X_SCL,M,MK_VEC,OR_VEC,SURV,
     &            WT_FUN,WT_PAR,Y_SCL,PAR_OUT,ERR)

      INTEGER*2   N,M,K,I,J,L,R,C,OR_VEC(M),ERR,
     &            IER,END_TST,REP
      REAL*4      X_LIST(N),X_SCL(N),Y_SCL(N),X_MIN,X_MAX
      REAL*8      PAR_OUT(M),WT_FUN,B(10),MAT(10,10),
     &            MAT_INV(10,10),WT_PAR(10),MK_1,MK_2,
     &            CUM_TOT,CUM(250),SURV,WT,ADJ,
     &            MK_VEC(M)

      EXTERNAL    WT_FUN

      DO I=1,N
        Y_SCL(I)=X_SCL(I)
      END DO

      END_TST=0
      ERR=0
      REP=0

      DO WHILE(END_TST.EQ.0)
        L=1
        REP=REP+1
        IF(REP.GT.5)THEN
           ERR=1
           RETURN
        END IF
        DO WHILE(Y_SCL(L).LE.0.)
          L=L+1
          IF(L.GT.N)THEN
            ERR=1
            RETURN
          END IF
        END DO

        IF((N-L).LT.4)THEN
           ERR=1
           RETURN
        END IF

        X_MIN=X_LIST(L)
        X_MAX=X_LIST(L)
        CUM_TOT=Y_SCL(L)
        CUM(L)=Y_SCL(L)
        MK_1=Y_SCL(L)*X_LIST(L)
        MK_2=Y_SCL(L)*X_LIST(L)*X_LIST(L)

        DO I=(L+1),N
          IF(X_LIST(I).LT.X_MIN)THEN
            X_MIN=X_LIST(I)
          END IF
          IF(X_LIST(I).GT.X_MAX)THEN
            X_MAX=X_LIST(I)
          END IF
          CUM_TOT=CUM_TOT+Y_SCL(I)
          CUM(I)=CUM(I-1)+Y_SCL(I)
          MK_1=MK_1+Y_SCL(I)*X_LIST(I)
          MK_2=MK_2+Y_SCL(I)*X_LIST(I)*
               X_LIST(I)
        END DO

        MK_1=MK_1/CUM_TOT
        MK_2=MK_2/CUM_TOT

        DO I=1,M
          B(I)=0.D0
          PAR_OUT(I)=0.D0
          DO J=1,M
            MAT(I,J)=0.D0
          END DO
        END DO

        DO I=1,M
```

```
        R=OR_VEC(I)
        DO J=1,N
          IF(Y_SCL(J).GT.0.)THEN
             WT=WT_FUN(X_LIST(J),X_MIN,
     &                  X_MAX,Y_SCL(J),CUM(J),
     &                  CUM_TOT,MK_1,MK_2,WT_PAR)
             B(I)=B(I)+Y_SCL(J)*(X_LIST(J)**R)
             DO K=1,M
               C=OR_VEC(K)
               MAT(I,K)=MAT(I,K)+(X_LIST(J)**(R+C))/WT
             END DO
          END IF
        END DO
        B(I)=B(I)-SURV*MK_VEC(I)
      END DO
      CALL IVERA(M,MAT,MAT_INV,IER)
      IF(IER.NE.0.)THEN
         ERR=1
         RETURN
      END IF

      DO I=1,M
        DO J=1,M
          PAR_OUT(I)=PAR_OUT(I)+MAT_INV(I,J)*B(J)
        END DO
      END DO

      END_TST=1
      DO I=1,N
        IF(Y_SCL(I).GT.0.)THEN
           WT=WT_FUN(X_LIST(I),X_MIN,X_MAX,
     &                Y_SCL(I),CUM(I),CUM_TOT,
     &                MK_1,MK_2,WT_PAR)
          ADJ=0.
          DO J=1,M
            ADJ=ADJ+PAR_OUT(J)*(X_LIST(I)**OR_VEC(J))
          END DO
          Y_SCL(I)=Y_SCL(I)-ADJ/WT
          IF(Y_SCL(I).LT.0.)THEN
             Y_SCL(I)=0.
             END_TST=0
          END IF
        END IF
      END DO

    END DO

    RETURN
    END
C
C SUBROUTINE WT_MORT
C
C Function subprogram for calculating the value of the
C weight function for the least square mortality
C allocation model. - Model 1.
C
C SUBROUTINE WT_MORT INPUT ARGUMENTS
C
C     WT_PAR = REAL*8(10) array of parameters passed to
C              LSQ_MORT/LP_MORT.
C     SIZE   = REAL*4 scalar containing individual size.
C     X_MIN  = REAL*4 scalar containing the minimum size in
C              X_LIST
C     X_MAX  = REAL*4 scalar containing the maximum size in
C              X_LIST
C     NUM    = REAL*4 scalar containing the number of
C              individuals of size SIZE at time 0.
C     CUM    = REAL*8 scalar containing the cumulative
C              number of individuals less then or equal to
C              size SIZE at time 1.
C     CUM_TOT= REAL*8 scalar containing the
C              total number of individuals at
C              time 0.
C     MK_1   = REAL*8 scalar containing the
C              first order moment of the size
C              of individuals at time 0.
C     MK_2   = REAL*8 scalar containing the
C              secon order moment of the size
C              of individuals at time 0.
C
C SUBROUTINE WT_MORT OUTPUT ARGUMENTS
C
C     WT_MORT= REAL*8 function return value of
C              the value of the weight.
C              Note the value of the weight
C              must be greater than 0.
C
C SUBROUTINE WT_MORT SOURCE CODE
C

      REAL*8 FUNCTION WT_MORT
     &                (SIZE,X_MIN,X_MAX,NUM,
     &                 CUM,CUM_TOT,MK_1,MK_2,
     &                 WT_PAR)
      REAL*4  SIZE,X_MIN,X_MAX,NUM
      REAL*8  WT_PAR(10),CUM,CUM_TOT,MK_1,MK_2

      WT_MORT=(NUM**WT_PAR(1))*(CUM**WT_PAR
     &        (2))*(CUM_TOT**WT_PAR(3))*
     &        (SIZE**WT_PAR(4))*(MK_1**WT_PAR
     &        (5))

      RETURN
      END
C
C End of Program A
C
C -------------------------- Program B
C
C SUBROUTINE LSQ_PROJ
C
C Least squares alogorithm for allocating size
C increase to a size class list adjusted for
C mortality.
C
C SUBROUTINE LSQ_PROJ INPUT ARGUMENTS
C
C     X_LIST = REAL*4(N) array of individual
C              size classes at time 0.
C     X_SCL  = REAL*4(N) array containing the
C              number individuals represented
C              by each size class in array
C              X_LIST adjusted for mortality.
C              X_SCL must not have any elements
C              less than or equal to 0.
C     N      = INTEGER*2 scalar containing the
C              number of elements in arrays
C              X_LIST, and X_SCL.
C     MK_ORD1= REAL*8 scalar containing the
C              first order moment of future
C              size.
C     MK_ORD2= REAL*8 scalar containing the
C              second order moment of future
C              size.
C     MAX_IT = INTEGER*2 scalar containing the
C              maximun number of iterations.
C     IT_REP = INTEGER*2 scalar containing the
C              maximum number of algorithm
```

```
C              allowed.
C     WT_FUN = REAL*8 function for evaluating the
C              weighting function. The return
C              value must be greater than 0.
C     WT_PAR = REAL*8(10) array of parameters to
C              be passed to WT_FUN.
C
C
C SUBROUTINE LSQ_PROJ OUTPUT ARGUMENTS
C
C    Y_LIST = REAL*4(N) array of future sizes for each size
C             in array X_LIST having moments equal to those
C              in MK_VEC.
C    PAR_OUT= REAL*8(2) array containing the values of the
C             Lagrangian multipliers satisfying the moment
C              constraints in MK_VEC.
C     ERR    = INTEGER*2 scalar error code. If ERR = 0 the
C              allocation was successful. Otherwise if ERR
C              = 1 the allocation is in error.
C
C SUBROUTINE LSQ_PROJ SOURCE CODE
C
      SUBROUTINE LSQ_PROJ
     &           (N,X_LIST,X_SCL,MK_ORD1,MK_ORD2,
     &            MAX_IT,IT_REP,WT_FUN,WT_PAR,Y_LIST,
     &            PAR_OUT,ERR)

      INTEGER*2   N,M,I,J,K,L,MAX_IT,IT_REP,ERR,IT_CNT,
     &            REP_CNT,END_TST,IER
      REAL*4      X_LIST(N),X_SCL(N),Y_LIST(N),XX,XS
      REAL*8      PAR_OUT(2),WT_FUN,TOL,WT,DIV,MK_ORD1,
     &            MK_ORD2,CUM(250),MK_1,MK_2,CUM_TOT,
     &            WT_PAR(10),AX,BX,XC,LAMDA,DELTA,
     &            D_SUM,Q_SUM,LAM_ADJ,DEL_ADJ,XDEL,DMIN1,
     &            WORK_1,WORK_2,WORK_3,F,G,F_LAM,F_DEL,
     &            G_LAM,G_DEL,BASE,LAM_CHG,DEL_CHG,DABS,
     &            XTOL,D0,Q0,D1,Q1,DSQRT
      EXTERNAL    WT_FUN
      SAVE TOL
      DATA TOL/.00000001/


      ERR=0
      IT_CNT=0
      REP_CNT=0
      END_TST=0
      DIV=2.D0
      XTOL=TOL

      X_MIN=X_LIST(1)
      X_MAX=X_LIST(1)
      CUM_TOT=X_SCL(1)
      CUM(1)=X_SCL(1)
      MK_1=X_SCL(1)*X_LIST(1)
      MK_2=X_SCL(1)*X_LIST(1)*X_LIST(1)

      DO I=2,N
        IF(X_LIST(I).LT.X_MIN)THEN
          X_MIN=X_LIST(I)
        END IF
        IF(X_LIST(I).GT.X_MAX)THEN
          X_MAX=X_LIST(I)
        END IF
        CUM_TOT=CUM_TOT+X_SCL(I)
        CUM(I)=CUM(I-1)+X_SCL(I)
        MK_1=MK_1+X_SCL(I)*X_LIST(I)
        MK_2=MK_2+X_SCL(I)*X_LIST(I)*X_LIST(I)
      END DO

      MK_1=MK_1/CUM_TOT
      MK_2=MK_2/CUM_TOT

      D_SUM=CUM_TOT*MK_ORD1
      Q_SUM=CUM_TOT*MK_ORD2

      D0=MK_1
      Q0=DSQRT(MK_2)
      D1=MK_ORD1
      Q1=DSQRT(MK_ORD2)

      DO I=1,N
        WT=WT_FUN(X_LIST(I),X_MIN,X_MAX,
     &            X_SCL(I),CUM(I),CUM_TOT,
     &            MK_1,MK_2,WT_PAR)
        IF(I.EQ.1)THEN
          XDEL=WT
        ELSE
          XDEL=DMIN1(WT,XDEL)
        END IF
      END DO


      DO WHILE(REP_CNT.LE.IT_REP.AND.END_TST.
     &  EQ.0)
        REP_CNT=REP_CNT+1
        DIV=DIV/2.D0
        XTOL=TOL*DIV
        BX=(Q1-D1)/(Q0-D0)
        AX=D1-BX*D0
        XX=D0
        XS=CUM_TOT/N
        XC=CUM_TOT/2.
        WT=WT_FUN(XX,X_MIN,X_MAX,XS,XC,
     &            CUM_TOT,MK_1,MK_2,WT_PAR)
        DELTA=WT*(1.-BX)/BX
        LAMDA=-AX*(WT+DELTA)
        DO WHILE(IT_CNT.LE.MAX_IT.AND.
     &   END_TST.EQ.0)
          IT_CNT=IT_CNT+1
          F=-D_SUM
          G=-Q_SUM
          F_LAM=0.
          F_DEL=0.
          G_LAM=0.
          G_DEL=0.
          IF(DABS(XDEL+DELTA).LE.1.D-6)THEN
            DELTA=-XDEL+1.D-6
          END IF

          DO I=1,N
            WT=WT_FUN(X_LIST(I),X_MIN,X_MAX,
     &                X_SCL(I),CUM(I),
     &                CUM_TOT,MK_1,MK_2,
     &                WT_PAR)
            WORK_1=WT*X_LIST(I)-LAMDA
            WORK_2=DELTA+WT
            WORK_3=WORK_1/WORK_2
            F=F+X_SCL(I)*WORK_3
            G=G+X_SCL(I)*(WORK_3**2)
            F_LAM=F_LAM-X_SCL(I)/WORK_2
            F_DEL=F_DEL-X_SCL(I)*WORK_1/
     &              (WORK_2**2)
            G_LAM=G_LAM-2.*X_SCL(I)*WORK_1/
     &            (WORK_2**2)
            G_DEL=G_DEL-2.*X_SCL(I)*
     &            (WORK_1**2)/(WORK_2**3)
          END DO
```

```
          BASE=DIV*(F_LAM*G_DEL-G_LAM*F_DEL)
          LAM_ADJ=LAMDA-(F*G_DEL-G*F_DEL)/BASE
          DEL_ADJ=DELTA-(G*F_LAM-F*G_LAM)/BASE

          IF(IT_CNT.LE.1)THEN
             LAMDA=LAM_ADJ
             DELTA=DEL_ADJ
          ELSE
             LAM_CHG=DABS(LAMDA-LAM_ADJ)
             DEL_CHG=DABS(DELTA-DEL_ADJ)
             IF(LAM_CHG.LE.XTOL.AND.DEL_CHG.LE.XTOL)
     &             THEN
               LAMDA=LAM_ADJ
               DELTA=DEL_ADJ
               PAR_OUT(1)=LAMDA
               PAR_OUT(2)=DELTA
               DO I=1,N
                 WT=WT_FUN(X_LIST(I),X_MIN,X_MAX,
     &                     X_SCL(I),CUM(I),
     &                    CUM_TOT,MK_1,MK_2,WT_PAR)
                Y_LIST(I)=(WT*X_LIST(I)-LAMDA)/(WT+DELTA)
               END DO
               END_TST=1
               RETURN
             ELSE
               LAMDA=LAM_ADJ
               DELTA=DEL_ADJ
             END IF
          END IF

        END DO
      END DO

      ERR=1

      RETURN
      END


C SUBROUTINE WT_GRW
C
C Function subprogram for calculating weight value for
C LSQ_PROJ.
C
C SUBROUTINE WT_GRW INPUT ARGUMENTS
C
C     WT_PAR = REAL*8(10) array of parameters passed to
C              LSQ_PROJ.
C     SIZE   = REAL*4 scalar containing individual size.
C     X_MIN  = REAL*4 scalar containing the minimum size
C              in X_LIST.
C     X_MAX  = REAL*4 scalar containing the maximum size
C              in X_LIST.
C     NUM    = REAL*4 scalar containing the number of
C              individuals of size SIZE at time 0.
C     CUM    = REAL*8 scalar containing the cumulative
C              number of individuals less then or equal
C              to size SIZE at time 1.
C     CUM_TOT= REAl*8 scalar containing the total number
C              of individuals at time 0.
C     MK_1   = REAL*8 scalar containing the first order
C              moment of the size of individuals at time 0.
C     MK_2   = REAL*8 scalar containing the second order
C              moment of the size of individuals at time 0.

C
C SUBROUTINE WT_GRW OUTPUT ARGUMENTS
C
C     WT_GRW = REAL*8 function return value of the weight.
C
C SUBROUTINE WT_GRW SOURCE CODE
C

      REAL*8 FUNCTION WT_GRW(SIZE,X_MIN,X_MAX,
     &                       NUM,CUM,CUM_TOT,
     &                       MK_1,MK_2,WT_PAR)
      REAL*4  SIZE,X_MIN,X_MAX,NUM
      REAL*8  WT_PAR(10),CUM,CUM_TOT,MK_1,MK_2

      WT_GRW=(NUM**WT_PAR(1))*(CUM**WT_PAR(2))
     &       *(CUM_TOT**WT_PAR(3))*
     &       (SIZE**WT_PAR(4))*(MK_1**
     &       WT_PAR(5))

      RETURN
      END
C
C
C End Program B
C
C
C -------------------------- Program C
C
C SUBROUTINE IVERA
C
C Matrix inversion program used by SUBROUTINE
C LSQ_MORT.
C
C SUBROUTINE IVERA INPUT ARGUMENTS
C
C     N      = INTEGER*2 scalar containing the
C              dimension of the matrix to be
C              inverted.
C     A      = REAL*8(10,10) arrary containing
C              the matrix to be inverted. The
C              matrix is destroyed by the
C              inversion.
C
C
C SUBROUTINE IVERA OUTPUT ARGUMENTS
C
C     AI     = REAL*4(10,10) array containing
C              the inverse of A.
C     IER    = INTEGER*2 scalar containing 0 if
C              the matrix was nonsingular or
C              one if the matrix could not be
C              inverted.
C
C SUBROUTINE IVERA SOURCE CODE
C


      SUBROUTINE IVERA(N,A,AI,IER)
      INTEGER*2 IU(10),N,IER
      REAL*8 DABS,DFLOAT,AI(10,10),A(10,10),
     &     AMAX,AUX,D,V
      DO 10 J=1,N
      DO 10 I=1,N
        AI(I,J)=0.D0
   10   IF (I.EQ.J) AI(I,J)=1.D0
      NN=0
      NNN=0
      CALL CTRAK(NN,NNN,N,IU)
      M=N-1
      DO 100 K=1,M
        KX=K
        CALL MAX(KX,L,LL,N,A,AMAX)
      IF (AMAX.EQ.0.D0)THEN
```

```
      IER=1
      RETURN
    END IF
      CALL CTRAK(KX,LL,N,IU)
      DO 30 I=1,N
       AUX=A(K,I)
       A(K,I)=A(L,I)
       A(L,I)=AUX
       AUX=AI(K,I)
       AI(K,I)=AI(L,I)
 30    AI(L,I)=AUX
       DO 40 I=1,N
         AUX=A(I,K)
         A(I,K)=A(I,LL)
 40      A(I,LL)=AUX
      D=A(K,K)
      DO 50 I=1,N
 50   AI(K,I)=AI(K,I)/D
        DO 60 I=K,N
 60       A(K,I)=A(K,I)/D
      MM=K+1
      DO 100 I=MM,N
        V=A(I,K)
        IF (V.NE.0.D0)THEN
          DO 70 J=1,N
 70       AI(I,J)=AI(I,J)-V*AI(K,J)
          DO 80 II=K,N
 80       A(I,II)=A(I,II)-V*A(K,II)
100     END IF

    IF (A(N,N).NE.0.D0)THEN
      DO 120 I=1,N
120     AI(N,I)=AI(N,I)/A(N,N)
        A(N,N)=1.D0
        DO 140 I=1,M
          N1=N+1-I
          DO 140 J=I,M
            N2=N-J
            IF (A(N2,N1).NE.0.D0)THEN
              DO 130 K=1,N
130             AI(N2,K)=AI(N2,K)-A(N2,N1)*AI(N1,K)
            END IF
140 CONTINUE
    CALL CITX(N,IU,AI)
    IER=0
    RETURN
    END IF
    IER=1
    RETURN
    END

    SUBROUTINE CTRAK(L,LL,N,IU)
    INTEGER*2 IU(10)
    IF (L.LT.1)THEN
    DO 10 I=1,N
 10 IU(I)=I
    RETURN
    END IF
    JJ=IU(L)
    KK=IU(LL)
    IU(LL)=JJ
    IU(L)=KK
    RETURN

    END


    SUBROUTINE MAX(K,L,LL,N,A,AMAX)
    REAL*8 DABS,DFLOAT,A(10,10),AMAX
    AMAX=DABS(A(K,K))
    DO 20 J=K,N
    DO 20 I=K,N
    IF (DABS(A(I,J)).GE.AMAX) GO TO 10
    GO TO 20
 10 AMAX=DABS(A(I,J))
    L=I
    LL=J
 20 CONTINUE
    RETURN
    END


    SUBROUTINE CITX(N,IU,AI)
    INTEGER*2 IU(10)
    REAL*8 DABS,DFLOAT,AI(10,10),AUX,AUX1
 10 I=0
 20 I=I+1
    IF (I.EQ.N) GO TO 50
    IF (IU(I).GT.IU(I+1)) GO TO 30
    GO TO 20
 30 L=I
    LL=I+1
    JC=IU(LL)
    KC=IU(L)
    IU(LL)=KC
    IU(L)=JC
    J=0
 40 J=J+1
    AUX=AI(L,J)
    AUX1=AI(LL,J)
    AI(LL,J)=AUX
    AI(L,J)=AUX1
    IF (J.GE.N) GO TO 10
    GO TO 40
 50 RETURN
    END
C
C End Program C
```

PERSONAL KNOWLEDGE AS PRIOR INFORMATION IN VOLUME ESTIMATES

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INTRODUCTION

An individual's prior knowledge of probable volumes is usually not incorporated in sample based volume estimates except to set sample size or define strata. This is the case even if volume estimates for similar areas are available or the forester feels his or her knowledge of the area is sufficient to make an educated guess about the volume.

Bayesian methods can provide a means of incorporating this prior knowledge of volume into a sample estimate. Several authors have demonstrated that information from past or concurrent samples can be used to improve sample estimates. Burk and Ek (1987) combined a current cruise with a distribution of past sample estimates in similar stands to reduce the error in volume estimates. Burk and Ek (1982) demonstrated how Bayesian methods could be used to improve volume estimates for small areas within a larger area inventory by using the larger area inventory results to adjust the small area estimates. Bleier et al. (1986) used aerial photo based volume estimates as prior information for adjusting ground based volume estimates. Other forestry applications of Bayesian methods have been shown by Gertner (1984) and Green and Strawderman (1985).

The intent of this paper is to demonstrate how personal knowledge of probable volume can be combined with traditional volume estimates using Bayesian methodology. This knowledge can be based on past volume estimates in similar timber types or only on knowledge the forester has acquired through experience. Specification of the prior distribution of volume is accomplished using a technique from management science. Combination of the prior distribution of volume per unit area, $g(v)$, and the sample data distribution of volume per unit area, $f(t)$, to arrive at a posterior distribution of volume per unit area, $h(v)$, as

$$h(v) \propto g(v) \cdot f(t)$$

is done using an algorithm described by Burk and Ek (1980).

PRIOR DISTRIBUTION SPECIFICATION

Mathematically, Bayesian methods are not controversial. However, prior distribution specification has been the subject of much discussion ever since Bayesian methods were proposed. The crux of the difficulty is that improper specification of the prior distribution will lead to an incorrect posterior distribution, and, in the case of forest volume estimates, incorrect estimates of volume (Swindel, 1972).

In many fields of study correctly specifying a prior distribution may be difficult or impossible. For example, estimating disease occurrence to develop a prior distribution may be very difficult, and any posterior distribuiton based on the prior may be worthless.

In forestry, however, we have a wealth of information on which to base prior distribution specification. Volume estimates are routinely made for stands across a wide range of site classes

and for many species. Any one company or individual may have access to years of cruise results for any species, site and age class combination desired. Pine plantations of a given age and site are very similar in volume. Growth and yield models can provide probable volumes for prior distributions even if previous cruise results are not available. In addition, most foresters have seen enough trees and stands and cruise results to be able to reliably estimate the probable volumes without explicitly referencing past cruises or growth and yield estimates. Therefore, we do not have a problem with information sufficient to specify a prior distribution of volume for a given situation. All we need is a means of constructing the prior distribution from the information available.

One possible method of constructing a prior distribution is to develop the distribution from all past cruise results in stands similar to the one at hand. The resulting distribution would be discrete with possibly many zeros in the range of the distribution, but it could be used as a prior distribution. However, this requires many previous samples and does not allow the forester's experience to be incorporated in the prior.

The prior distribution construction method proposed is to use whatever knowledge exists, past results or growth and yield estimates or experience, to construct a Beta distribution for the prior. The Beta distribution parameters are estimated using a technique from management science. The particulars of the method can be found in Levin et al. (1986). The form of the Beta distribution used here is

$$g(\mathrm{x}) = \frac{\Gamma(p+q)}{\Gamma(p)\,\Gamma(q)} \frac{(\mathrm{x}-\alpha)^{p-1}(\beta-\mathrm{x})^{q-1}}{(\beta-\alpha)^{p+q-1}} \quad , \quad \alpha < \mathrm{x} < \beta \ , \ p, q > 0 \ .$$

The Beta distribution has several characteristics that make it desirable as a prior distribution. The endpoints are finite; there are no tail probabilites past the smallest and largest possible observations, α and β, respectively. The flexible shape can range from inverse-J to approximately normal to uniform, and is unimodal for most combinations of the parameters p and q. The parameters are easily estimated with only the specification of the minimum, maximum, and most probable values using the method of moments.

METHODS

Beta Distribution Parameter Estimates

In the context of forest sampling for volume estimation, the parameters of the Beta distribution can be estimated by specifying the minimum possible volume per unit area, α, the maximum possible volume per unit area, β, and the most probable volume per unit area, m.

The mean, $\bar{\mathrm{x}}$, and variance, s^2, are then estimated as

$$\bar{\mathrm{x}} = [\alpha + 4m + \beta]/6$$

$$\mathrm{s}^2 = [\alpha - \beta]^2/36$$

The parameters p and q are estimated using the method of moments:

$$p = \left[\frac{\bar{\mathrm{x}}-\alpha}{\beta-\alpha}\right]\left[\frac{(\beta-\bar{\mathrm{x}})(\bar{\mathrm{x}}-\alpha)}{\mathrm{s}^2} - 1\right]$$

$$q = p\left[\frac{\beta-\bar{\mathrm{x}}}{\beta-\alpha}\right]$$

It is important to note that the α and β specified above are the minimum and maximum values for the volume per unit area for the stand being considered. Although individual plot observations may have a wide range, the minimum and maximum volume per unit area would have a considerably smaller range, and would be much easier to choose than the minimum and maximum observations.

Sample Data and Posterior Distribution

The sample data distribution is represented by a t-distribution:

$$f(t) \propto \left[1 + \frac{t^2}{(n-1)} \right]^{-\frac{n}{2}} \quad , -\infty < t < \infty,$$

$$t = \frac{x_i - \bar{x}}{s_{\bar{x}}}$$

where n is the number of sample plots, x_i is the ith observation of volume per unit area from the sample, and $\bar{x}$ and $s_{\bar{x}}$ are the sample mean and standard error of the mean, respectively. This would be the representation of the sample data if the usual estimation procedures were followed.

The posterior distribution can be developed analytically, but a numeric solution along the lines presented by Burk and Ek (1980) is simple, can be implemented on a spreadsheet program, and allows easy calculation of the mean and confidence intervals. This method was used for all examples that follow. Details of the procedure can be found in the cited reference.

EXAMPLE APPLICATION

Sample data to test the effect of prior distribution specification as described above was collected in a natural pine stand and a planted pine stand. Both stands are owned by Westvaco Corporation in Appomattox and Amherst counties, Virginia. In each stand .04 ha plots were located on a 40 x 60 m grid. Diameter at breast height by 2.5 cm classes and total height by 1.5 m classes were measured on all trees 13 cm dbh or greater. Thirty-three plots were taken in the natural pine stand and 50 plots were taken in the planted pine stand. The sample results are shown in table 1.

The specifications for developing the prior distributions are shown in table 2. These specifications were chosen to examine the effect of the various choices of minimum, maximum, and mode on the posterior distribution. Minimum and maximum values were selected to include and exclude the sample mean in each stand. The modal values were chosen as approximately one-third, one-half, and two-thirds the range of the specified minimum and maximum values.

RESULTS

The posterior distribution means and confidence interval widths for each stand and prior distribution specification are shown in table 2.

In all cases the posterior distribution confidence intervals were narrower than the sample data distribution confidence intervals. For α, β intervals centered on the same value (e.g. 59.5, 148.8 and 89.3, 119.0 in planted pine, both of which are centered on the sample data mean) narrower α, β specification results in narrower posterior distribution confidence interval widths, but the reduction is not as great as the reduction in α, β range. As the α, β range moves to

exclude the sample data mean, the posterior distribution confidence interval width becomes narrower. Ranges of α, β well beyond the range of the sample data will have very narrow posterior distribution confidence intervals, but would probably be worthless unless the forester feels strongly that the prior specification is correct. Making the α, β range narrower while maintaining β (5.95, 148.8 and 89.3, 148.8 in natural pine) also reduces the posterior distribution confidence interval width.

Differences in mode specification within an α, β range have little effect on the posterior distribution mean. Posterior distribution confidence intervals are narrower for modal values centered in the α, β range than near the ends of the range, but the width reduction is small unless the α, β range is wide.

For a given α, β range, posterior distribution confidence intervals are narrower for the sample data with the greater variability (59.5, 148.8 in planted pine and natural pine).

CONCLUSIONS

The minimum, maximum, and modal values of volume per unit area can be used to specify a Beta distribution. That distribution can be used as a prior distribution of volume per unit area to develop a posterior distribution using Bayesian methods.

A forester's knowledge of the volume per unit area can be used to choose the minimum, maximum, and mode for specifying the prior distribution. As long as the forester feels his or her knowledge is valid, it can be used to adjust a sample data estimate.

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Table 1. Results for samples in planted and natural pine stands.

	Planted	Natural
Sample size	50	33
	(cubic meters per hectare)	
Mean	103.9	85.3
Standard Error	6.9	10.3
Coeff. of Variation (%)	47.1	69.7
Minimum Observation	0	5.5
Maximum Observation	238.6	219.9
	95% Confidence Interval	
Lower Limit	89.8	64.2
Upper Limit	118.0	106.5

Table 2. Mean, 95% confidence interval lower and upper limits (LL, UL) and percent of posterior confidence interval width as a percent of sample data distribution confidence interval width (% Width) for planted and natural stand samples and prior distribution specifications. Units are cubic meters per hectare.

Min	Max	Mode	Mean	LL	UL	% Width
			PLANTED PINE			
			Sample Data Distribution			
			103.9	89.9	118.0	
Prior Distribution			Posterior Distribution			
59.5	148.8	89.3	102.3	89.1	115.2	93
59.5	148.8	104.1	103.9	91.0	116.9	92
59.5	148.8	119.0	105.6	92.6	118.8	93
89.3	119.0	95.2	100.0	91.7	109.3	62
89.3	119.0	104.1	104.0	95.7	112.4	59
89.3	119.0	113.1	108.1	98.8	116.5	63
119.0	178.5	130.9	123.6	119.3	130.9	54
119.0	178.5	148.8	127.3	121.0	136.3	41
119.0	178.5	166.6	129.2	122.0	139.2	61
			NATURAL PINE			
			Sample Data Distribution			
			85.3	64.2	106.5	
Prior Distribution			Posterior Distribution			
59.5	148.8	89.3	87.4	71.1	105.1	80
59.5	148.8	104.1	86.1	74.8	108.8	80
59.5	148.8	119.0	94.0	77.5	112.4	83
89.3	148.8	101.2	98.4	90.4	111.1	58
89.3	148.8	119.0	104.9	94.3	119.0	49
89.3	148.8	136.9	109.0	96.5	125.6	69

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BIOMETRICAL DESCRIPTION OF STANDS STRUCTURE FOR MOUNTAINEOUS PERMANENT PLOTS IN PORTUGAL

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SUMMARY

The layout of a set of permanent plots to study the structure and yield of the most important forest tree species at the mountainous region of Central Portugal was realized in 1988-89. In these permanent plots, maritime pine (*Pinus pinaster*) has a special importance and particular attention is given to the comparison between its stands and stands of other species, concerning their growth and yield capacities.

As these stands are nearly at the cutting age, it is important the measuring and the detailed study of the natural regeneration, particularly chestnut, oak and pine. In this paper we present a first biometrical description of stand structure of these maritime pine stands, refering to their natural regeneration using some ecological parameters and multivariate analysis methods.

Keywords: natural regeneration; stand structure.

INTRODUCTION

With a total area of about 1 250 000 ha the maritime pine is the most important forest tree of Portugal. Specially frequent along the Atlantic coast north of Tejo river, it also grows at the mountains of Central and Northern Portugal, up to 800/900 meters of altitude. In the National Afforestation Program at about 1940, the maritime pine was the most employed species, which can explain its actual age class distribution, where the class "more than 40" attains more than 106 700 ha (TABLE 1).

TABLE 1. Age class distribution for *Pinus Pinaster* in Portugal according to the National Forest Inventory.

AGE CLASS	AREA(ha)
0 - 9	304 700
10 - 19	66 900
20 - 29	243 400
30 - 39	185 000
>40	106 700
Unevenaged	343 300
TOTAL	1 250 000

Taking in account the usual rotation age for maritime pine between 50 and 60 years, it becomes clear the need of research for a sound silviculture in particular concerning the reproduction methods. Shelterwood and seed-tree reprodution methods should be experimented if the use of the natural regeneration is decided. On the other hand the management and regeneration of stratified mixtures, in particular with the native hardwoods, must be examined when mixed structures are the management goal.

In this report we present the first results of the research project "Biometrical and Silviculture Research for Mixed Stands in Portugal" supported by the Alexander von Humboldt Foundation, Germany and by the Centro de Estudos Florestais (I.N.I.C.), Portugal.

The project began with the layout of a set of permanent plots for the most important forest types in the mountainous region of Central and Northern Portugal and with the characterization of their structure, growth and yield. Particular importance was given to the natural regeneration and development as an opportunity to get mixed structures generally accepted as the most suitable for a multiple use forestry.

MATERIAL AND METHODS

Study Area

The study area 'Cantão das Hortas' lies in Administração Florestal da Lousã, Serra da Lousã, 30 km southeast of Coimbra (*Fig.1*). With a total area of 475.4 ha, Cantão das Hortas was used for agriculture till 1939/40 when the afforestation was decided to protect the soil against erosion.

The altitude of Serra da Lousã attains 1200 meters. The average annual precipitation (P) is 1200 mm with a maximum in January/February and a minimum in July/August. The average annual temperature (T) is 13.6º with monthly maximum

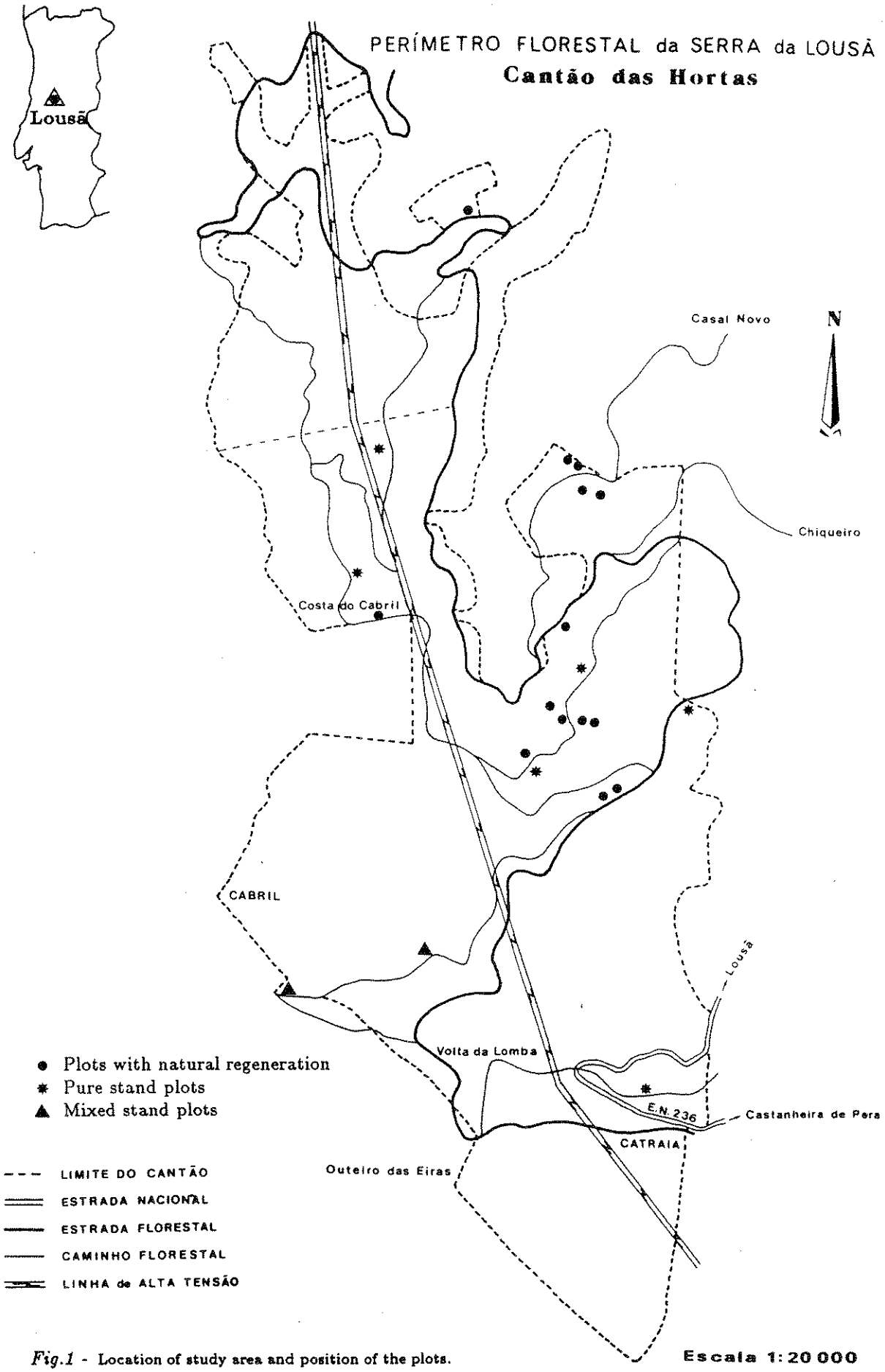

Fig.1 - **Location of study area and position of the plots.**

of 27º C in August. Between the beginning of July and the end of August there exists a dry period of two months ($P < 2T$) and a transition period is found between the middle of June and the end of September ($P < 3T$).

Geologically Serra da Lousã belongs to the Paleozoic Era and the soil parent rock is state clay. The soils are classified as 'litólicos húmicos' or as Cambissols according to the F.A.O. classification (Cardoso et al.,1973). Within the portuguese ecological classification Serra da Lousã chiefly belongs to the Subatlantica (SA) zone, and at the lower altitudes to Mediterraneo-Atlantica (SAxMA) (Albuquerque, 1954). The forest vegetation of Serra da Lousã comprises the chestnut (*Castanea sativa*), the maritime pine (*Pinus pinaster*), the *Quercus pyrenaica*, the yew (*Taxus baccata*) and *Betula celtiberica*. At lower altitudes we can also find the common oak (*Quercus robur*), the cork oak (*Quercus suber*) and the stone pine (*Pinus pinea*).

A complete forest inventory was made in Cantão das Hortas in 1973 (D.G.S.F.A.,1973). The maritime pine occupies 48.6% of the total area while other coniferous get only 17.5%. Only 1.5% of the total area is occupied by hardwood pure stands, while mixed structures were present in 15.6%. Forest roads need 10.2% and 6.6% are not forested.

Field Procedures

In 1988 and 1989 twenty two plots (22) permanent plots were laid out in Cantão das Hortas (*Fig.1*). The plots can be classified in the following groups.

1. Ten plots with understory and natural regeneration layers:
Old stands of maritime pine with understory of sweet chestnut, common oak and maritime pine, and with natural regeneration of all three species (plots nºs 108/1-4; nºs 109/1-4 and nºs 199/1-2). While the maritime pine overstory is about 50 years old, the understory plants can get to 20-25, with heights between 2.5 and 18 meters. As natural regeneration was considered the strata lower with height $< 2.51m$.

2. Four plots with natural regeneration of almost the same species as the old stand and without understory layer:
Two maritime pine plots (nº 206/6 and nº 202/12, the last one has only natural regeneration after clearcutting); one *Castanea crenata* plot (nº 209/2) and one common oak plot (nº 209/9).

3. Six plots with neither natural regeneration nor understory:
Two maritime pine plots (nº 207/4 and nº 206/8); two *Pinus sylvestris* plots (nº 209/1 and nº 213/5); one *Pinus nigra* plot (nº 210/7) and one *Cupressus lusitanica* plot (nº 209/3).

4. Two mixed stand plots:
One plot with *Quercus rubra*, *Betula alba* and *Castanea sativa* (nº 311/10) and one

plot with *Pseudotsuga menziesii*, *Castanea sativa* and others coniferous and hardwoods (n° 311/11).

The field measurements were the BHD (d,mm); tree height (h,m); canopy height (Kra,dm); tree rectangular coordinates ($x/y,dm$); tree classification according to KRAFT (Bkl); four canopy radius (Krr,dm); density using ($5 \times 5m$) horizontal squares, in particular for the natural regeneration and the understory layers and the last 5 years height increment (i_h) for the natural regeneration. For stem analysis 18 dominant trees were felled (11 maritime pine; 2 *Pinus sylvestris*; 1 *Pinus nigra*; 1 *Cupressus lusitanica*; 1 *Castanea creanata* and 2 common oaks). All plots were photographed according to special methodology (Preuhsler,1988).

DATA ANALYSIS

For the biometrical description of stand structure the first group of plots (n°s 108, 109 and 199) were choosen.

It is important to know what species are present, their distribution and the relative degree of abundance of each species; so, the structure of vegetation, floristic composition and some species diversity will be described.

Structure of vegetation

The structure of any kind of vegetation is defined by three components: the stratification of the vegetation, the spatial distribution of individuals and the abundance of each species (Kershaw,1985).

Stratification of vegetation

Stratification of vegetation means the vertical arrangement of species. The method of description of the forest vegetation stratification is due to Davis and Richard (1933-34), and it has been called profile diagram.

In these plots we may consider three layers: the overstory, the understory and the young natural regeneration. The overstory layer is composed only by *Pinus pinaster*, aged about 50, total height about $20m$. They had been seeded. The other two layers have resulted from natural regeneration, and they are similar with regard to the species (*Pinus pinaster, Castanea sativa, Quercus robur* and *Prunus avium*); they are distinguished by the total height of the individuals: if it is $\leq 2.50m$ they belong to the young natural regeneration layer and to the understory layer if it is $> 2.50m$. In respect of age, it is somewhat variable with species and plots. The plants at the

understory layer may have started up to 20-25 years ago; and up to 15 years ago for the young natural regeneration plants.

At two plots, we made narrow rectangular subplots, with $40m$ in length and $10m$ in width. Two profile diagrams were drawn for this work. The following data, per species, were used for

- overstory and understory layer: total and canopy heights, canopy radius and X-Y coordinates;
- young natural regeneration: the number of plants in each $(5 \times 5m)$ square.

Fig.2 shows these profile diagrams, where it is possible to see the vertical arrangement of the vegetation and the species composition of the three layers in two different situations.

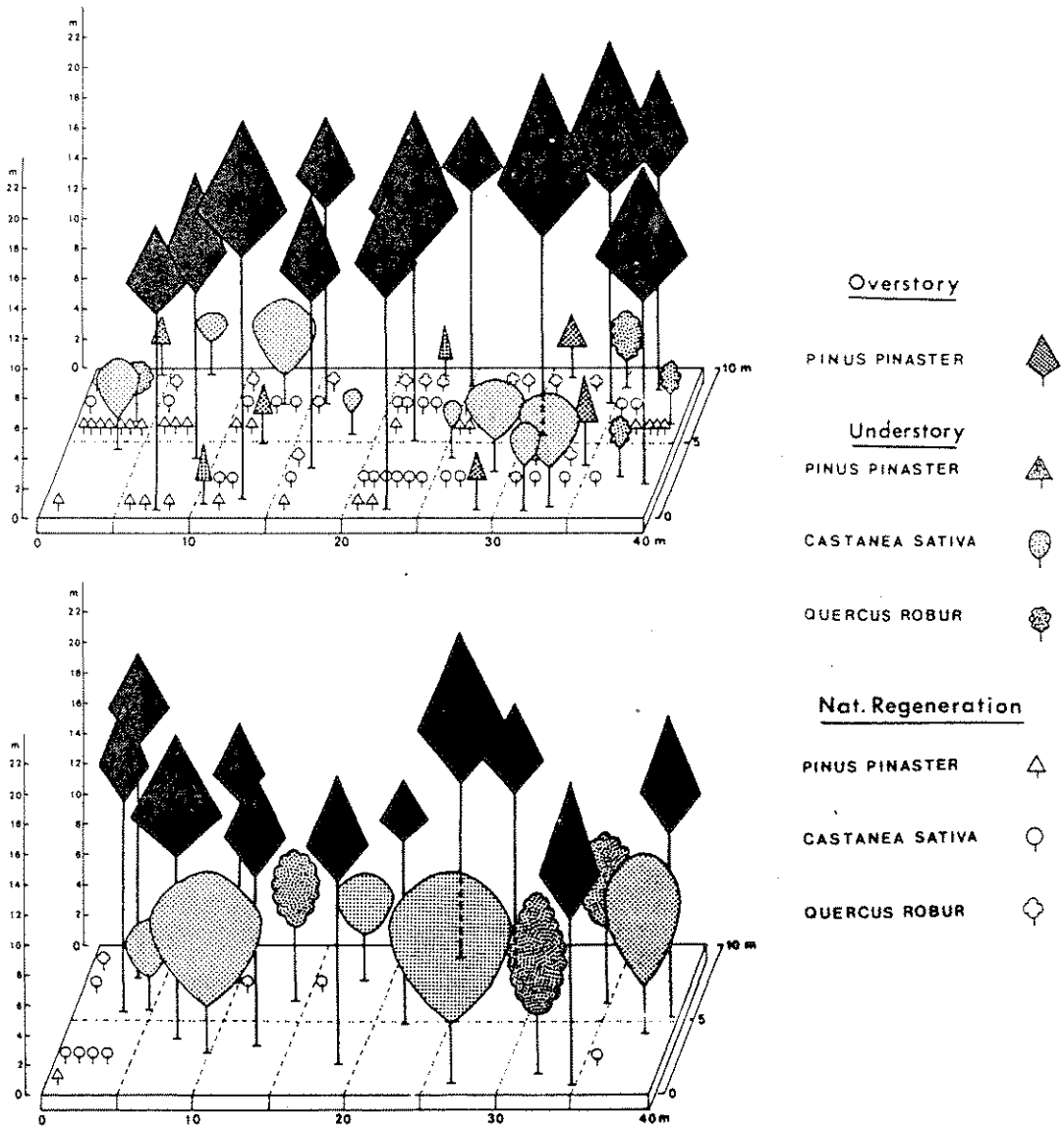

Fig.2 - **Two scaled profile diagrams of subplots with 40m long and 10m width.**

Spatial distribution of individuals

Spatial distribution of individuals means the horizontal arrangement of species. Per species and with the X-Y coordinates of all trees, it was possible to make a detailed mapping of each plot for the overstory and understory layers ($h > 2.50m$). An example of these maps is shown at *Fig.3A*. The horizontal distribution of the plants in the young natural regeneration layer is presented per species, by the number of plants in each ($5 \times 5m$) square, as we may see at *Fig.3B*, where one line in each square represents one plant.

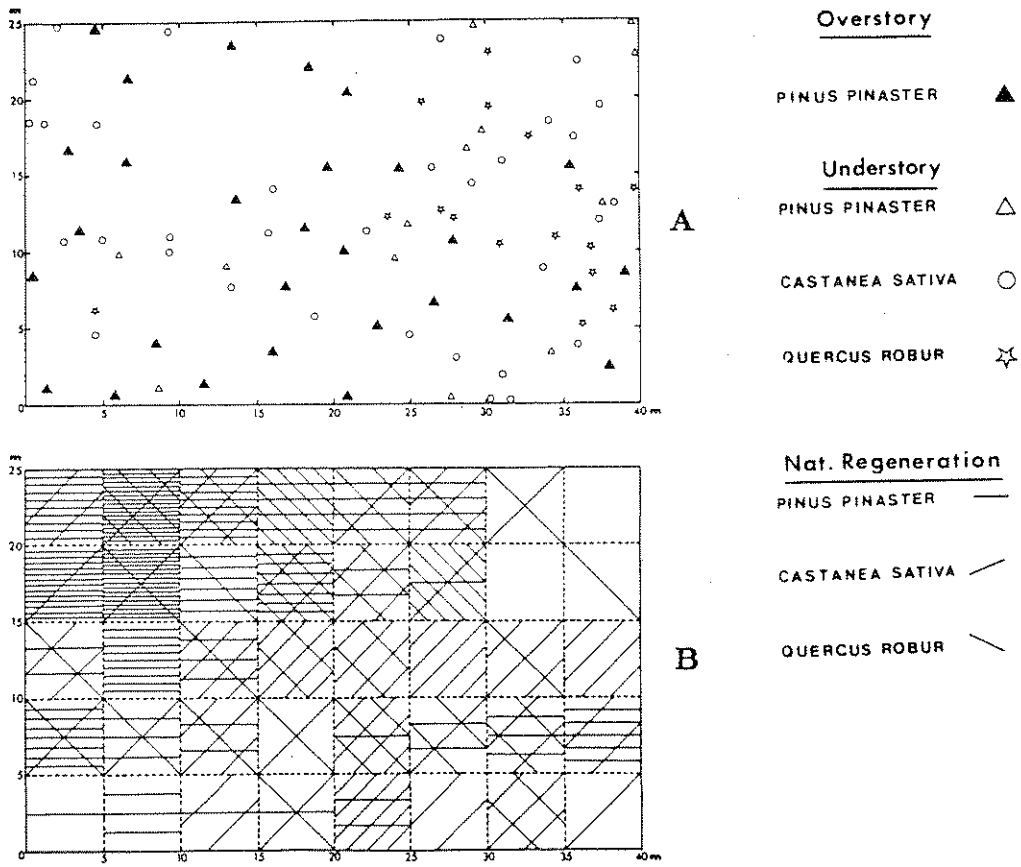

Fig.3 - A) Stems map of the overstory and understory layers of one plot. *B)* Horizontal distribution of the young natural regeneration plants, in each ($5 \times 5m$) square of the same plot.

Relative degree of abundance of each species

In our case we chose, for each plot, the following measures for the abundance of species

- overstory layer: *density*, *frequency* and *basal area*;
- understory layer: *relative density*, *relative frequency* and *relative basal area*;
- young natural regeneration layer: *relative density* and *relative frequency*.

For the understory and young natural regeneration layers, we calculated *Importance Percentage* (IP) values for each species, by using those measures. With these

IP values, we calculated, per species, the *Percentage Similarity* (*PS*) between the understory and young natural regeneration layers.

• Overstory layer

The values of *density*, *frequency* and *basal area* for the overstory layer of all plots are described in TABLE 2.

TABLE 2. Abundance values of *Pinus pinaster* at the overstory layer.

	Plots									
	108				109				199	
	1	2	3	4	1	2	3	4	1	2
Density	29	37	40	31	38	34	37	51	30	30
Frequency(%)	65.0	75.0	72.5	70.0	77.5	77.5	75.0	95.0	62.5	65.0
Basal area(m^2)	2.6	3.2	3.1	2.6	3.5	3.3	3.0	4.0	3.1	2.6

To have an idea about the relationship among the 10 plots, and considering these three abundance measures together, in *Fig.4A* and *Fig.4B* we may see their similarity matrix in a shaded form (higher densities of shading for larger similarities), and the respective dendrogram using the furthest neighbour cluster analysis (Digby,1987).

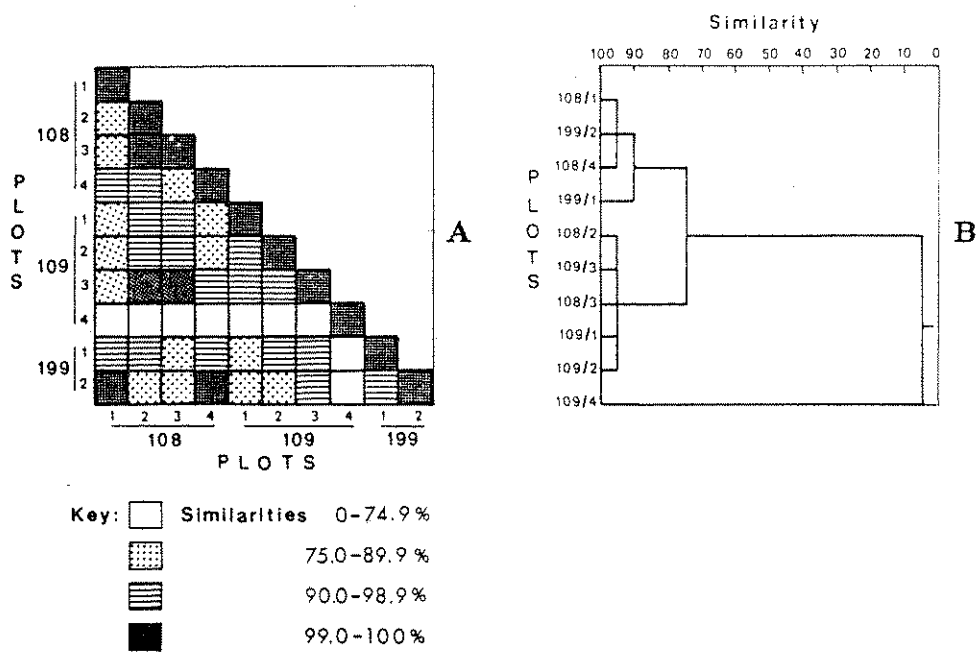

Fig.4 - A) Shaded similarity matrix based on the abundance values of *Pinus pinaster* at the overstory layer of the plots, using the Euclidean measure of distance. *B)* The respective dendrogram derived by furthest neighbour clustering, after reordering the plots.

• Understory and young natural regeneration layers

Importance Percentage (*IP*) value of a species at the understory layer is the sum of *relative* values of *density*, of *frequency* and of *basal area* divided by three; for the young natural regeneration layer *IP* value of a species was calculated as *relative density* plus *relative frequency* divided by two.

Percentage Similarity (PS) between the two layers of a plot is derived from Coefficient of Similarity suggested by Czekanowski, similar to Greig-Smith(1983), and in our case is defined as

$$PS = \sum_{S} min(IP_u, IP_r),$$

where IP_u and IP_r are, respectively, the *Importance Percentages* of each species at the understory and young natural regeneration layers of each plot; S is the total number of species in the two layers.

In *Fig.5* it is shown the IP values and in TABLE 3 you may see the PS values of the species in these two layers.

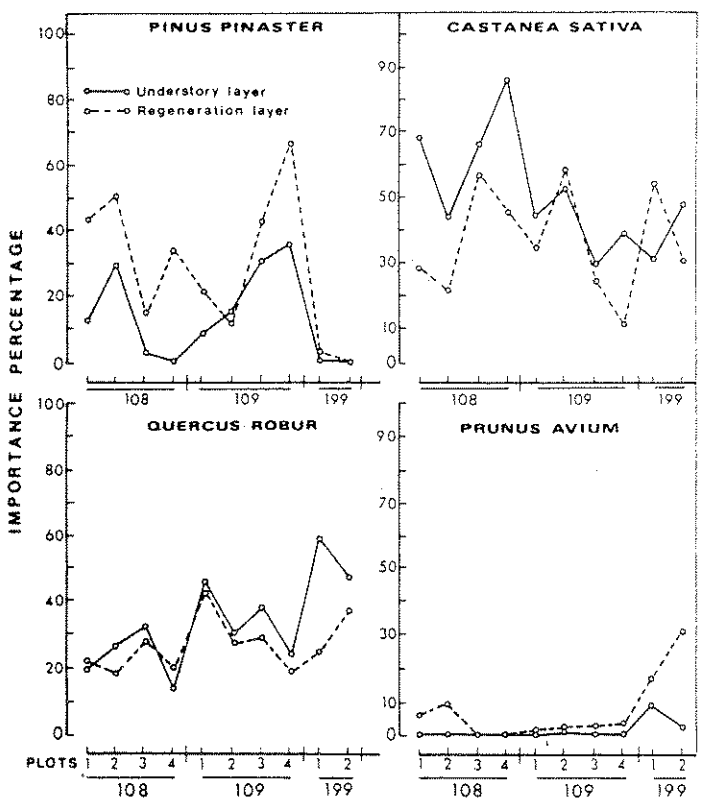

Fig.5 - Importance Percentage values of the species at the understory and natural regeneration layers.

TABLE 3. Percentage Similarity between understory and natural regeneration layers.

	Plots									
	108				109				199	
	1	2	3	4	1	2	3	4	1	2
Percentage Similarity(%)	61	70	88	60	87	93	85	66	66	72

Floristic composition

In our case, the comparison between the understory and young natural regeneration layers in respect of species present was calculated by the *Coefficient of Community (Sorenson's Index)*, that is defined, similar to Pielou(1977), as

$$CC = \frac{2S_{ur}}{S_u + S_r} \times 100,$$

where S_{ur} is the number of species common to the two layers, and S_u and S_r are, respectively, the number of species found at the understory layer, and at the young natural regeneration layer. The results obtained for our plots are described in TABLE 4.

TABLE 4. Coefficient of Community between the understory and young natural regeneration layers.

	Plots									
	108				109				199	
	1	2	3	4	1	2	3	4	1	2
Sorenson's Index(%)	86	86	100	80	86	100	86	86	86	100

Density and Basal Area Diversity

Diversity is a parameter of community structure involving species and their abundance. There are various ways of defining and measuring diversity, but two components are always involved: *richness* (the number of species) and *equitability* (the distribution of individuals or other measures of abundance among the species) (Pielou,1977).

To measure the *diversity* of the understory and regeneration layers in each plot, we used the *Shannon-Wiener's Index* that is derived from information theory (Greig-Smith,1983). It is defined as

$$H'_k = -\sum_{i=1}^{S_k} p_i log_e p_i,$$

where k is the understory or young natural regeneration layer; S_k is the total number of species, p_i the proportional abundance of the ith out of S_k species in the layer k.

To measure the *equitability* we used,

$$E_k = \frac{H'_k}{max(H'_k)} = \frac{H'_k}{log_e S_k},$$

(Pielou,1977) where H'_k is the *Shannon-Wiener's Index* value.

We calculated diversity for density data of the understory and young natural regeneration layers, and for basal area data of the understory layer. The results are described in TABLE 5.

TABLE 5. Density diversity of the understory and natural regeneration layers, and basal area diversity of the understory layer of the plots.

		Plots									
		108				109				199	
		1	2	3	4	1	2	3	4	1	2
Density	*Diversity*(H')										
	:understory	0.90	1.09	0.77	0.42	0.95	1.05	1.07	1.09	0.93	0.82
	:regeneration	1.06	1.07	0.94	1.01	1.08	0.95	1.09	0.66	1.07	1.10
	Equitability(E)										
	:understory	0.82	0.99	0.70	0.61	0.86	0.76	0.98	0.99	0.85	0.75
	:regeneration	0.77	0.77	0.85	0.92	0.78	0.68	0.78	0.47	0.77	1.00
	Richness										
	:understory	3	3	3	2	3	4	3	3	3	3
	:regeneration	4	4	3	3	4	4	4	4	4	3
Basal area	*Diversity*(H')										
	:understory	0.59	0.97	0.36	0.17	0.79	0.73	1.03	0.93	0.72	0.70
	Equitability(E)										
	:understory	0.54	0.89	0.33	0.24	0.72	0.53	0.94	0.85	0.66	0.64

Classification of the plots with some structural variables

To make the classification of the plots in respect of the overstory, understory and natural regeneration layers, we used the Automatic Classification based on the contribution of the factorial axes of the Principal Component Analysis, using the package *SPAD*(1985). We tried to group the 10 plots in 2, 3 and 4 classes, successively, and after that, to choose the best grouping and the most responsible structural variables for that ordination.

We considered the variables: overstory and understory basal area; density of all layers; understory and regeneration richness; Importance Percentage values of all species at the understory and natural regeneration layers; Percentage Similarity and Coefficient of Community; understory and natural regeneration density diversity and equitability; understory basal area diversity and equitability.

With a classification in 4 classes we found:

Class 1 - Plot 108/4 was distinguished from the plots 108/3 and 109/2(these 3 plots belonged to the same class at the classification in 3 classes) because of its high density and basal area of *Castanea sativa* at the understory, and as result of that, the density and the basal area are unequally distributed by the species; it is characterized by a low richness in this layer.

Class 2 - Plots 108/1, 108/2, 109/1, 109/3 and 109/4. The understory layer of these plots are characterized by a low basal area, almost equally distributed by the different species; the density is almost equally distributed too; a high *IP* value of *Pinus pinaster* is an important characteristic of the understory layer of these plots. Their young natural regeneration layers have high density, mainly because the big importance of *Pinus pinaster*; *Castanea sativa* has a low *IP*.

Class 3 - Plots 108/3 and 109/2, characterized for having the same species at the understory and regeneration layers, and a high similarity between these two layers in respect of the *IP* values of the different species; the high *IP* value of Castanea at the regeneration layer is an important feature of this class.

Class 4 - Plots 199/1 and 199/2. The understory layer of these plots is characterized by a high *IP* value of *Prunus avium* and *Quercus robur*, with a low density and a high basal area, that shows that the trees may be older than ones in the other plots. These plots do not have Pinus at the understory layer. Their young natural regeneration layers have a high *IP* of Prunus and the lowest *IP* value of *Pinus pinaster*.

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KONSTRUKTION VON WUCHSMODELLEN FÜR REIN- UND MISCHBESTÄNDE AUS DEN DATEN LANGFRISTIGER VERSUCHSFLÄCHEN

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SUMMARY

Modeling the growth of pure and mixed stands on the data base of long term experimental plots.

For simulating the growth in pure and mixed stands the distance-dependent single tree simulator SILVA 1 is developed. The transition from simulation models for pure stands to models refering to single trees which should also be applicable to multilayer mixed stands requires new methods to register the environmental parameters of the trees and to control their development. Compared to pure stands the neighbourhood relationships in mixed stands vary on a much wider scale. The environmental parameters, controling tree growth and providing important information for growth simulation, are therefor more difficult to obtain.

The essential parts of the growth simulator SILVA 1 are the modules for numerical discription of a trees lateral competition (ε) and its shading by the neighbours (ω). These modules support the construction of models by deriving specific information about growth and structure from the results of long term experimental plots. During the simulation run these modules provide information about neighbourhood-effects and competition of all trees and control their development. They simulate and display the three dimensional stand structure. Thus they are the essential modules of the simulation program SILVA 1.

Presented are the submodels for simulating growth of Bavarian pure and mixed stands of spruce and beech.

KEYWORDS: Mixed-species model, three dimensional approach, shading, Bavarian mixed stands of spruce and beech, SILVA 1 model.

ZUSAMMENFASSUNG

Für die Simulation des Wachstums von Rein- und Mischbeständen wird der abstandsabhängige, einzelbaumorientierte Wachstumssimulator SILVA 1 entwickelt. Der Übergang von den bisher gängigen Simulationsmodellen für Reinbestände zu einzelbaumbezogenen Modellen, die auch für mehrschichtige Mischbestände anwendbar sein sollen, erfordert neue Verfahren, um die Umgebungsparameter der Bestandesglieder zu erfassen und deren Entwicklung zu steuern. Denn im Unterschied zum gleichaltrigen Reinbestand variieren die Nachbarschaftsverhältnisse in Mischbeständen in viel breiterem

Rahmen und die Umgebungsparameter, die das Baumwachstum steuern und eine wichtige Information für die Wachstumssimulation darstellen, sind in solchen Beständen schwerer zu erfassen.

Zentrale Elemente des Wachstumssimulators SILVA 1 sind Programmodule zur numerischen Erfassung der seitlichen Kroneneinengung (ε) und der Beschattung (ω) von Einzelbäumen. Die Progrmmodule unterstützen die Modellerstellung, indem sie neue Leistungs- und Strukturdaten aus Ergebnissen langfristig beobachteter Versuchsflächen erschließen. Während des Simulationslaufes stellen sie Informationen über Nachbarschaft und Konkurrenz aller Bestandesglieder bereit und steuern darauf aufbauend deren Entwicklung. Sie bilden den räumlichen Bestandesaufbau modellhaft nach und sind das Kernstück des Simulationsprogrammes SILVA 1.

Vorgestellt werden die wichtigsten Teilmodelle eines Simulators für bayerische Fichten-Buchen-Mischbestände.

Schlagwörter: Mischbestandsmodell, dreidimensionales Modellkonzept, Beschattung, bayerische Fichten-Buchen-Mischbestände, Wuchsmodell SILVA 1.

WECHSELWIRKUNGEN ZWISCHEN RÄUMLICHER BESTANDESSTRUKTUR UND ZUWACHS DES EINZELBAUMES

Das Wuchsverhalten des Einzelbaumes im Bestand kann nur dann richtig beurteilt und prognostiziert werden, wenn seine räumliche Wuchskonstellation hinreichend genau bekannt ist. Beispielsweise wird eine nachwachsende Tanne im Plenterwald auf Beschattung anders reagieren als auf eine Einengung ihrer Krone von der Seite und wieder anders, wenn beide Faktoren gleichzeitig auftreten.

Die räumliche Umgebungsstruktur des Einzelbaumes und sein Wuchsverhalten stehen in enger Wechselwirkung miteinander. Dieser Tatbestand soll in *Abbildung 1* verdeutlicht werden:

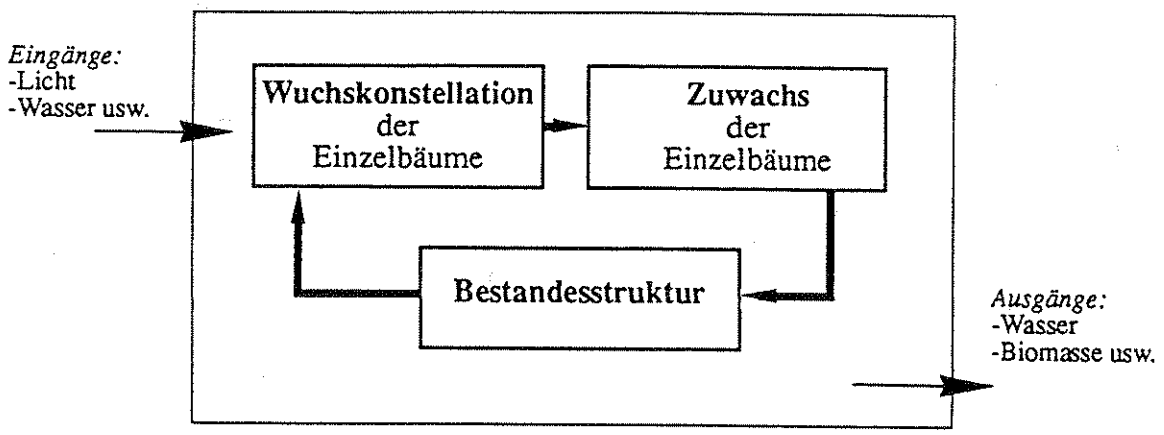


Abb. 1 Schematische Darstellung der Wechselwirkungen zwischen dem Zuwachs der Einzelbäume, der Bestandesstruktur und der Wuchskonstellation der Bäume in einem Waldbestand.

Die Bäume in einem Waldbestand beeinflussen ihre eigene Wuchskonstellation und die ihrer Nachbarn in dem langsam wirkenden Regelkreis: Wachstum des Einzelbaumes-> Bestandesstruktur->Wuchskonstellation->Wachstum des Einzelbaumes. Durch den Zuwachs verändert sich die Struktur von Baum und Bestand. Die Strukturänderungen stellen die

Wuchskonstellation des Baumes und die seiner Nachbarn neu ein, wodurch wieder das Zuwachsverhalten gesteuert wird. Beispielsweise hat ein Baum mit günstigem Lichtangebot aufgrund seiner höheren Stoffproduktion bessere Entwicklungsmöglichkeiten. Er kann sich gegenüber seinen Nachbarn leichter durchsetzen und die Bestandesstruktur und seine Wuchskonstellation zu seinen Gunsten verändern: Jeder Baum wird in seiner Entwicklung von den Bestandesnachbarn gesteuert und wirkt selbst als Regler auf seine Nachbarn.

In einem Wuchsmodell, das für ein breites Spektrum von Waldaufbauformen, für Rein- und Mischbestände, einsetzbar sein soll, kommt dem dargestellten Regelkreis eine zentrale Bedeutung zu. Als Ergebnis dieses Regelprozesses wächst die eingangs erwähnte Tanne im Plenterwald unter günstigen Bedingungen rasch aus dem Unterstand hinaus und hat mit 100 Jahren eine Höhe von 40 Metern erreicht, unter schlechteren Voraussetzungen wird sie bei minimalem Höhenwachstum im Unterstand 100 Jahre alt, wie wir das von Tannen und Fichten kennen, die von MAGIN (1959) und PREUHSLER (1979) untersucht wurden. Das konzipierte Wuchsmodell muß ausreichend flexibel sein, um das ganze Spektrum der Entwicklungsmöglichkeiten darstellen zu können.

Zur Steuerung des Wachstums in einem einzelbaumorientierten Wuchsmodell müssen also für jeden Baum die räumlichen Umgebungsparameter erfaßt und ihre Regelwirkung auf das Wuchsverhalten quantifiziert werden. Eine wichtige Voraussetzung dafür ist die Analyse der räumlichen Umgebungsstruktur des Baumes.

ANALYSE DER RÄUMLICHEN BESTANDESSTRUKTUR

Allgemeine Aussagen über die Struktur des Kronenraumes

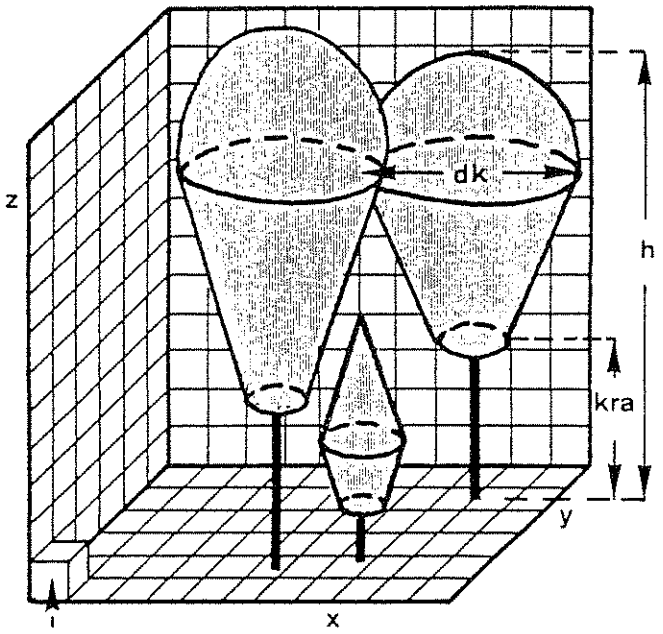
Um die räumliche Struktur des Kronenraumes und das nachbarliche Umfeld jedes Bestandesgliedes charakterisieren zu können, wurde das Programmodul RAUM erstellt.

Tragendes Element des Moduls RAUM ist eine dreifach dimensionierte Matrix, in welche für jeden Kubikmeter Bestandesraum einer Versuchs- oder Testfläche ein Informationsgehalt eingespeichert werden kann *(Abb. 2)*. Beispielsweise wäre für die räumliche Erfassung eines Untersuchungsbestandes mit den Abmessungen 20 m * 20 m und einer Maximalhöhe von 25 m eine Matrix mit der Dimensionierung (20, 20, 25) erforderlich, d.h. der Bestandesraum würde sich aus 10 000 Zellen der Größe 1 cbm aufbauen. Den Zentren der Zellen können beliebige Informationen zugeordnet werden.

Alle Bestandesglieder einer zu untersuchenden Testfläche werden nacheinander in die Matrix eingeführt: Auf der Basis der Stammfußkoordinaten, der Kronenradien, der Dimensionsgrößen Höhe und Kronenansatzhöhe und der baumartenspezifischen Kronenformen wird die räumliche Ausdehnung der Bäume in kartesische Koordinaten umgesetzt und in der Raummatrix lokalisiert.

Aus den Eingangsdaten Baumhöhe, Kronenansatzhöhe und den Kronenradien wird die räumliche Ausdehnung nach den Kronenformmodellen von ASSMANN, BADOUX und BURGER bestimmt: Der Lichtkronenabschnitt wird bei der Fichte als Kegel, bei der Tanne als quadratisches Paraboloid und bei der Buche als kubisches Paraboloid dargestellt. Der Schattkronenbereich wird durch Kegelstümpfe nachgebildet.

Nach dem Trefferprinzip wird für alle Zellenmittelpunkte der Raummatrix bestimmt, von welchen Bäumen, welchen Baumarten und mit welcher Häufigkeit sie getroffen werden. Die Ergebnisse der Trefferabfrage werden in der Raummatrix abgespeichert. Sie enthält dann ein gerastertes, modellhaftes Abbild der tatsächlichen Bestandesstruktur und stellt weitreichende Informationen, z.B. über die Ausnutzung des Kronenraumes durch die Baumkronen und die Präsenz verschiedener Baumarten in unterschiedlichen Höhenbereichen, bereit.



Zelle (1, 1, 1)
mit den Mittelpunktkoordinaten (0.5, 0.5, 0.5)

Abb. 2 Erfassung der räumlichen Ausdehnung eines Baumes durch Trefferabfrage. In einer dreidimensionalen Matrix werden für jeden Kubikmeter Bestandesraum Informationen u.a. über die Präsenz verschiedener Baumarten und die Beschirmungsdichte abgespeichert.

h:	Baumhöhe	kra:	Kronenansatzhöhe
dk:	mittlerer Kronendurchmesser	x, y:	Stammfußkoordinaten

Die *Abbildungen 3 - 5* zeigen die Ergebnisse der Strukturanalyse am Beispiel eines Tannen-Fichten-Mischbestandes bei Starnberg (Versuchsfläche WOL 97/3), eines Fichten-Buchen-Mischbestandes in Zwiesel (Versuchsfläche ZWI 111/3) und eines Fichten-Tannen-Buchen-Mischbestandes in Freyung (Versuchsflächen FRY 129/2) in grafischer Form.

Das Modul RAUM fertigt Aufrißzeichnungen für beliebige Bestandesausschnitte an. *Abbildung 3 (oben)* zeigt die Bestandesstruktur des Fichten-Tannen-Buchen-Plenterwaldes bei Freyung. Hier mischen sich einzelbaumweise und in Gruppenstruktur Fichten, Tannen und Buchen verschiedener Alter und Dimensionen auf engstem Raum. Es tritt z.B. sehr anschaulich hervor, daß sich die Tannen an den Druckstand anpassen, indem sie relativ breite Kronen ausbilden, um so ihre Lichtaufnahme zu erhöhen. Auf *Abbildung 3 (Mitte und unten)* sind zwei Entwicklungsphasen (1954 und 1982) des Fichten-Buchen-Mischbestandes in Zwiesel dargestellt. Zu Beginn des betrachteten Wachstumszeitraumes *(vgl. Abb. 3, Mitte, Bestandesalter 60-70 Jahre)* war die Bestockung relativ dicht und demzufolge der Konkurrenzdruck zwischen den Bäumen groß. Drei Jahrzehnte später *(vgl. Abb. 3, unten)* hat der Bestand einen homogeneren Aufbau; die Stammzahl hat infolge der Selbstdifferenzierung mit konkurrenzbedingten Stammausfällen in der Unter- und Mittelschicht und der hochdurchforstungsartigen Eingriffe in der Oberschicht abgenommen.

Aus den Informationen, die in der Raummatrix abgespeichert sind, können beliebige Horizontal- und Vertikalschnitte durch den Kronenraum erstellt werden.

Abbildung 4 (a) zeigt Horizontalschnitte durch den Kronenraum des Fichten-Buchen-Mischbestandes (ZWI 111/3) in 20 und 25 m Höhe für das Jahr 1982. Die Buche *(hellgrau)* dominiert - nahezu unbehindert durch die Fichte *(dunkelgrau)* - im unteren Kronenraum (Bestandeshöhe 10 bis 20 m). Im mittleren Höhenbereich kommen Buchen und Fichten etwa gleich häufig vor und nutzen den vorhandenen Wuchsraum fast vollständig aus.
In dem Fichten-Tannen-Buchen-Plenterbestand *(vgl. Abb. 4, b)* sind die beteiligten Baumarten über alle Höhenschichten verteilt. Die Buchen sind *hellgrau*, Tannen *grau* und Fichten *dunkelgrau* dargestellt.

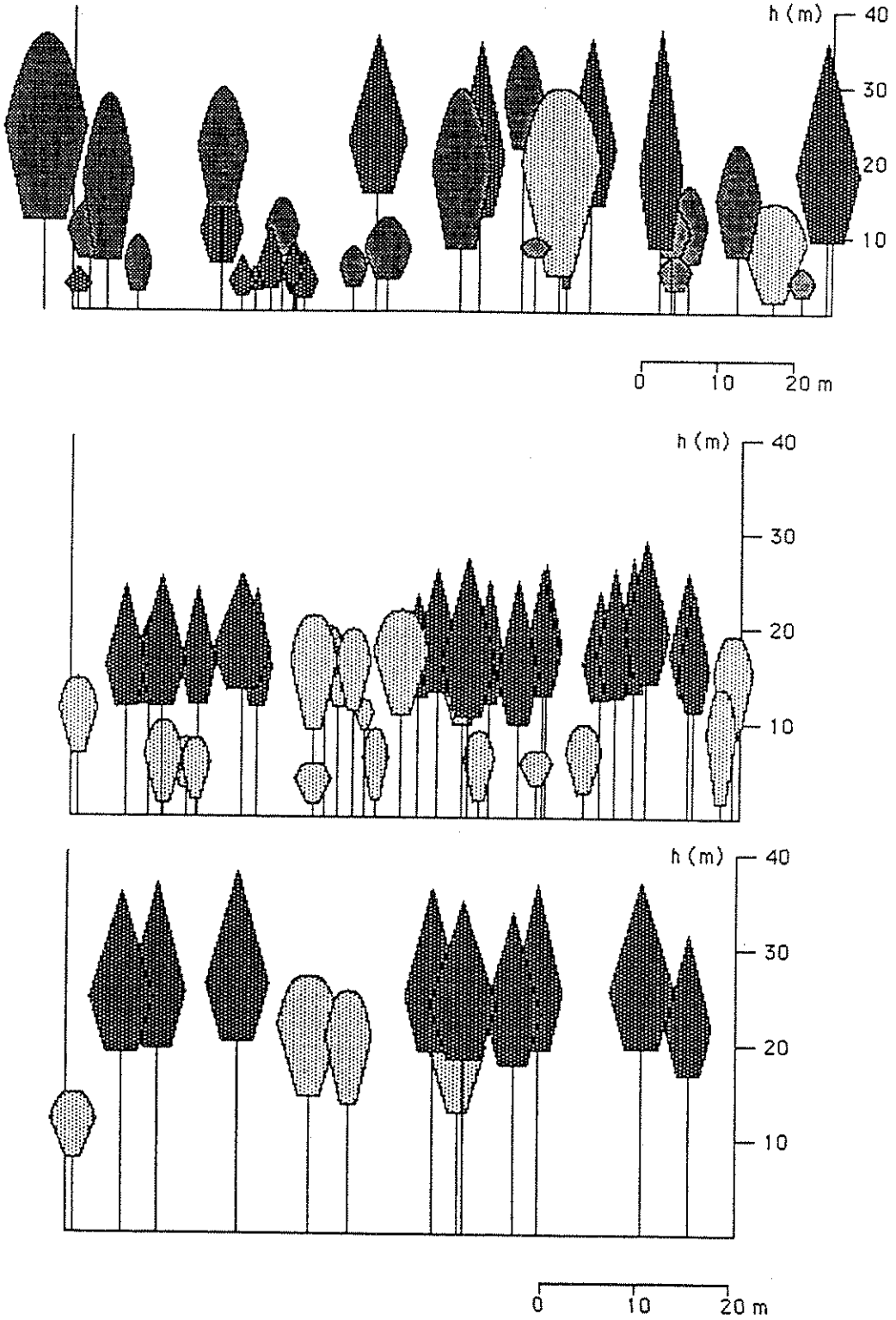


Abb. 3 Bestandesaufrißzeichnungen, ausgeführt vom Modul RAUM des Wachstumsmodells SILVA 1.

oben Fichten-Tannen-Buchen-Mischbestand (Versuchsfläche FRY 129/2) im Jahr 1980 (Zone 20 - 25 m).

Mitte und unten Fichten-Buchen-Mischbestand (Versuchsfläche ZWI 111/3) im Jahr 1954 und 1982 (Zone 25 - 30 m).

hellgrau Buche *dunkelgrau* Fichte
grau Tanne

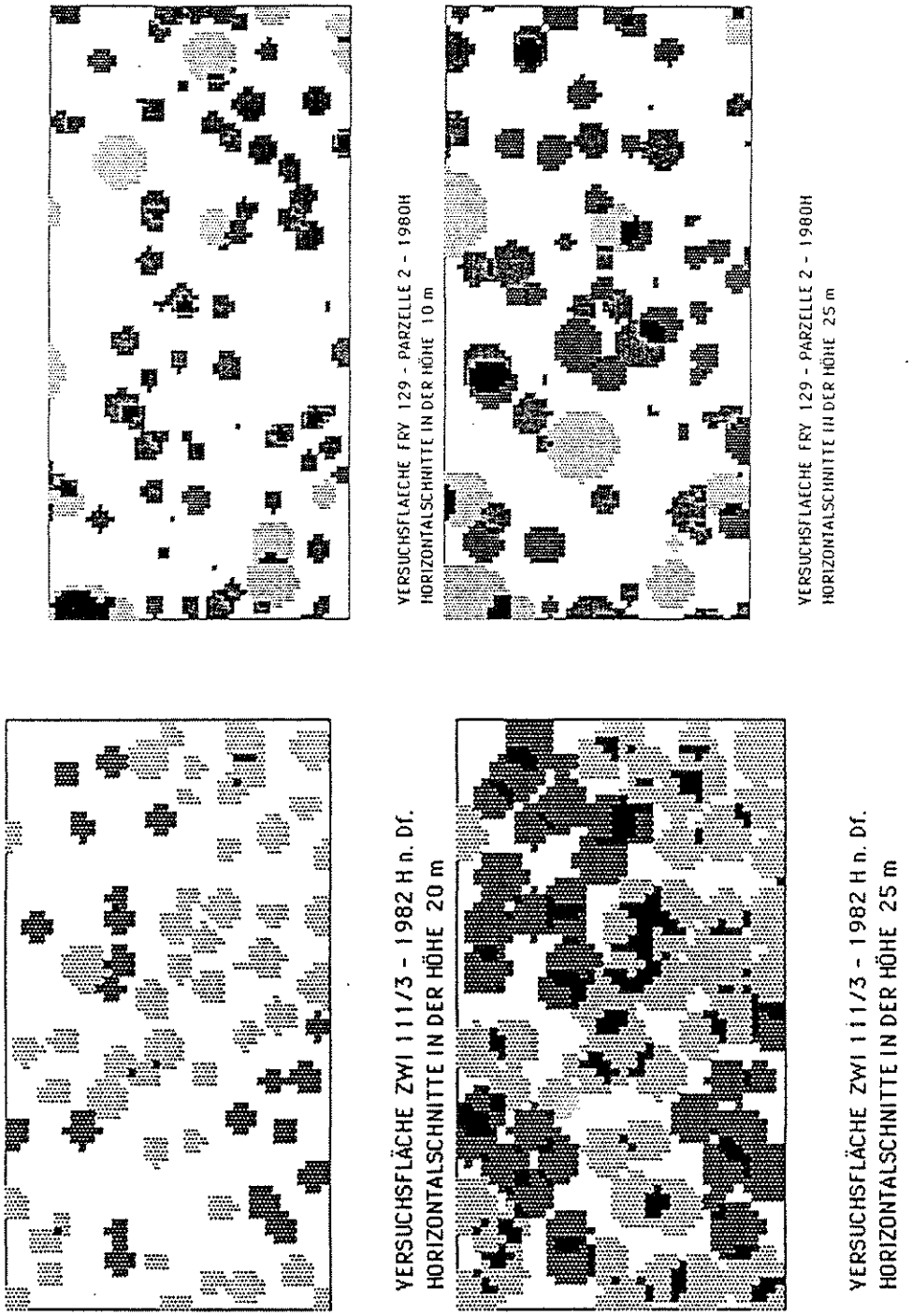


Abb. 4 Horizontalschnitte durch den Kronenraum der Fichten-Buchen-Mischbestandsfläche Zwiesel 111/3 *(a)* und der Plenterwald-Versuchsfläche Freyung 129/2 *(b)*.
ZWI 111/3, Schnitte in 20 m und 25 m Höhe, Aufnahme Herbst 1982 n. Df.
FRY 129/2, Schnitte in 10 m und 25 m Höhe, Aufnahme Herbst 1980.

hellgrau	Buche	*dunkelgrau*	Fichte
schwarz	Mehrfachüberschirmung		

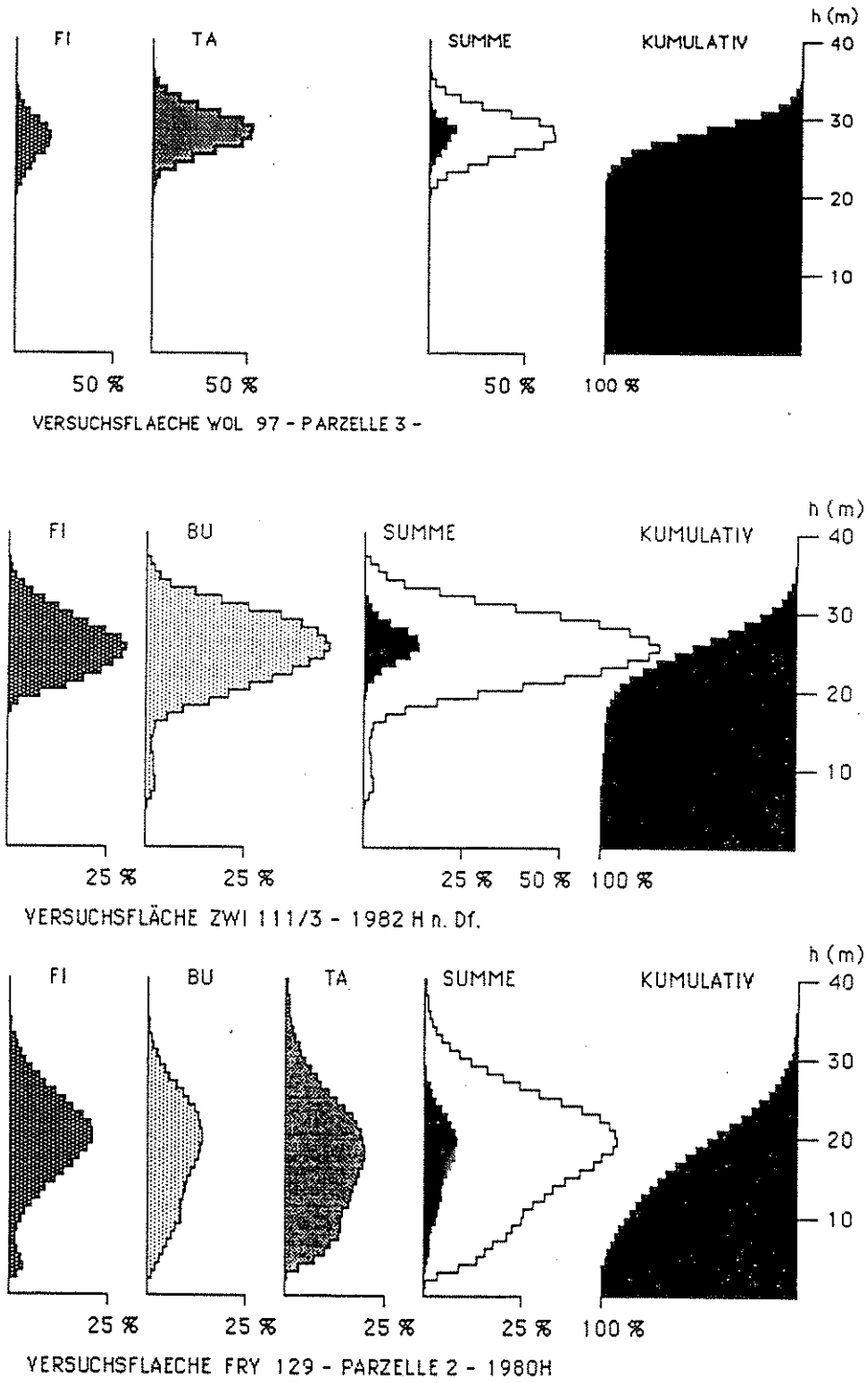


Abb. 5 Strukturanalyse des Kronenraumes mit dem Modul RAUM für einen Tannen-Fichten-Mischbestand (WOL 097/3, Herbst 1986), Fichten-Buchen-Mischbestand (ZWI 111/3, Herbst 1982) und einen Fichten-Tannen-Buchen-Plenterwaldbestand (FRY 129/2, Herbst 1980).
Getrennt nach Baumarten und für alle Baumarten zusammen *(Einfachbesetzungen weiß, Mehrfachbesetzungen schwarz)* sind die prozentischen Anteile am Kronenraum angegeben.
Im rechten Teil der Abbildung ist die kumulative Zunahme des Kronenschlusses vom Wipfelraum bis zum Boden gezeigt.

Aufbauend auf den Ergebnissen der Trefferabfrage kann eine Statistik über die Vertikalverteilung der Kronen im Bestandesraum angefertigt werden *(vgl. Abb. 5)*. Erkennbar wird dadurch, welche Prozentanteile am Bestandesraum die verschiedenen Baumarten in 1-Meter-Höhenschichten einnehmen. Im rechten Teil der Grafiken ist jeweils ergänzend die kumulative Häufigkeit für die Präsenz der Baumkronen angegeben. Die kumulative Häufigkeitsverteilung zeigt den Gradienten der Überschirmungszunahme vom Wipfelraum bis zum Boden an. Dieser Gradient spiegelt in etwa den Kurvenverlauf der Lichtextinktion innerhalb des Kronenraumes wider.

Im oberen Teil von *Abbildung 5* sind die Ergebnisse für einen einschichtigen Tannen-Fichten-Mischbestand (WOL 97/3) dargestellt; die Kronen von Tannen und Fichten befinden sich in einem relativ schmalen Höhenbereich. Im Fichten-Buchen-Mischbestand *(Mitte)* sind die Häufigkeitsverteilungen über der Höhe breiter und zweigipfelig; das kumulative Häufigkeitsdiagramm ist trapezförmig. Im Plenterwald aus Fichten, Tannen und Buchen verteilen sich die Baumarten über den ganzen Höhenbereich, das kumulative Häufigkeitsdiagramm hat Dreiecksform und weist auf einen stärkeren Lichteinfall bis in die unteren Höhenschichten hin.

Aussagen über die Wuchskonstellation des Einzelbaumes

Bisher wurden ausschließlich bestandesbezogene Aussagen über die Struktur des Kronenraumes getroffen, und es sollte hierbei vor allem das Prinzip der räumlichen Strukturanalyse vorgestellt werden. Nach demselben Prinzip der Trefferabfrage können beliebige Informationen über die räumliche Umgebungsstruktur des Einzelbaumes aus Aufnahmedaten erschlossen werden. Wir gehen jetzt also über zur einzelbaumweisen Betrachtung:

Der einzelbaumbezogene Untersuchungsansatz ist in den letzten Jahren insbesondere von ABETZ mehrfach favourisiert worden (ABETZ, 1985 und 1987). PETRI (1966) hat in den sechziger Jahren in diese Richtung vorgedacht und einzelbaumbezogene Begriffe wie "Kontaktzone zwischen Nachbarn", "Überlagerung", "Versperrung" und "Ablenkung" geprägt.

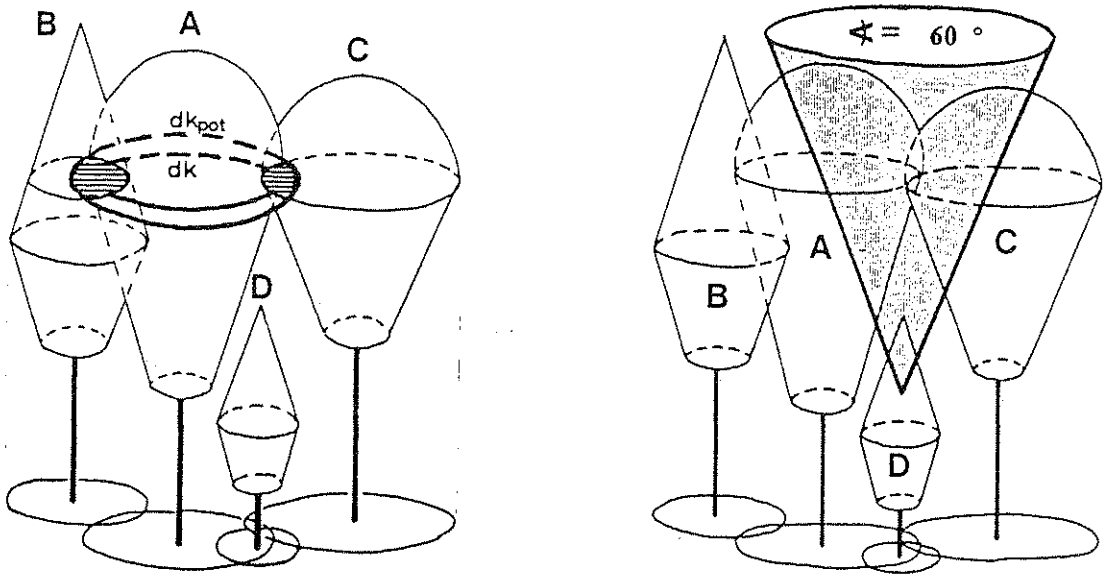
Für die Steuerung des Wachstums von Einzelbäumen haben sich vor allem zwei Strukturparameter als wichtig erwiesen: die seitliche Einengung der Krone und ihre Beschattung durch Nachbarn.

Bestimmung der seitlichen Kroneneinengung ε

Die seitliche Kroneneinengung ε wird nach dem folgenden Verfahren bestimmt *(vgl. Abb. 6, links)*:

1. Für den zu beurteilenden Baum A wird aus einer zuvor abgeleiteten Beziehung der potentielle Kronendurchmesser dk_{pot} bestimmt, der bei optimaler Kronenentwicklung zu erwarten wäre. 2. Von allen Nachbarn der gleichen Kronenschicht wird, getrennt nach Baumarten, die Überlappungsfläche mit dem Zentralbaum A berechnet. 3. Die Bestimmung der Überlappungsflächen erfolgt nach dem oben vorgestellten Prinzip der Rasterabfrage in Höhe der Grenze zwischen Licht- und Schattenkrone des zu beurteilenden Baumes. Die Überlappungsflächen werden in Beziehung zur Gesamtfläche bei potentiellem Kronendurchmesser gesetzt; damit ergibt sich ein relatives Maß für die seitliche Kroneneinengung.

Es ist hervorzuheben, daß diese Überlappungsanalyse auf der Grundlage des räumlichen Modellansatzes in der Baumhöhe durchgeführt werden kann, in dem eine seitliche Einengung durch den Nachbarn wirksam wird (Höhenbereich mit dem größten Kronendurchmesser von Baum A). Würde die Überlappungsanalyse unter Vernachlässigung der Baumhöhe, nur auf Basis der Kronenprojektionskarten erfolgen, so würden in höhenstrukturierten, mehrschichtigen Beständen häufig Überlappungen diagnostiziert, die nicht bestehen *(vgl. Abb. 6, Bäume A und D)*.



ε_{vor} = Kroneneinengung vor Durchforstungseingriff

ε_{nach} = Kroneneinengung nach Durchforstungseingriff

$\Delta\varepsilon$= Durchforstungsbedingte Freistellung

ω_{vor} = Beschattung vor Durchforstungseingriff

ω_{nach} = Beschattung nach Durchforstungseingriff

$\Delta\omega$= Durchforstungsbedingte Auflichtung

Abb. 6 Bestimmung der seitlichen Kroneneinengung ε *(links)* und der Beschattung ω *(rechts)* von Einzelbäumen.
links Bestimmung der seitlichen Kroneneinengung ε von Baum A durch Rasterabfrage.
rechts Bestimmung der Beschattung ω von Baum D nach der Lichtkegelmethode.

Durch Wiederholung des Rechenganges für die Bestandesstruktur nach einem Durchforstungseingriff können ε_{vor}, ε_{nach} und die durchforstungsbedingte seitliche Freistellung $\Delta\varepsilon = \varepsilon_{vor} - \varepsilon_{nach}$ quantifiziert werden.

Bestimmung der Beschattung der Krone ω

Zur Bestimmung der Beschattung eines Baumes konstruieren wir, ähnlich wie SLOBODA und PFREUNDT (1989), in 70 Prozent der Baumhöhe einen Lichtkegel mit einem Öffnungswinkel von 60 Grad *(Abb. 6, rechts)*. Je weiter sich die Nachbarbäume in den Lichtkegel des Zentralbaumes ausbreiten, umso ungünstiger sind sein Lichtangebot und seine Wuchskonstellation. Die Überschirmungsverhältnisse innerhalb des Lichtkegels werden über den Kennwert ω erfaßt.

Die Herleitung des Kennwertes ω baut auf dem bereits beschriebenen Rasterverfahren auf und umfaßt folgende Schritte (stark vereinfacht dargestellt):

1. Für den Zentralbaum werden der Lichtkegel und die in den Kegel hineinreichenden Konkurrenten bestimmt. 2. Von den Zellenmittelpunkten innerhalb des Lichtkegels wird nach einem speziellen Algorithmus abgefragt, ob sie Biomasse enthalten. Wenn das der Fall ist, fließt der Raumpunkt mit in den Beschattungsindex ω ein. Der errechnete Beschattungsindex ω ist ein relatives Maß für die Beschattung eines Baumes durch seine Nachbarn.

Es ist hervorzuheben, daß nicht nur die Beschattung ω, sondern auch ihre Zusammensetzung nach Baumarten ermittelt wird. In einem Fichten-Buchen-Mischbestand könnte das Ergebnis der Beschattungsanalyse z. B. wie folgt lauten: $\omega=3$, mit einer Zusammensetzung von $\omega_{FI}=1$ und $\omega_{BU}=2$; d.h. der zu beurteilende Baum wird stärker von benachbarten Buchen als Fichten beschattet.

Abbildung 7 zeigt die Beschattungsverhältnisse innerhalb des Lichtkegels eines Randbaumes auf einer Versuchsfläche. Dargestellt sind die Schnitte durch den Lichtkegel in den Höhen 13.3 m, 15.3 m und 17.3 m mit den Ergebnissen der Rasterung. Die Linie zeigt die Grenze der Versuchsfläche an. Mit zunehmender Höhe vergrößern sich die Öffnungsweite des Lichtkegels und der zu analysierende Umgebungsbereich von Baum A, der im Kreiszentrum steht. Die Treffer bei der Rasterabfrage auf Beschattung sind für die Fichte dunkelgrau und für die Buche weiß dargestellt.

Die Beschattungsanalyse wird, ähnlich wie bei der Ermittlung von ε, für die Strukturverhältnisse vor und nach Durchforstungsmaßnahmen durchgeführt.

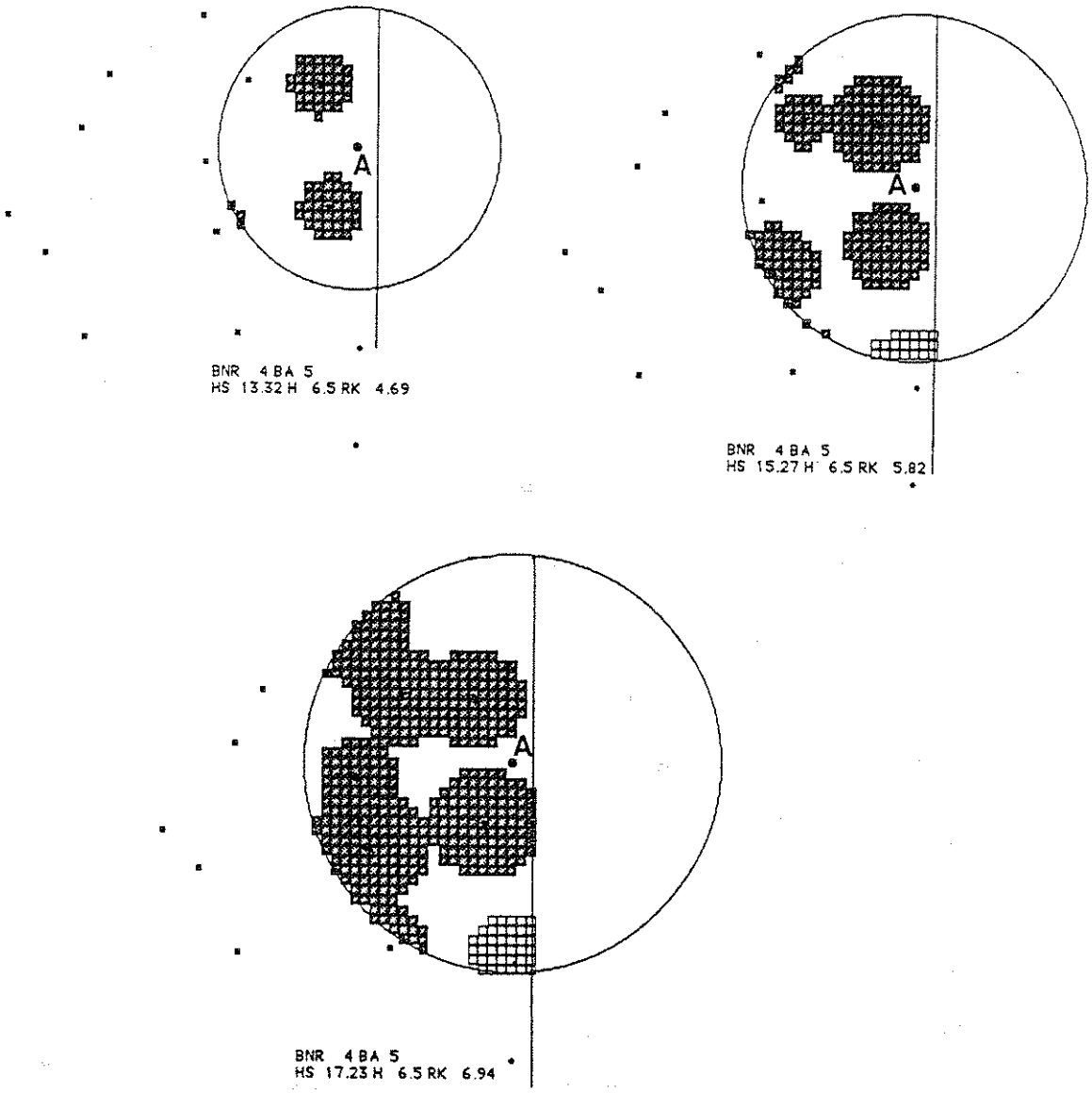


Abb. 7 Beschattungsanalyse durch Rasterabfrage innerhalb des Lichtkegels von Baum A in den Höhen 13.3 m, 15.3 m und 17.3 m. In den Lichtkegel des Baumes A reichen die Kronen benachbarter Fichten *(graue Raster)* und Buchen *(weiße Rasterquadrate)*.

KONZEPTION DES GESAMTMODELLS

Die Strukturparameter ε und ω bilden die zentralen Steuergrößen in unserem Wuchsmodell und haben darin folgende Funktionen: Der Ausgangszustand eines Waldbestandes vor Beginn der Vegetationszeit wird im Modul RAUM in die Raummatrix eingelesen. Anschließend werden für jeden Baum die Kennwerte ε, Δε, ω und Δω zur Charakterisierung seiner Wuchskonstellation berechnet. Die Strukturparameter steuern seinen Zuwachs in der Vegetationsperiode und legen damit den strukturellen Ausgangszustand für das Folgejahr und den nächsten Simulationszyklus fest *(vgl. Abb. 8)*. Sie sind also die Steuergrößen in dem eingangs eingeführten Regelkreis: Wachstum des Einzelbaumes-> Bestandesstruktur->Wuchskonstellation->Wachstum des Einzelbaumes *(vgl. Abb. 1)*.

Die Strukturparameter steuern den Höhenzuwachs, Kronenbreitenzuwachs, die Verlagerung der Kronenansatzhöhe, den Grundflächenzuwachs und außerdem die Mortalität. Die Strukturveränderung eines Bestandes wird somit als Wanderungsbewegung in der Raummatrix nachgebildet.

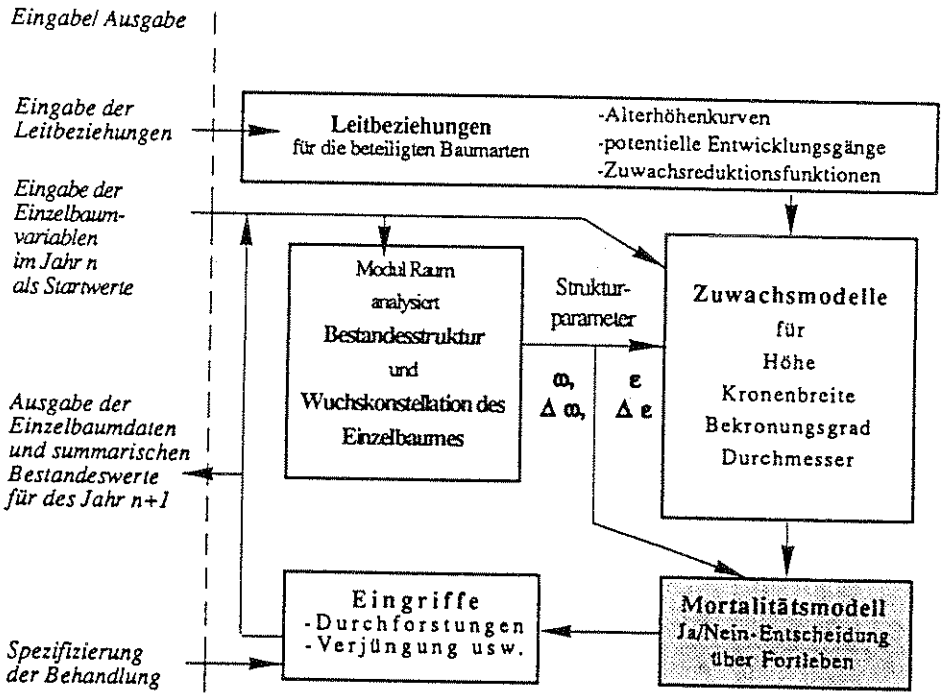


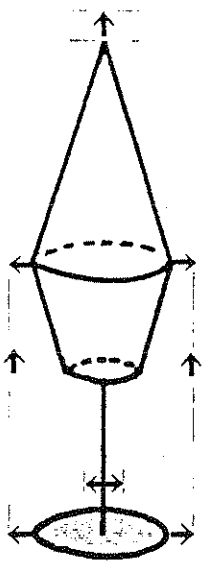
Abb. 8 **Flußdiagramm des Wuchsmodells SILVA 1.**

DATENGRUNDLAGE ZUR PRÜFUNG DER MODELLHYPOTHESEN

Ein Wuchsmodell dieser Architektur wurde für Fichten-Buchen-Mischbestände realisiert. Als Datengrundlage wurden Versuchsflächen in Rein- und Mischbeständen im Harz, Bayerischen Wald und Allgäu ausgewählt. Es handelt sich dabei um sieben Versuchsreihen in Rein- und Mischbeständen aus Fichten und Buchen mit insgesamt 27 Versuchsparzellen der Größe 0.2 bis 0.5 ha. Die Flächen liegen in den Forstämtern Wieda, Zwiesel und Schongau.

Die vorgestellten Verfahren der dreidimensionalen Strukturanalyse wurden zur Gewinnung einzelbaumbezogener Umgebungskennwerte aus den Versuchsflächendaten verwendet. In dieser Hinsicht sind die Informationen, die in unseren Versuchsflächendaten stecken, noch bei weitem nicht voll erschlossen. Für alle Bäume eines Bestandes können die Kennwerte ε und ω Werte ermittelt werden.

Von der Mehrzahl der Bestandesglieder sind der Zuwachs an Höhe und Kronenbreite, die Verschiebung des Kronenansatzes und die Veränderung des Brusthöhendurchmessers für mehrere 5-Jahres-Perioden bekannt. Diese Zuwachsinformationen können mit den Strukturparametern in Verbindung gebracht und zur Verifizierung von Modellhypothesen benutzt werden *(vgl. Abb. 9)*.



$$\hat{zh} = f\,(zh_{pot}, \alpha, \varepsilon, \omega_{FI}, \omega_{BU}, \Delta\varepsilon, \Delta\omega)$$

$$\hat{zkd} = g\,(zkd_{pot}, \varepsilon, \omega_{FI}, \omega_{BU}, \Delta\varepsilon, \Delta\omega)$$

$$\Delta\hat{b}^{o} = i\,(h, zh, \omega_{FI}, \omega_{BU})$$

$$\hat{zd} = h_1\,(\hat{ig}) = h_2\,(g_a, M, V, \omega_{FI}, \omega_{BU}, \Delta\omega)$$

Mortalitätsmodell:

$$\hat{T} = j\,(\varepsilon, \omega, M, V, g_a, \hat{zg})$$

Abb. 9 Modellansätze für die wichtigsten Dimensionsveränderungen des Einzelbaumes - vorläufige Ergebnisse:

Ziel- bzw. Schätzgrößen:		Steuergrößen:	
$\hat{zh}$:	Höhenzuwachs	ω, ωFI, ωBU:	Beschattung durch Fichte und Buche
$\hat{zkd}$:	Kronendurchmesser-Zuwachs	ε:	seitliche Kroneneinengung
$\Delta\hat{\alpha}$:	Veränderung des Bekronungsgrades	$\Delta\omega$, $\Delta\varepsilon$:	durchforstungsbedingte Änderung von ω, ε
$\hat{zd}$:	Durchmesserzuwachs	α:	Bekronungsgrad
$\hat{zg}$:	Grundflächenzuwachs	zh_{pot}:	potentieller Höhenzuwachs
$\hat{T}$:	Mortalität bzw. Überlebensstatus	zkd_{pot}:	potentieller Kronendurchmesser-Zuwachs
		h:	Baumhöhe
		M:	Kronenmantelfläche
		V:	Kronenvolumen
		g_a:	Ausgangsgrundfläche

EINZELBAUMORIENTIERTES WUCHSMODELL FÜR REIN- UND MISCHBESTÄNDE AUS FICHTE UND BUCHE

Die Dimensionsveränderungen des Baumes werden über die auf *Abbildung 9* dargestellten Modellansätze beschrieben. Am Beispiel der Modelle für den Höhen-, Kronendurchmesser- und Grundflächenzuwachs und des Mortalitätsmodells wird gezeigt, wie die Strukturparameter in die Modellgleichungen einfließen und den Zuwachs steuern.

Höhen-Zuwachsmodell

Der aktuelle Höhenzuwachs ($\hat{zh}$) eines Baumes wird geschätzt aus seinem potentiellen Höhenzuwachs zh_{pot}, der unter Optimalbedingungen zu erwarten wäre. Dieser wird bei Abweichung des Baumes von den Optimalbedingungen, die in den Kenngrößen α, ε, $\Delta\varepsilon$, ω und $\Delta\omega$ zum Ausdruck kommen, reduziert *(vgl. Abb. 10)*. Wir werden bei der folgenden Besprechung einzelner Teilmodelle jeweils nur auf den Effekt eingehen, den ε und ω auf die Zuwachsgrößen haben.

Seitliche Einengung der Krone ε und Beschattung der Krone ω

Das Ziel der zweifaktoriellen Beschreibung der Wuchskonstellation über die Kennzahlen ε und ω ist es, horizontale und vertikale Konkurrenz zu trennen und ihre spezifische Wirkungen auf den Zuwachs zu quantifizieren.

Abbildung 10 zeigt für Fichte und Buche die Kombinationswirkung von seitlicher Einengung ε und Beschattung ω auf den Multiplikator hSTEU (hSTEU=1.0 bedeutet potentieller Zuwachs). Zwischen dem Grad der seitlichen Einengung (ausgedrückt durch die Kennzahl ε) und dem Höhenzuwachs von Fichte und Buche besteht eine Optimum-Beziehung: Maximale Höhenzuwächse werden bei mittlerem seitlichen Konkurrenzdruck erreicht. Mit zunehmender Beschattung der Krone fällt der Höhenzuwachs - bei Konstanthaltung aller anderen Einflußfaktoren - exponentiell ab. Der Schattbaumcharakter der Buche kommt dadurch zum Ausdruck, daß ihr Kurvenabfall im Vergleich zur Fichte wesentlich geringer ist *(vgl. Abb. 10, a und b)*. Beispielsweise sinkt der Zuwachs der Buche bei einer Beschattung von ω=1 auf 70 Prozent (hSTEU=0.7) und bei der Fichte auf 40 Prozent (hSTEU= 0.4) des potentiellen Höhenzuwachses (hSTEU= 1.0) ab.

Die ω-Werte für Fichte ω_{FI} und Buche ω_{BU}, d.h. die Beschattung durch benachbarte Fichten und Buchen fließen separat, also mit verschiedenen Parametern, in die Teilmodelle ein *(vgl. Abb. 10, c und d)*. Aus der Grafik kann für Fichten *(c)* und Buchen *(d)* abgelesen werden, wie der Höhenzuwachs reagiert, wenn sich ihr Umfeld ausschließlich aus Buchen ($\omega_{BU}>0$, $\omega_{FI}=0$) bzw. aus Fichten ($\omega_{FI}>0$, $\omega_{BU}=0$) aufbaut; in diesem Fall erfolgt die Ablesung an den zur x- und y-Achse parallelen Basislinien der Regressionsflächen, welche Reinbestandsverhältnisse repräsentieren. Außerdem kann die Wirkung beliebiger Mischungsverhältnisse zwischen Buche und Fichte auf den Zuwachs abgegriffen werden ($\omega_{FI}>0$ und $\omega_{BU}>0$, z.B.: $\omega_{FI}=1$, $\omega_{BU}=3$); die Ablesung der Zuwachsreaktion erfolgt dann bei der entsprechenden Wertekombination.

Im Modell wird also nicht nur berücksichtigt, daß die Zuwachsreaktion auf Beschattung baumartenspezifisch ist (Buche toleranter als Fichte), sondern auch, daß der Beschattungsbeitrag baumartenabhängig ist (Fichte lichtundurchlässiger als Buche). Durch die Zusammenfassung dieser beiden Effekte in den Modellgleichungen wird aus waldwachstumskundlicher Sicht - wie FRANZ es formuliert - "[...] der Reinbestand zum Grenzfall des Mischbestandes [...]".

Höhenzuwachsmodelle für Fichte und Buche:

(e)

$$\hat{z}h = f\,(zhpot,\ \alpha,\ \varepsilon,\ \omega_{FI},\ \omega_{BU},\ \Delta\varepsilon,\ \Delta\omega)$$

$$\hat{z}h = zh_{pot} * \alpha^{\,b0} * b_1 * e^{\,(b2*\varepsilon + b3*\varepsilon^2)} * e^{\,(b4*\omega_{FI} + b5*\omega_{BU})} * (1.0 + b6*\Delta\varepsilon + b7*\Delta\omega)^{b8}$$

(f)

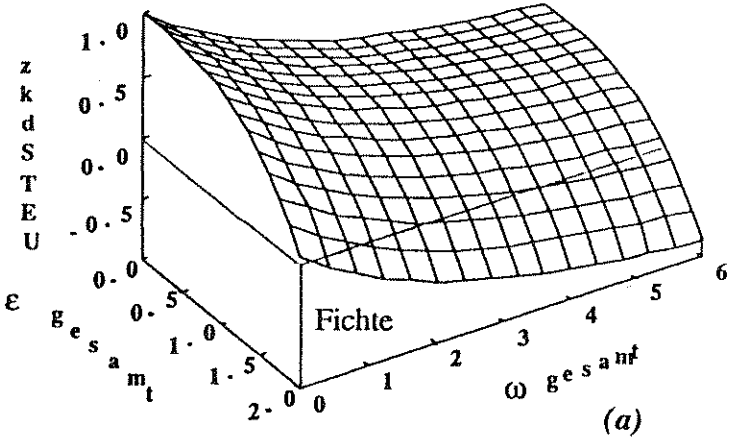
Baumart	Stichprobenumfang	b0	b1	b2	b3	b4	b5	b6	b7	b8	r^2
Fichte	622	0.100	0.840	0.378	-0.208	-0.629	-0.465	0.839	0.113	-0.810	0.47
Buche	568	0.088	0.890	0.247	-0.144	-0.437	-0.208	0.060	0.114	-0.423	0.35

Abb. 10 Einfluß der Strukturparameter ε und ω auf den Höhenzuwachs - vorläufige Ergebnisse:

(a und b) Kombinationswirkung von ε und ω auf den Höhenzuwachs von Fichte *(a)* und Buche *(b)*.

(c und d) Baumartenspezifische Beschattungswirkung von Fichte und Buche auf den Höhenzuwachs von Fichte *(c)* und Buche *(d)*.

(e und f) Funktionsgleichungen *(e)* und Parameter *(f)* der Höhenzuwachsmodelle für Fichte und Buche.



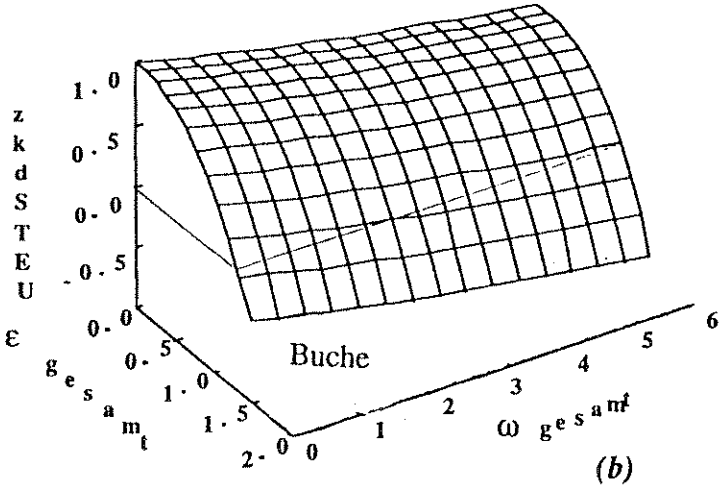


Abb. 11 Einfluß der Strukturparameter ε und ω auf den Kronendurchmesserzuwachs - vorläufige Ergebnisse:
(a und b) Kombinationswirkung von ε und ω auf den Kronendurchmesserzuwachs von Fichte *(a)* und Buche *(b)*.
(c und d) Funktionsgleichungen *(c)* und Parameter *(d)* der Kronendurchmesser-Zuwachsmodelle für Fichte und Buche.

Kronendurchmesser-Zuwachsmodelle für Fichte und Buche:

(c) $\hat{zkd} = g\,(zkd_{pot}, \varepsilon, \omega_{FI}, \omega_{BU}, \Delta\varepsilon, \Delta\omega)$

$$\hat{zkd} = zkd_{pot} * (e^{(b0*\omega FI+b1*\omega BU)} - b2 * \varepsilon^{b3}) * (1.0+b4*\Delta\varepsilon+b5*\Delta\omega)^{b6}$$

(d)

Baumart	Stichproben-umfang	b_0	b_1	b_2	b_3	b_4	b_5	b_6	r^2
Fichte	442	-0.458	-0.338	-0.166	3.043	1.868	0.490	-0.789	0.62
Buche	466	-0.160	-0.076	-0.538	2.438	1.368	0.672	-1.526	0.94

Kronendurchmesser-Zuwachsmodell

Das Kronendurchmesser-Zuwachsmodell gleicht im Konstruktionsprinzip dem Höhenzuwachsmodell: Ausgehend von der aktuellen Baumhöhe wird aus der Beziehung $zkd_{pot} = f(h)$ der potentielle Kronendurchmesserzuwachs zkd_{pot} bestimmt, den Bäume im Bestand bei optimaler Wuchskonstellation erreichen. Dieser potentielle Zuwachswert für den Kronendurchmesser wird dann je nach Abweichung des Baumes von der optimalen Wuchskonstellation auf den aktuellen Zuwachswert zkd reduziert.

Beschattung der Krone ω

Ohne Beschattung (ω=0) beträgt kd_{STEU}=1 (d. h. es würde der potentielle Zuwachs gebildet). Mit Zunahme der Beschattung (ω, bzw. ω_{FI} und ω_{BU} steigend), nimmt der Zuwachs des Kronendurchmessers bei der Fichte stärker, bei der Buche weniger stark, ab *(vgl. Abb. 11)*.

Seitliche Einengung der Krone ε

Außerdem reduziert der Baum seinen Kronendurchmesserzuwachs, wenn die seitliche Einengung ε ansteigt. *Abbildung 11* zeigt den Einfluß von ω und ε auf den Kronendurchmesserzuwachs für Fichte *(a)* und Buche *(b)*. Die Grafiken lassen erkennen, daß die Regressionsfläche ab einer bestimmten baumartenspezifischen ω-, ε-Wertekombination die Nullinie unterschreitet; die Kombination von Beschattung und Einengung bewirkt unterhalb dieser Grenzlinie eine Rückbildung der Krone.

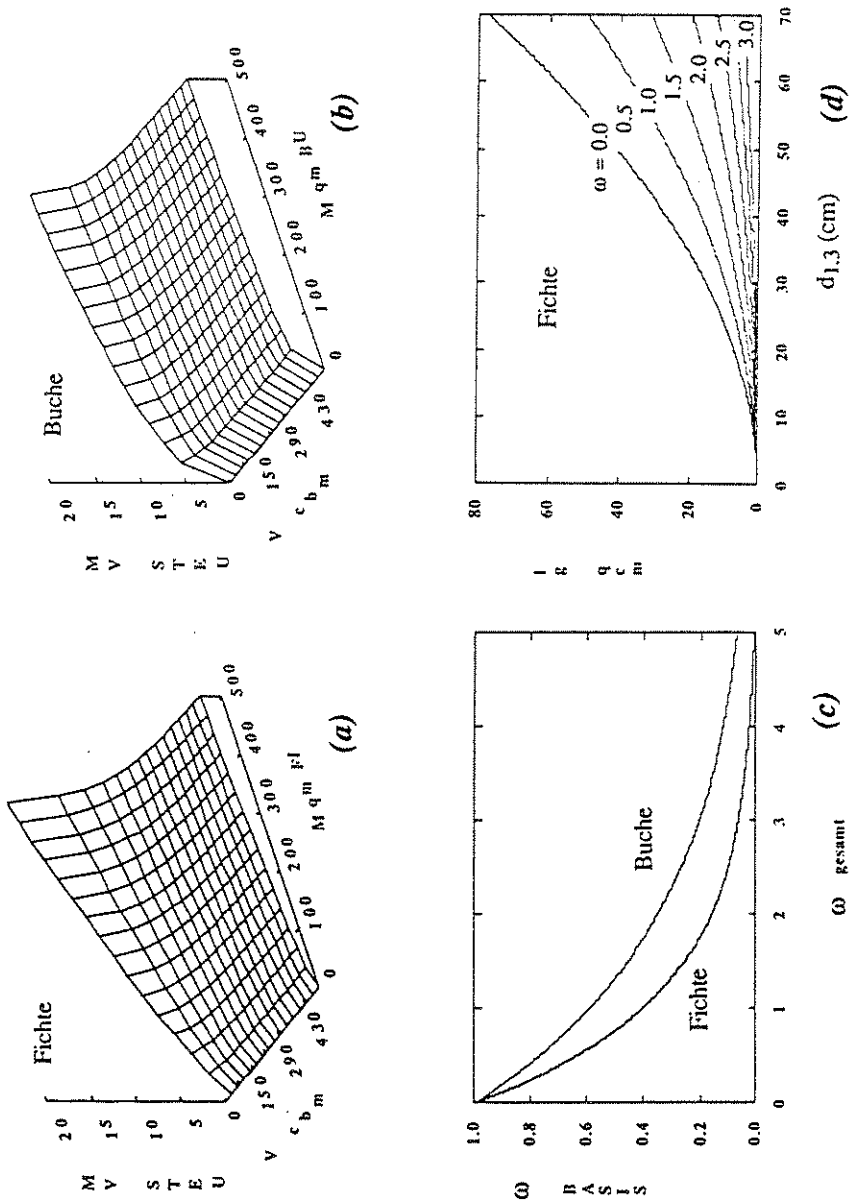
Grundflächen-Zuwachsmodell

Der Durchmesserzuwachs wird indirekt über den Grundflächenzuwachs bestimmt. Bei gegebener Anfangsgrundfläche g_a zu Beginn der Zuwachsperiode wird der Grundflächenzuwachs (ig) durch die Variablen Kronenmantelfläche (M), Kronenvolumen (V), Beschattung (ω) und durchforstungsbedingte Veränderung der Beschattung ($\Delta\omega$) determiniert. Kronenmantelfläche und Kronenvolumen werden auf der Grundlage der eingangs dargestellten Kronenformmodelle hergeleitet *(vgl. Abb. 12)*.

Mit steigender Kronenmantelfläche nimmt der Grundflächenzuwachs nach einer allometrischen Beziehung zu. Diese Zunahme ist umso stärker ausgeprägt, je günstiger das Verhältnis zwischen Kronenmantelfläche und Kronenvolumen ist, d.h. je günstiger die Relation zwischen Oberfläche und Inhalt für die Assimilationsbilanz des Baumes wird.

Mit Zunahme der Beschattung fällt der Grundflächenzuwachs mit baumartenspezifischen Koeffizienten exponentiell ab. *Abbildung 12* zeigt für die Fichte den zu erwartenden Grundflächenzuwachs in Abhängigkeit des Brusthöhendurchmessers für verschiedene Beschattungsverhältnisse (unterstellt wurde dabei eine Kronenmantelfläche von 50 qm und ein Kronenvolumen von 50 cbm).

In alle genannten Teilmodelle fließen außer den Strukturparametern ε und ω zu Beginn der Zuwachsperiode noch die durchforstungsbedingten Veränderungen $\Delta\varepsilon$ und $\Delta\omega$ ein. Hierdurch wird berücksichtigt, daß der Baum auf ε- und ω-Werte, die durch abrupte Freistellung entstanden sind, anders reagiert als auf eine Wuchskonstellation gleicher ε- und ω-Werte, die allmählich entstanden ist.



Grundflächenzuwachsmodelle für Fichte und Buche:

(e) $\hat{zg} = h\,(g_a, M, V, \Delta\omega, \omega_{FI}, \omega_{BU})$

$\hat{zg} = a0 * g_a^{\,a1} * M^{(a2+a3*\ln(V))} * (1.0+a4*\Delta\omega)\; a5*_e\; (a6*\omega_{FI}+a7*\omega_{BU})$

(f)

Baumart	Stichprobenumfang	a0	a1	a2	a3	a4	a5	a6	a7	r^2
Fichte	480	0.1275	0.9510	1.2180	-0.1193	0.6400	-0.5703	-0.9251	-0.7794	0.86
Buche	832	1.4770	0.6699	0.4373	-0.0407	0.0725	1.1690	-0.7486	-0.5231	0.94

Abb. 12 Basisbeziehungen des Grundflächenzuwachs-Modells - vorläufige Ergebnisse:

(a und b) Einfluß von Kronenmantelfläche (M) und Kronenvolumen (V) auf den Grundflächenzuwachs von Fichte *(a)* und Buche *(b)*.

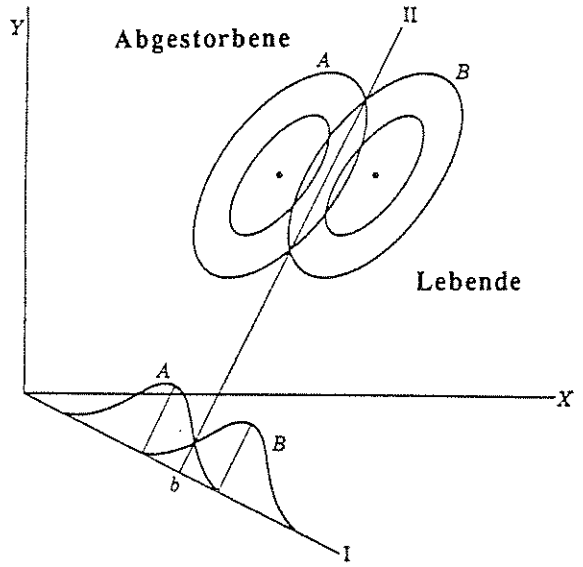
(c) Exponentieller Abfall des Grundflächenzuwachses bei Fichte und Buche mit Zunahme der Beschattung.

(d) Jährlicher Grundflächenzuwachs der Fichte in Abhängigkeit vomAusgangsdurchmesser für verschiedene Beschattungsverhältnisse (ω=0 bis 3.0) bei M=50 qm und V=50 cbm.

(e und f) Funktionsgleichungen *(e)* und Parameter *(f)* der Grundflächenzuwachsmodelle für Fichte und Buche.

Modell für die Mortalität

Das Mortalitätsmodell prognostiziert, ausgehend von der Vitalität und Wuchskonstellation eines Baumes, ob dieser die folgende Wuchsperiode überlebt oder abstirbt. Diese "Zwei-Gruppen-Klassifikation: abgestorben/lebend" erfolgt über baumartenspezifische Diskriminanzfunktionen, die auf der Basis von Versuchsflächendaten entwickelt wurden. Der Variablensatz Beschattung ω, seitliche Einengung ε, Kronenmantelfläche M, Kronenvolumen V, Grundfläche g_a und Grundflächenzuwachs ig gewährleistet in 93 bis 95 Prozent der Fälle (Angaben für Fichte bzw. Buche) eine korrekte Gruppenzuordnung.



FISHER' sche Klassifikationsfunktionen für die Gruppen "Lebend" und "Abgestorben":

Fichte

$$K_{leb}=-26.171+24.377*\omega+5.534*\varepsilon+0.410*M-0.219*V-2.690*g_a+1.743*\hat{zg}$$

$$K_{tot}=-21.909+41.542*\omega+0.489*\varepsilon+0.232*M-0.091*V-5.096*g_a+2.713*\hat{zg}$$

Buche

$$K_{leb}=-10.996+1.311*\omega+3.519*\varepsilon+0.087*M-0.071*V-0.663*g_a+1.540*\hat{zg}$$

$$K_{tot}=-5.052+1.762*\omega+1.360*\varepsilon+0.096*M-0.046*V-1.514*g_a+0.744*\hat{zg}$$

Allgemeine Zuordnungsvorschrift für den Überlebensstatus:

$$\text{Überlebensstatus} = \begin{cases} \text{lebend, wenn } K_{leb} \geq K_{tot} \\ \text{tot, wenn } K_{leb} < K_{tot}. \end{cases}$$

Abb. 13 Mortalitätsschätzung über lineare Diskriminanzfunktionen - vorläufige Ergebnisse:
oben Zwei-Gruppen-Diskriminanzanalyse zur dichotomen Klassifizierung lebend versus tot in schematischer Darstellung (nach COOLEY und LOHNES, 1971, S. 245).
Mitte FISHER' sche Klassifikationsfunktionen der Gruppen "Lebende" und "Abgestorbene" für Fichte und Buche (92.5 % der Fichten und 94.7 % der Buchen werden korrekt zugeordnet).
unten Allgemeine Zuordnungsvorschrift.

SCHLUSS

Durch Kompilation der Teilmodelle entsteht ein Wachstumssimulator für Fichten-Buchen-Mischbestände. Mit diesem kann das Wuchsverhalten bei verschiedenen Mischungsanteilen und Mischungsformen studiert werden, und es lassen sich verschiedene Durchforstungseingriffe simulieren. Der Wachstumssimulator soll die Grundlage zur Leistungstabellierung für Mischbestände bilden.

Der Modellansatz stellt einen Mittelweg dar zwischen den bestandesbezogenen, statistischen Wuchsmodellen ASSMANN/FRANZ'scher Prägung (vgl. ASSMANN/FRANZ, 1965) und physiologisch-erklärenden Modellen von SLOBODA (1989), MÄKELÄ (1986) oder MÄKELÄ und HARI (1986). Aufgebaut wird auf der Forschungslinie von BOTKIN (1972), MONSERUD (1975) und MITCHELL (1975) und neueren Arbeiten u.a. von KOOP (1989), KRUMLAND (1982), STERBA (1989), WENSEL und KOEHLER (1985).

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MEASURING YIELD DATA ON LONG TERM EXPERIMENTAL PLOTS OF MIXED STANDS WITH NATURAL REGENERATION

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SUMMARY

Mixed stands investigations need a substantial higher complexity and intensity of recording the data of stand structure than in pure stands. If natural regeneration is also included, there apears a second and completely different kind of stand information. But nevertheless the results of old stand and of regeneration analysis should be compareable and should show the relationship between them.

In plots of mixed stands the complete measuring of all trees in the old stand as well as in the regeneration enables the discovery of a most accurate testimony of present structure and change by growing or treatment.

The data of old stand single trees are basic for developing and testing growth models - for example by neighbouring or concurrent parameters. They also allow social stratification or performance-orientated groups in the stand.

In the natural regeneration the size of mapping out is predicted by the size of canopy of old trees and also by practicability and repetition.

Some experiences and methods are shown at the results of a long term observation of the mountain forest mixed stand plot RUHPOLDING in Upper Bavaria, Germany.

Keywords: Mixed stands, natural regeneration, stands structure, long term yield sample plots, stocking map

INTRODUCTION

Mixed stands contains at least two species of trees. Often they are unevenaged, within one species as well as between them. The mixed stands species may compete for the best position in the productive main stand layer. But in the same way they may complete their

growing rhythms, or a serving species may accompany a dominant species with high productivity and definite main influence on the stand structure.

The **portion of mixture** shows the part of mixed species at the total crop. Usually it is expressed as percentage of numbers of trees, basal area, volume or increment, but also of canopy or growing space. It may be referred to the whole stand as well as to a part of it like layers of age or height or social tree classes and their productivity.

The **type of mixture** is predicted by the spatial distribution of species. Single tree mixture and small or greater groups of mixed species are normaly combined in the same stand.

The **structure of areal distribution** in mixed stands is mostly determined by different ages of the species. It may also be influenced by the pretention to site, growing space and light and by the different growing rhythms of the species. It may be consistent during the whole life of a stand and may be characteristic of it, but it also may change the mixture towards a pure stand by competition.

Therefore an evenaged pure stand seems to be the extreme variation of a "normal" unevenaged mixed stand (FRANZ, 1989).

The importance of mixed stands for a high stability of ecological silviculture was postulated already by Karl GAYER (1878), Professor of silviculture at Munich. But in his time yield science had had plenty to do to find out the rules of growings in pure stands. The more complicated question for growing systems in mixed stands was started half a century later. The investigations of WIEDEMANN and his colleagues found their expression in first yield tables of mixed stands in the time between 1939 and 1946. But these are only "one side stepping, univariate models" for special relationships of growing (FRANZ, 1987) of mostly two evenaged species at fixed mixture and treatment. The management of these old trials was close to the old thinning experiments and was not able to get the variety of mixed stand structure and growing.

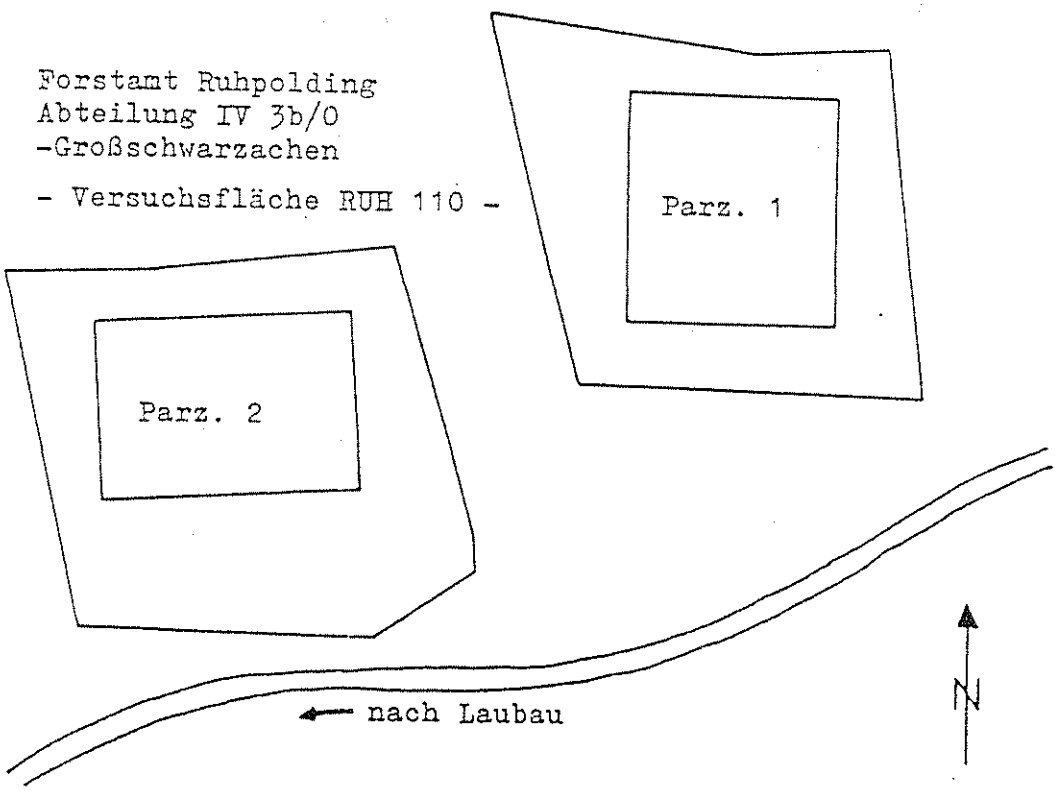


Fig. 1: Site plan of the mountain mixed forest stand trial with natural regeneration RUH 110, including the former total plot area from 1953 and the new areas for intensive measurements of 1977.

At the beginning of the fifties MAGIN (1959) started his investigations in mixed, unevenaged and structured mountain forest stands in the Bavarian Alpes. These had been virgin forests and his aim was to get first yield information about natural growings in the primeval stands without human influence.

ASSMANN (1954, 1961) extended the question to the whole life of man managed forest stands from the regeneration up to harvesting time. He attached importance to the "overlapping period", which was defined by him for that time in the life of a stand, when the natural regeneration is already growing below the shelter of the old stand.

His followers and those of his successor FRANZ at Munich established some research plots for this question; for example JOHANN (1968) in natural regeneration stands of spruce of "Laugna" or PREUHSLER (1979) in mountain mixed forest stands of spruce, fir, beech and maple with natural regeneration in "Kreuth".

One of the first long term trials with natural regeneration in mixed stands of Bavaria was started by ASSMANN in the Bavarian forest district of RUHPOLDING at 1953. His aim was to find out the differences within the growing of natural regeneration at old stands with different density.

LAYOUT OF THE RESEARCH PLOT

ASSMANN installed two parcels of 0.4 ha and 0.5 ha. The parcels included the whole stands, surrounded by a forest road, other stands with different structure or age and a forest aisle between them. **Fig. 1** shows the nonregular layout of the plot.

At the following measurement of 1977 new intensive measuring areas had been installed in the middle of the old plots for a total complete recording of the old stand and the natural regeneration. This had been necessary because:

- The old plots have had no enclosuring strips. Therefore a bordering sphere was influenced by the neighbouring areas.
- It was not possible to reconstruct the corners of the old plot with complete accuracy. This is of high importance particularily for measuring the regeneration. Simple geometric figures should be the best lay out of plots. To minimize the surrounding influence the circumference should be as small as possible, which means a circle. But in this case at each recording the area must be reestablished and the transformation from the slope to the horizontal area is somewhat difficult. Beside a square the rectangular form is practicable, while the basic line should follow the horizontal line. The proportion of the bordering lines should be 1 : 2 in maximum.
- To design a stocking map it is necessary to include those trees, which are standing outside the area but with their crowns reaching inside (bordering trees) and influencing the canopy.

The size of plots is predicted by the aim, the plane and the duration of an investigation and mostly by stand structure. The common proving of spatial distribution of old stand and regeneration plants in mountain mixed forest stands by using the CHI-SQARE-DISTRIBUTION - test shows distinct clusterings within horizontal as well as vertical strata (PREUHSLER, 1979, 1981, 1989/1). To enable approximate true answers on the question of stand structure, the minimum size of an intensively measured old stand with regeneration is about 0.15 ha. If the stand shows more diversity in structure - like "Plenterwald" - it may bee 0.5 up to 1.0 ha.

CALCULATING STAND STRUCTURE

Within 35 years of observation the two parcels of the trial plot had been measured seven times, completed by three additive measurings. **Tab. 1** shows the multiple variety of these activities with 16 different measurements.

Tab. 1: Measurements of the mixed mountain forest stand plot RUHPOLDING (RUH 110) from 1953 to 1988

Measurement	Year (thick = yield measurement of the plot)									
	1953	1954	1957	1959	1960	1963	1971	1977	1982	1988
Installing the plots										
Parcel 1 = 0.4140ha	X									
Parcel 2 = 0.4664ha	X									
Intens.Parcel 1 = 0.1596ha								Y		
Intens.Parcel 2 = 0.1488ha								Y		
Main stand										
Diameter	X		X			X	X	X	X	X
Height	(x)					(x)	(x)	Y	(y)	Y
Canopy height								Y		Y
Tree coordinates								Y		
Crown radius (4x)								Y		Y
Stump registration								Y		
Age of stumps	(x)				(x)			Y		
Cutting for regeneration			X			X	X	X	X	X
Scaling			(x)					(y)		(y)
Stem analysis			x					(y)		(y)
Tree classification	(x)									Y
Forest decline										Y
Regeneration										
Density (number/species)	(x)	(x)	(x)	(x)				Y		Y
Height classes	(x)	(x)	(x)	(x)				Y		Y
Height increment								(y)		(y)
Photographic documentation								Y		Y

total measurement = **X** great old area **(x)** = representative measurement
total measurement = **Y** Intensive measuring area **(y)** = representative measurement

In 1988 JAKOBI with his diploma work did a summarized evaluation of all measurements since 1953. The developement of yield data in old stand and natural regeneration is described in his results. During all measurements only the diameter was obtained consequently. The total measuring of the other yield data from all trees was done for the first time at the intensive areas in 1977 and repeated in 1988.

Some graphical descriptions for this mixed stands will be shown.

Main Stand

The structure of stands is based on horizontal and vertical components, which together illustrate the growing conditions. The object to bee measured is the single tree, but it is not possible to look at it in isolation, as it is the result of the surroundings as well as its influences on the neigbourhood itself.

The primary simple measurings are tree height and canopy height and their changing by growing. At **Fig. 2** the height curves of the plot of RUHPOLDING characterize in a clear way the concurrent growing of spruce and fir. The shift of the curves between 1953 and 1988 and especially the height increment of single trees, influenced by the spacing effect is not calculable: The total measurement of all tree heights was only done in 1977 and 1988 at the intensiv areas. At the other terms a representative group of trees which always changed its composition by dieing or by cuttings had been measured. This is the reason, that complex data of single trees (for example the h/d ratio) as well as concurrent and neigbourhood data are not available for the whole time.

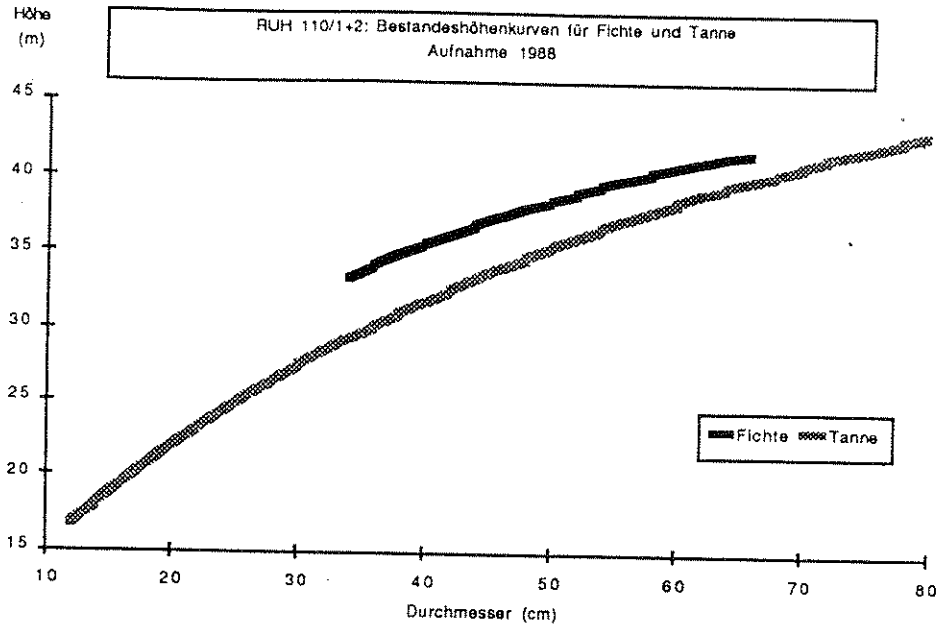


Fig. 2: Height curve of spruce and fir of the trial RUH 110, measurement of 1988.

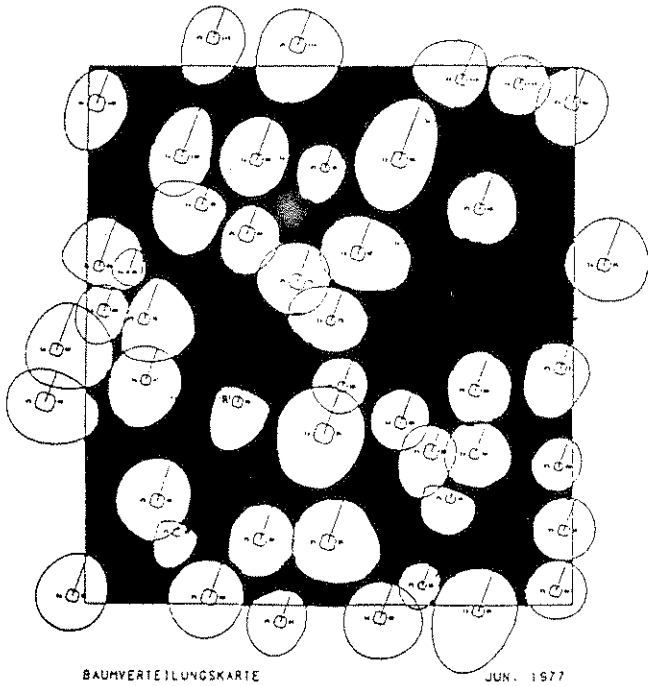
The horizontal structure is shown at the canopy maps at **Fig. 3**. They are designed with the data of the tree coordinates, canopy radius and diameter. In our mountain forest plots we take only four radius according to the slope because of the difficulties of measuring at the slope. They will be fitted graphically and also calculated for the crown cover by four quarters of ellipsis.

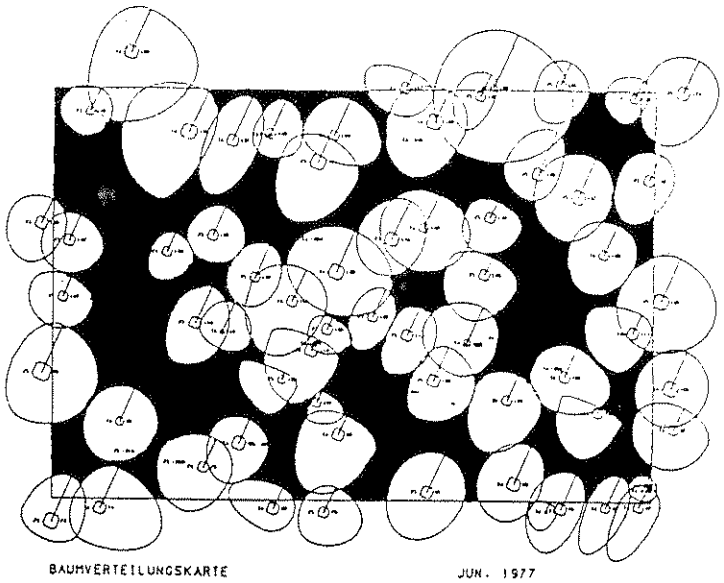
The above pictures at **Fig. 3** show the trees of 1977 before cuttings. The trees after cutting and remaining until 1988 (below) show the larger growing space and the crown increment after 11 years. The data of canopy including the double and more covered parts are to be seen in **Tab. 2.**

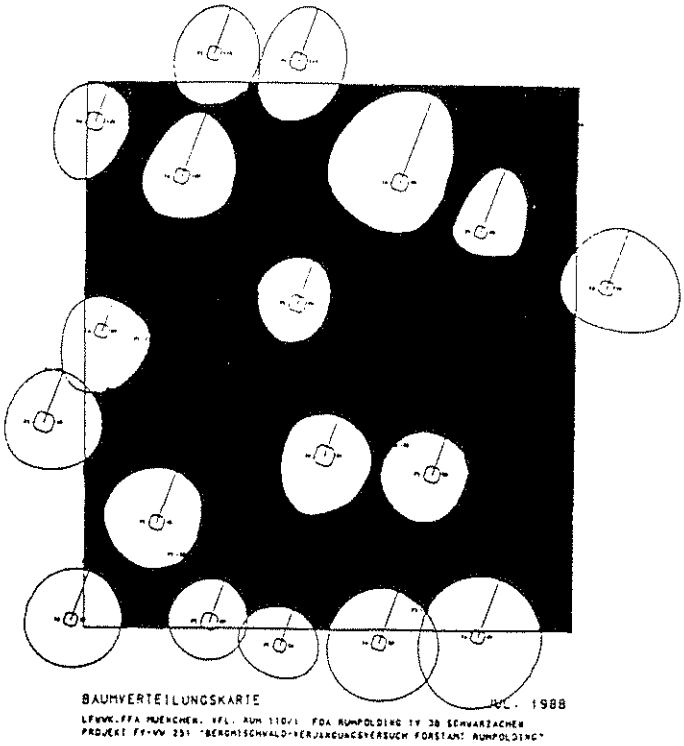
Tab. 2: Percentage of canopy at RUH 110/1+2 at the measurements of 1977 and 1988 (total crop)

Parcel	Measurement	canopy (% of the area of the parcel)			
		1 x	2 x	3 x	total
110/1	1977	47.5	2.4	-	49.9
	1988	29.6	0.1	-	29.7
110/2	1977	55.2	10.7	0.2	66.1
	1988	55.2	15.3	0.9	71.4

Most of our mountain mixed forest stands with three or more species have up to 8 times crown covered areas (PREUHSLER, 1979)







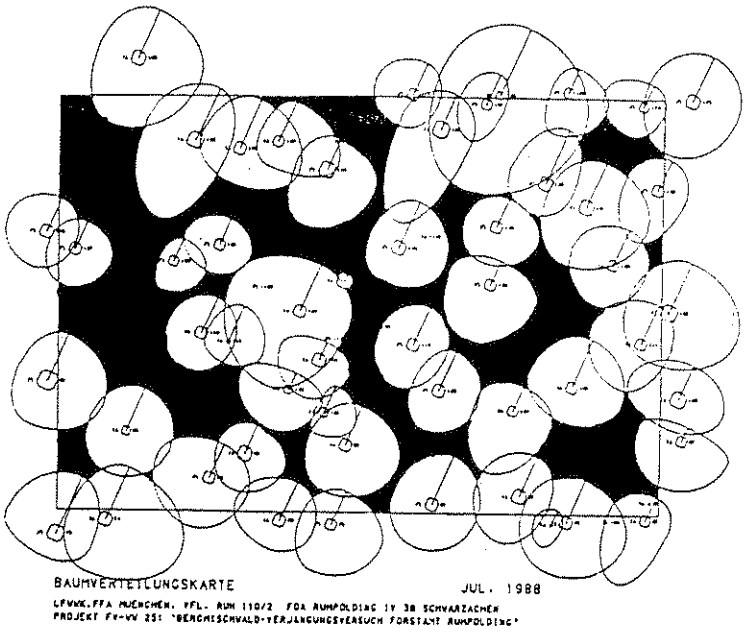


Fig. 3: Canopy map of RUH 110/1(left) and RUH 110/2 (right) at 1977 (above) and 1988 (below)

Natural Regeneration

In the natural regeneration with up to 50ooo plants/hektar the observation of the single trees like in the old stand is not possible. Therefore we selected 5x5m horizontal squares covering the total area for intensive measurement. This lowest information unit is defined by a fixed net which enables the measurement to be repeated.

The size of 5x5m was chosen because
- it is nearly the size of the mean growing space of 400/ha remaining trees at the final stand density in the upper layer of a stand,
- it is possible to keep the overview while counting and measuring the plants on that unit, and
- it is possible to show on our plots the variety of structure by using about 60 squares.

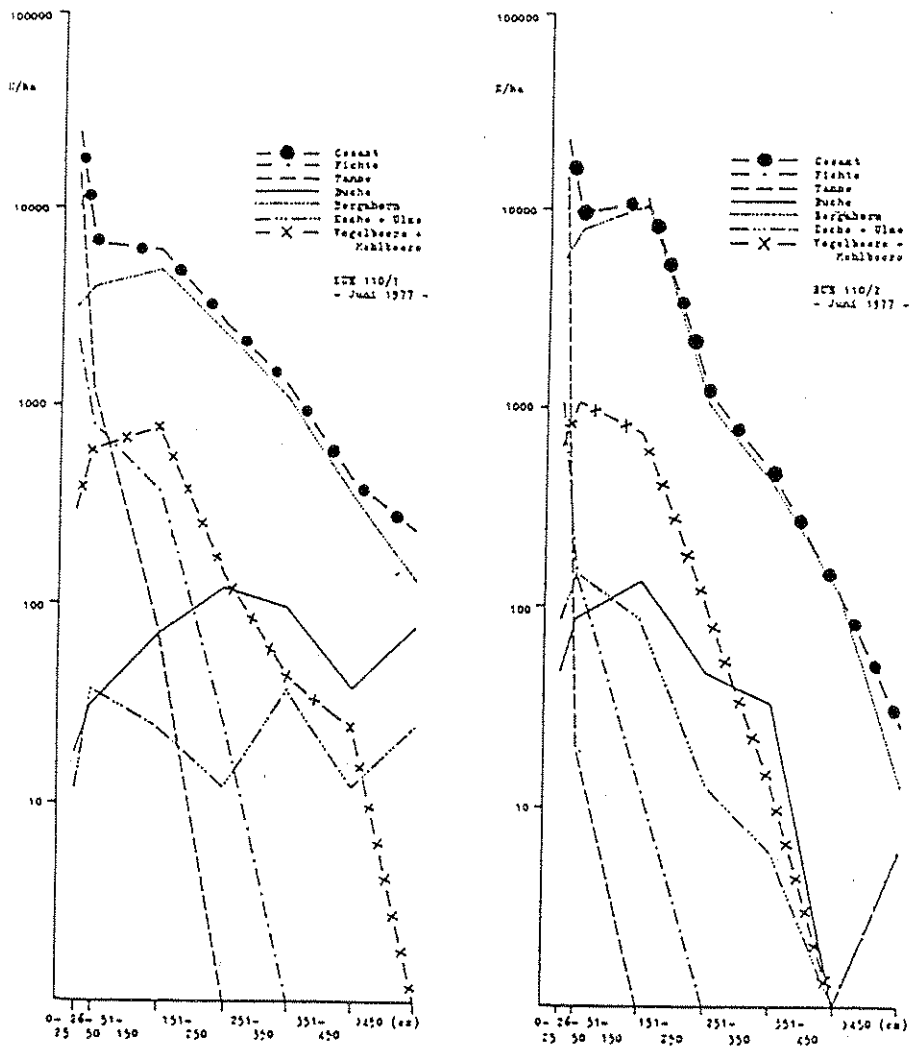


Fig. 4: Quantity (N/ha) of species of natural regeneration in height classes for RUH 110/2 at 1977.

The measurement on each square embrace the species, height classes and some more information like bitten by deer or others.

Simple information of vertical structure are shown by the absolute quantity in height classes at **Fig. 4.** or at the relative quantity of species at **Fig. 5.**

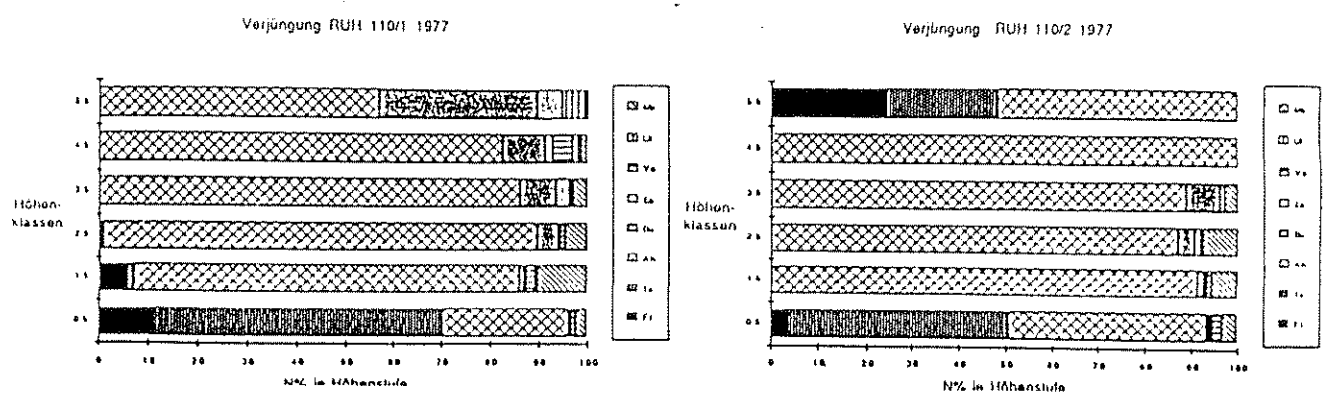


Fig. 5: Relative quantity of species in height classes of the natural regeneration on plot RUH 110/2 at 1977.

Information of the horizontal structure is shown at the map of spatial distribution of natural regeneration at **Fig. 6.** Within the net of squares one symbol represents four plants.

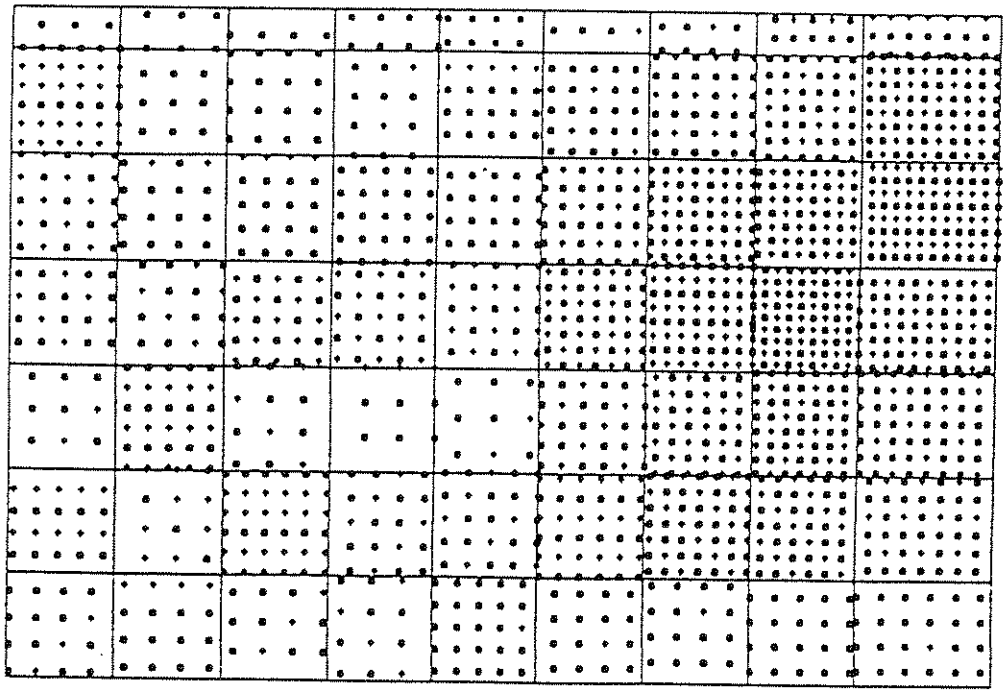


Fig. 6: Map of spatial distribution of natural regeneration for RUH 110/2 at 1977.

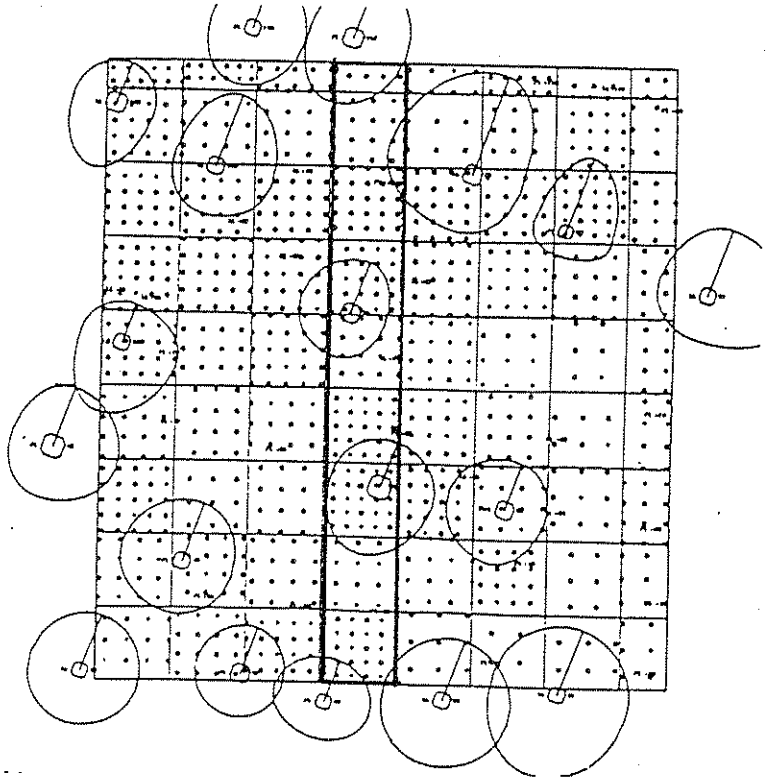


Fig. 7: Stocking map of RUH 110/1 at 1988

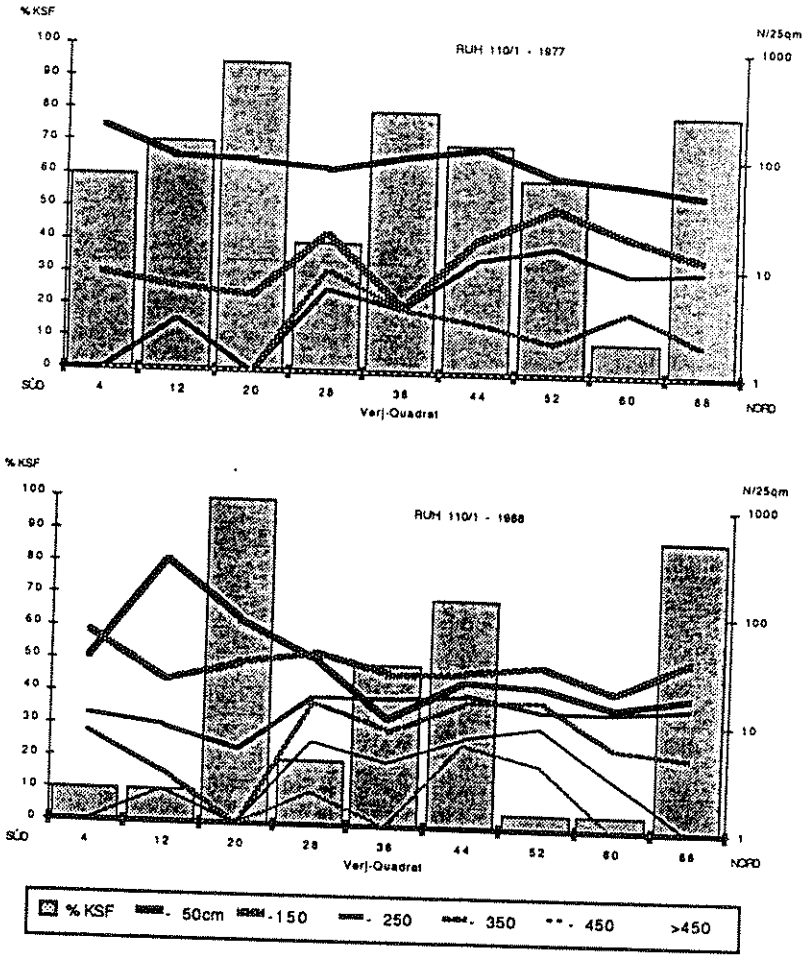


Fig. 8: Canopy and number of plants in height classes at the squares in the fouth column of Parcel RUH 110/1 in 1977 and 1988.

Comparison of Main stand and Natural Regeneration

If canopy map and regeneration map are put together the chance to compare both parts of the stand even at the the small measuring unit is to be seen at the stocking map on **Fig. 7**.

On this figure the map is signed at the fourth column. For these 9 squares you may see on **Fig. 8** a graphical comparison of the canopy on the squares with the numbers of plants in height classes. The influence of canopy on the structure and the growing of regeneration may be analysed by statistical methods on the total area as well as on the squares. The shifted global radiation may be considered according to FISCHER (1980).

The next cuttings within the plan of treatment for regeneration in October 1988 are to be seen at the canopy map and the regeneration map of parcel RUH 110/1 at **Fig.9**. The effect of the felling trees to the horizontal and vertical structure of regeneration for the whole area as well as for the concerned squares was calculated and demonstrated in this way (PREUHSLER, 1989/2)

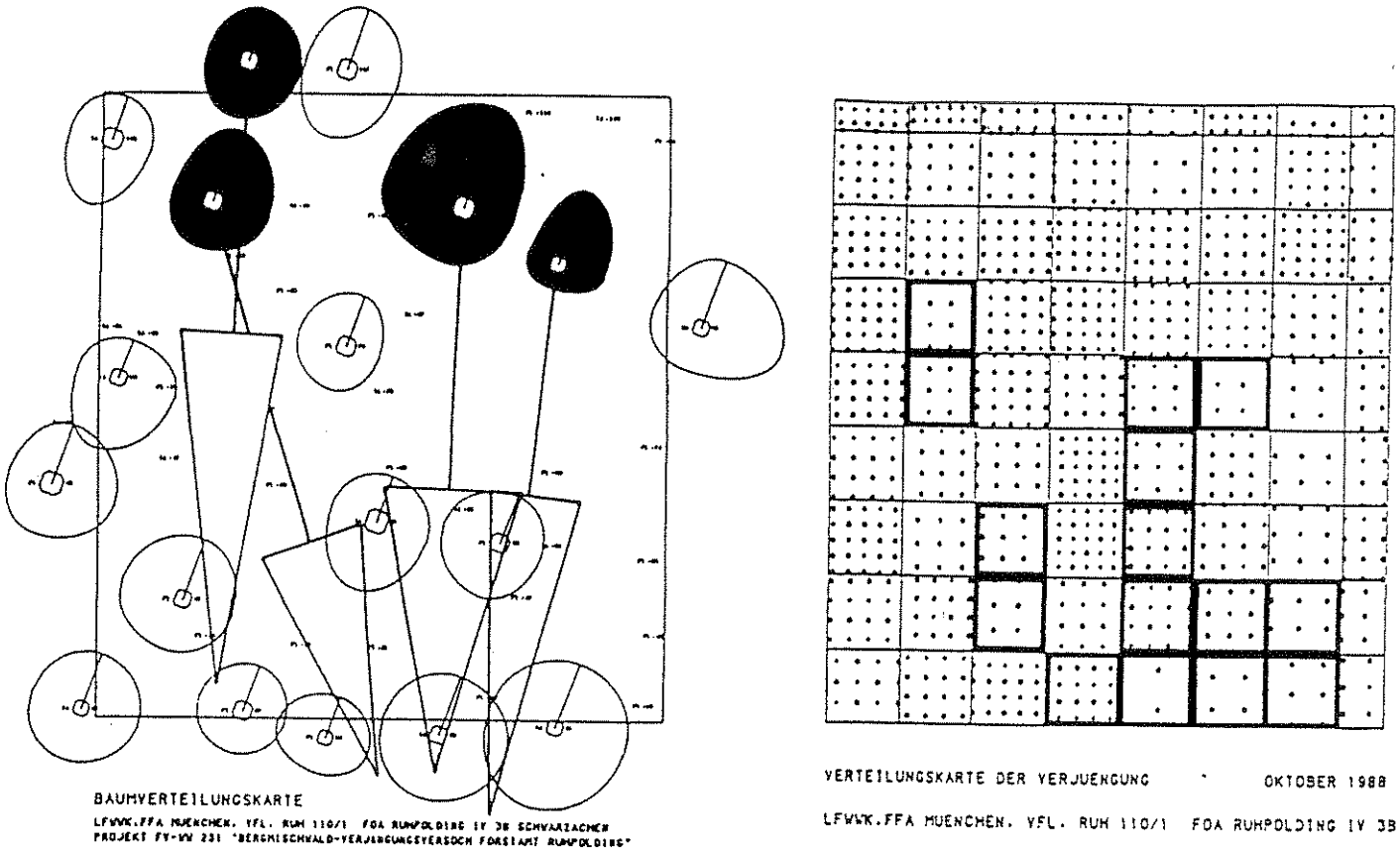


Fig. 9: Cuttings for regeneration on parcel RUH 110/1 in October 1988

CONCLUSIONS

Mixed stands have a great variety of structure, which is determined by the species, the portion and the type of mixture and the spatial and areal distributions. The registration of the actual situation and the dynamic of developement is done in the best way by total measurement on long term trial plots. Beside the usual single tree yield data diameter, height, canopy height and the parameters of quality and social position the total measurement should contain also the data of coordinates and crown radius. Especially in mixed stands the total measurement of natural regeneration enables the discovery of the variability of structure and the growing process dependent on the growing of the old stand during the "overlapping period". The layout of 5x5m squares for the regeneration measurement was well tried. The results of both layers are easy to proove and to show in stocking maps.

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A SIMPLE MEASURE OF STEM FORM

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Summary

The height ratio ($R_{.67D}$) -- the ratio of height to two-third breast height diameter ($H_{.67D}$), and the total tree height (H), both measured above breast-height -- is strongly correlated with the upper-stem (above breast-height) cylindrical form-factor. The estimated form-factor yields a precise volume estimate and a fully compatible taper model for the upper-stem. Comparison of this model with some volume models shows that the MSE of the predicted volume is only about 20% of the traditional volume prediction models. The volume prediction model uses predicted inside bark breast-height diameter. Use of stand specific inside bark breast-height diameter prediction models, together with a measure of stem form should minimize (if not eliminate) volume prediction bias when volume tables are used in specific stands. The simple taper model, being fully compatible with the volume model, also is more consistent in tracking stem profile than the complex taper models.

$R_{.67D}$ is not only useful in predicting volume or in the prediction of upper stem diameters, it is likely to play an important role in: (a) the assessment of inter-tree competition and, therefore, in the prediction of stand density and suppression mortality, and (b) in proper assessment of the response to silvicultural treatments on volume growth.

Keywords: form-factor, stem-taper, stem volume, height-ratio

Introduction

The estimation of stem volume and of recoverable product (lumber, veneer, etc.,) has been the focus of attention for the past century. As breast height diameter measurement is taken outside bark and the stem volume is generally estimated inside bark, four factors may be identified as being important in the estimation process. These are: breast-height diameter (DBH), total tree height (H), stem form, and bark thickness. Majority of the volume prediction models are based on DBH and H alone. Though the role of stem form has been recognized in the prediction of stem volume, very little attention has been given to bark thickness in the estimation process. As both stem-form and bark thickness may be influenced by site factors, it is not unusual for DBH and H based regional volume prediction models to provide biased estimates in specific stands.

The stem form can have substantial impact on tree volume. For example, for a given basal diameter and height, a paraboloid has 50% more volume than a cone of the same basal diameter and height. As trees with the same DBH and H can and do have different forms, DBH and H based volume prediction models generally have prediction error of 12% or more, even though R^2 may be high.

Using the cylindrical form-factor concept, the stem volume can be expressed by:

$$V = F*[k*dbh^2*H] \quad (1)$$

where F is the cylindrical form-factor, k = 0.00007854 (0.005454 in English units) and dbh is the inside bark breast-height diameter. The expression within the square bracket gives the volume of a cylinder of diameter (dbh) and height (H). There are several advantages in using this simple model:

1. The use of estimated dbh (instead of DBH) should eliminate bias due to differences in the bark thickness.
2. By inclusion of a measure of stem-form, the model recognizes its role in addition to that of dbh and H, as a predictor for stem volume. It allows the form-factor to vary independently of dbh and H.
3. Percent change in volume can be expressed as the sum of percent change in F, twice the percent change in dbh, and percent change in H. This should enable better monitoring of response to stand treatments such as thinning, pruning and fertilization.
4. Due to explicit recognition of the variability in stem form and bark thickness in different regions, a single volume function may have wider geographic application, and may even be applicable to more than one species.

It should be noted that unless F and dbh can be directly estimated from standing tree data, equation (1) is of little practical use. Though regional bark-thickness prediction models enable estimation of dbh, very few studies have focused on the influence of site factors and age on bark thickness (Dolph 1989). An error of 2.5% (1 cm error in 40 cm DBH) will contribute 5% to the error in total estimated volume and perhaps more in product recovery. This points to the need of greater attention to bark factors. Stand-specific rather than regional functions should improve the prediction accuracy of dbh.

The direct estimation of F in the past focused on the measurement of one upper stem diameter (Loetsch and others 1973). As this measurement proved to be rather difficult to make, and involved use of cumbersome equipment, it did not become popular in spite of the fact that the standard error of the prediction could be reduced by 50%. In this paper a partial height measurement is described. The ratio of this height to total tree height provides an excellent estimate of the cylindrical form-factor. Inclusion of this estimated form-factor produces dramatic improvement in the prediction of stem volume.

Finally, we show how this simple measure of stem form could be used for obtaining a fully compatible stem profile model, for monitoring response to silvicultural treatments, for monitoring inter-tree competition and, perhaps, in the prediction of suppression mortality.

Measures of Stem-Form: A Brief Review

Even though DBH and H jointly can explain over 97% of the variation in stem volume, it should be noted that standard error of predicted volume of about 17% is implicit in the R^2-value of 0.97. This is due to the fact that tree volume data used in the development of volume functions have a coefficient of variation of 100% or larger. Therefore, DBH and H based individual tree volume estimates can be off by more than 33%. Inclusion of an explicit measure of stem-form should reduce this large error of prediction. As the past investigations (see Loetsch and others 1973, pages 155-157) have shown, substantial reduction in the standard error of prediction is possible with one or more additional diameter and height measurements on standing trees.

The major emphasis of the past efforts has been on the measurement of another diameter at some fixed or relative height; or the measurement of the crown length (Loetsch and others 1973, Hann and others 1987). Inclusion of an upper-stem diameter in the volume prediction model can reduce the standard error of prediction by 50% or more. The crown length, on the other hand, has been found to have only marginal impact on the volume prediction accuracy (Laassasenaho 1982, Farrar 1985, Hann and others 1987).

A suitable measure of stem-form should have the following attributes.

1. Provide improvement in the prediction of stem volume.
2. Facilitate development of a simple, fully compatible stem-taper model giving a reasonable approximation of the true stem profile.
3. Require few and simple additional field measurements.

Two measures of stem form found in the literature are form quotient (ratio of an upper stem diameter to DBH) and crown length (or its ratio to H). An example of the former is Girard's form class (1933) where the ratio of the inside bark diameter at the top of the first (5 or 10 m) log to DBH provides a measure of stem form in the estimation of lumber. The use of crown ratio as a surrogate for stem form is an example of the latter (Farrar 1982, Hann and others 1987).

The ratio of two diameters measured at fixed intervals provides a measure of the rate of taper between these two points rather than a measure of the stem form. This can be illustrated by a simple example. Suppose there are two cones with the same basal diameter of 50 cm and height of 30 and 50 m respectively. If the second diameter is measured at 10 m above base, the form quotient for the first cone would be 0.67 whereas for the second cone it would be 0.80, thereby implying that the latter has better form. This is, of course, not true as both have the same (conical) form. This drawback can be overcome by measuring the upper stem diameter at the same relative height, but this proves to be a somewhat arduous task in the field. Use of slenderness ratio H/DBH (Bitterlich 1984), has also been proposed as a measure of stem form, but this still implies constant form for a given H/DBH ratio.

A major problem with the measurement of crown length or ratio is the subjective judgment involved in locating the crown base in shade tolerant species where the lower branches may persist even under high densities. Another problem arises when the crown base is raised by artificial pruning. Both the crown length and ratio, though statistically significant, have provided only limited improvement in the accuracy of volume prediction.

Height Ratio as a New Measure of Stem-Form

Rustagi and Loveless (1990) have shown that there is a strong linear relationship between the height ratio ($R_{.D}$ = ratio of height at a point where diameter outside bark is a constant fraction of DBH, and the total tree height) and the cylindrical form factor F. Use of this ratio as the predictor of form-factor not only improves the accuracy of the predicted volume, but also provides Ormerod's (1973) shape parameter (β) for a fully compatible taper model.

The relationship between $R_{.}$, F and β is based on the properties of geometric solids of revolution formed by rotating the function:

$$(2) \qquad y = \alpha x^{\beta}; \qquad = Y(x \mid X)^{\beta} \qquad \{0 \le x \le X; \alpha \ \& \ \beta > 0\}$$

around the x-axis, with $y = Y$ when $x = X$. If we define height ratio as:

$$(3) \qquad R_{.5Y} = \left[\frac{(X - x_{.5Y})}{X}\right]$$

where $x_{.5Y}$ is x-value corresponding to $y = 0.5Y$, the following relationship holds between the cylindrical form-factor and the height ratio:

$$(4) \qquad F = \frac{V}{(\pi Y^2 X)} \approx \left(\frac{2}{3}\right) * R_{.5Y}$$

for a fairly broad range of β-values. When β equals 1 or 0.5 (i.e., the solid is a cone or paraboloid), the relationship in (4) is exact.

Another interesting relationship in case of geometrical solids is that the form coefficient (β) is related with F as follows:

(5) $$\beta = \frac{(1-F)}{2F}$$

which should enable fitting of the simple geometric form once the form-factor is known.

Tree as a Geometric Solid of Revolution

In order to maintain compatibility between tree stems and simple solids of revolution, the tree stem may be viewed as consisting of two parts: the section below breast-height (lower-stem), and the section above breast-height (upper-stem). In that case the upper-stem may be equated with the simple solid of revolution in (2). The following correspondence exists between the inside bark upper-stem and the simple solid of revolution:

(6) $$d = dbh\left\{\frac{(H-h)}{H_a}\right\}^{\beta}$$

where $$X = H_a = (H-1.37); \quad x = (H-h);$$

$$Y = \frac{dbh}{2}; \quad y = \frac{d}{2};$$

Let us define the upper-stem cylindrical form-factor F_a as:

$$F_a = \frac{V_a}{\{k*(dbh)^2*H_a\}};$$

F_a may be predicted using the simple linear regression model:

(7) $$F_a = b_0 + b_1 + R_{.D} + e$$

where $R_{.D}$ is the ratio of height at a predetermined fraction of DBH to the total tree height (both measured above breast-height).

After comparative evaluation of height measurements to three different fractions of DBH (1/2, 2/3 and 3/4), the height to two-thirds of DBH was selected for the height ratio ($R_{.67D}$), and its use in (7) for the prediction of F_a. Even though the coefficient of variation in the form-factor is usually very small (under 15%), $R_{.67D}$ alone can account for 85% or more of the total variability in F_a.

With β estimated using (5), the upper-stem inside bark diameter (d_i) for any given height (h_i) may be computed as follows:

(8) $$d_i = (dbh)*\left[\frac{(H-h_i)}{H_a}\right]^{\beta}$$

Similarly, height (h_i) to any upper-stem diameter (d_i) may be computed as:

(9) $$h_i = H - H_a*\left(\frac{d_i}{dbh}\right)^{\frac{1}{\beta}}$$

The volume of the lower stem (V_b) may be estimated separately using DBH (or dbh). There is no real need to fit a taper model in the lower-stem as neither the lumber nor veneer output is affected by taper in this section.

The entire process of form-factor prediction and its use in volume and stem-taper prediction may be summarized as follows:

1. Predict the dbh from DBH. In order to minimize prediction bias, locally developed functions should be used for this purpose.
2. Predict the upper-stem form-factor (F_a) from $R_{.67D}$. Using height above breast-height (H_a), the predicted dbh, and $R_{.67D}$, compute the upper-stem volume V_a.
3. Obtain the shape parameter (β) from F_a, and use it to predict the upper-stem profile. For example, the upper end diameters of the logs of any given length, or log length (between any two diameters) could be determined with the help of (8) and (9).
4. Separately estimate the volume of lower stem (V_b) and add it to V_a to obtain the total stem volume.

Douglas-Fir Example

A sample of 98 Douglas fir trees from Oregon, Washington and British Columbia was used to develop the volume and taper models. The results of this study have been reported in Rustagi & Loveless (1990). The summary of the developed regression relations is:

$$(10)\quad dbh = -0.2897 + 0.9145 * DBH; \quad R^2 = 0.994; \quad MSE = 1.09\ \text{cm}^2$$

$$(11)\quad F_a = 0.13894 + 0.65712 * R_{.67D}; \quad R^2 = 0.794; \quad MSE = 0.00058$$

$$(12)\quad f_a = 0.08079 + 0.78177 * R_{.67D}; \quad R^2 = 0.864; \quad MSE = 0.00050$$

$$(13)\quad V_a = F_a * [k * dbh^2 * H_a]; \quad MSE = 0.01356\,(m^3)^2$$

$$(14)\quad V_a = f_a * [k * dbh^2 * H_a]; \quad MSE = 0.01310\,(m^3)^2$$

The difference between (13) and (14) is that in (13) F_a is based on true dbh, and should be used if stand specific functions are available to predict dbh from DBH. In (14) f_a utilizes the dbh estimated from a regional model, and should be used if stand-specific bark thickness models are not available.

The following simple model was derived for predicting the lower-stem volume:

$$(15)\quad V_b = 0.003887 + 0.00010189 * DBH^2; \quad R^2 = 0.984; \quad MSE = 0.000253\,(m^3)^2$$

As no correlation was observed between the residuals of V_a and V_b, the MSE for predicting total stem volume using (11) and (12) is 0.013662 $(m^3)^2$, which is about 20% of the best DBH and H based models for volume prediction. In fact, the mathematical model:

$$(16)\quad V_a = \left(\frac{2}{3}\right) * (k * dbh^2) * H_{.5D}$$

consistently performed better than any other DBH and H based regression model for predicting upper-stem volume. This shows that approximation of the tree stem by a geometric solid of revolution is a logical approach in tree stem modeling.

An independent sample of 20 Douglas-fir trees was used to compare the height ratio based model with two other volume and taper prediction models (Walters & Hann 1986, Kozak 1988). The differences (absolute as well as percent) between the actual and predicted volumes are presented in Figure 1. The form-factor based model predicted upper-stem volumes with far greater accuracy than either of these two models.

From these 20 trees, three trees were selected to represent poor, average and better form. For these trees, the upper-stem diameters were predicted using the form factor model and the taper models by Walters & Hann (1986) and Kozak (1988). The inside bark cross-sectional areas (actual and predicted) are plotted against tree height in Figure 2. Here again, the simple geometric solid form fits better than the other two taper models, except when the stem has prominent butt swell. In that case, Kozak's (1988) variable form taper model appears to fit better (Tree #12, Figure 2).

Discussion

The form-factor based volume and taper models have simple algebraic form, which makes it easy to estimate inside bark diameter at any height, height to any given inside bark diameter, and volume between any two heights (or between any two inside bark diameters) by integration. The compatibility between the two models guarantees that the sum of the sectioned volumes, when added over the entire upper-stem, will lead to the same volume as obtained directly by using the volume formula. The estimated upper-stem volume has small root mean square error (RMSE), but is slightly biased, the average bias for the calibration sample being less than one-third of one percent of the mean volume per tree. A larger average bias was noticed (about 1% of the mean volume) for the validation sample, but the RMSE was smaller even though the average upper-stem volume for the validation sample is about 50% larger than for the calibration sample. Table 1 summarizes the bias and RMSE of the calibration and validation samples.

Table 1: The summary of volume predictions using different models. The average bias is the mean difference and average RMSE is the square-root of the mean of the squared differences between the measured and predicted upper-stem volumes.

Prediction Model	Calibration Sample Average (98 Trees)			Validation Sample Average (20 Trees)		
	V_a (m^3)	Bias (m^3)	RMSE (m^3)	V_a (m^3)	Bias (m^3)	RMSE (m^3)
Form-factor[a]	1.551	-0.005	0.0114	2.168	-0.023	0.103
Form-factor[b]	1.548	-0.003	0.0115	2.145	0.000	0.088
Walters & Hann (1986)				2.293	-0.148	0.383
Kozak (1988)				1.906	0.240	0.306

[a] - Form-factor predicted from $R_{.67D}$ only.

[b] - Form-factor predicted from $R_{.67D}$ and H_a/dbh.

As the volume and taper models are based on the theory of geometric solids of revolution they are robust and less likely to provide seriously biased estimates than the empirical volume and taper models even for small data sets. In contrast, when tested on the validation sample of 20 trees, both Walters & Hann's (1986) and Kozak's (1988) taper models produced seriously biased results and very large RMSE. In an earlier study, Rustagi & Loveless (1990) found that the crown ratio based volume prediction model by Hann & others (1987) was no better than the DBH and H based volume prediction models, thereby confirming earlier observations that the crown ratio by itself helps little in reducing the RMSE. Kozak's model has the drawback that it provides average stem taper for a tree of given DBH and H. Another drawback of his taper model is that the predicted inside bark diameter at breast-height is subject to large error as it is estimated indirectly from the estimated inside-bark diameter at the inflection-point which is taken to be at 20% of the total tree height.

Inclusion of H_a/dbh ratio in the form-factor prediction model provided mixed results. For the calibration sample, the bias was reduced but the RMSE was slightly increased. On the other hand, both the bias and RMSE were reduced for the validation sample when the H_a/dbh-ratio was included in the model. Though further testing is needed, it appears that the H_a/dbh ratio in the form-factor prediction model may lead to reduction in bias in the prediction of upper-stem volume.

The form-factor based taper model is not without its drawbacks. First, it assumes the same geometric form for the entire upper-stem and therefore ignores the inflection and reflection points. This can cause fairly large errors in the predicted upper-stem diameters of trees with low (i.e., <0.35) form-factor and a high inflection point. For such trees, Kozak's taper model appears better suited, provided the height and inside bark diameter of the inflection point is precisely determined. Perhaps a modified Kozak's model, constrained to have the dbh predicted directly from DBH, and have same volume as the form-factor model, may perform better than either of the two stem profile models.

Second, the additional height measurement $H_{.67D}$ may involve measurement error. The parallel bands of the Speigel Relaskop provide a method of identifying the point where the diameter is two-thirds of the DBH and obtaining its height. Though further testing is needed, the author found that the errors in estimation of $H_{.67D}$ with a Relaskop were far less serious than the errors in estimating upper-stem diameters. A simple device for locating (2/3)DBH has been designed by the authors which appears promising but needs further testing. In order to lower the point of this height measurement, $R_{.75D}$ may be substituted for $R_{.67D}$, with little loss in precision. Once the need and usefulness of this fractional height measurement is clearly established, optical and/or electronic devices for this purpose will not be far behind.

Besides improvement in volume and stem profile prediction, the form-factor approach has several other advantages. For example, measurement of change in the form-factor should lead to more reliable estimation of response to silvicultural treatments such as thinning, fertilization and pruning. Another potential use could be in the estimation of stand density from the height-ratio. In high density stands, trees have more cylindrical boles and, therefore, larger height-ratios. There may even be a threshold height-ratio above which the suppression mortality may become more imminent. Thus, height-ratio may play a useful role in modeling inter-tree competition and in growth simulation.

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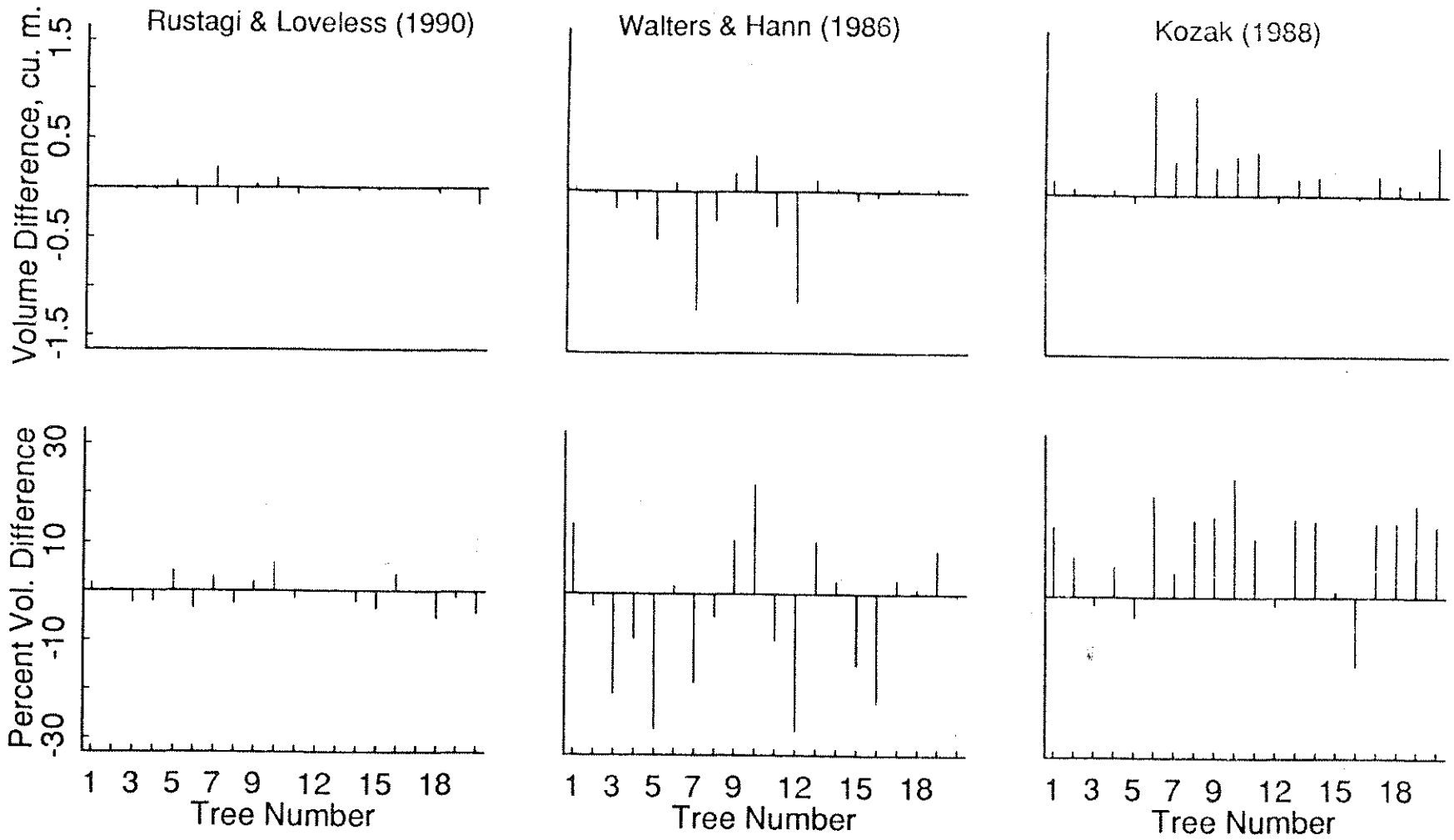

Figure 1. The difference between the actual and estimated volumes of validation sample trees using 1) Rustagi & Loveless (1990), b) Walters & Hann (1986), and c) Kozak (1988) taper models. The algebraic differences are shown in the upper and percent differences in the lower rows of plots.

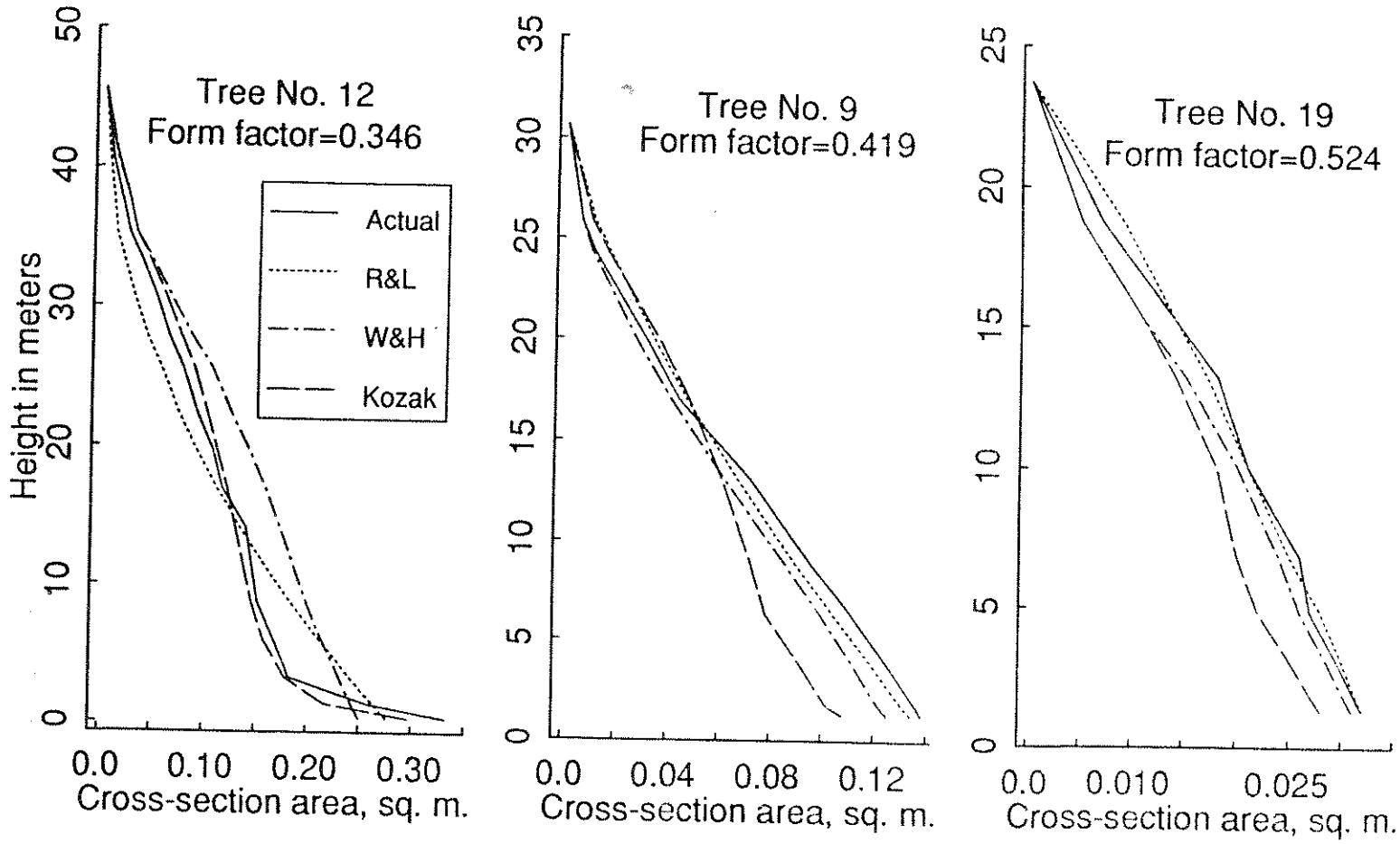

Figure 2. Actual and estimated profiles of three trees from the validation sample representing low, average, and high upper stem form factors. For visual representation of actual and estimated volumes, cross-sectional areas (instead of under bark diameters) have been shown along the X-axis.

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GROWTH, YIELD AND TIMBER DENSITY OF SHORT ROTATION COPPICE STANDS OF *EUCALYPTUS GRANDIS*

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SUMMARY

The effects of timing initial coppice reductions, number of reductions and final number of shoots per stool on growth and yield of *Eucalyptus grandis* are described. The basic data were obtained from three experiments situated in different climatic zones of the summer rainfall area of South Africa.

The influence of site differences, stand densities, mortality and factorial treatments on timber yield and timber density are compared at harvesting at about ten and fourteen years of age. The relationships of yield and timber density of the parent seedling crops and those of the coppice crops are investigated and the influence of different growth rates due to prevailing climate and management are analysed. The importance of basal area as a predictor variable of yield at harvesting for the wide range of treatment interactions is indicated.

On the basis of these results and observations in commercial plantations, practical recommendations are given for the management of *E.grandis* plantations grown from coppice with different management objectives.

Keywords : Diameter, basal area, height, bark thickness, stocking, tree form.

INTRODUCTION

Comparatively little research has been carried out and few research results published on the management of coppice stands of *Eucalyptus* species in general and of those of *E.grandis* in particular. The ability of eucalypts to coppice and the concepts of managing the reduction of the prolific coppice shoots have been described (WRI, 1972; Evans, 1982; Cremer *et al,* 1984; Opie *et al,* 1984 and Schönau and Coetzee, 1989). This paper summarizes the development and final results of three experiments with *E.grandis* designed to compare for short rotations, the survival, growth, form and timber yields of the plant crop with the first coppice crop under different systems of coppice reduction. The prime objective of their establishment was to prove or disprove the benefits of the many different practices applied in coppice management. Results up to about eight years were fully discussed by Stubbings and Schönau (1980).

DETAILS OF EXPERIMENTS

The location of the three experimental sites, their altitude, mean annual precipitation (MAP), mean annual temperature (MAT) and the duration of the experiments are given in Table 1. The soils at the three sites are all merely moderately leached and belong to the Hutton form (Rhodic Ferralsol - FAO; Haplustox - USDA) which become firm during dry periods. This indicates that the effective rainfall in relation to temperature at these sites is only just adequate for the sustained growth of *E.grandis.*

TABLE 1 : Details of experimental sites, and time and duration of experiments

Experiment No.	SE.4	SE.5	SE.7
Farm	Herbstlie	Broadmoor	Bloemendal
District	Piet Retief	New Hanover	Pietermaritzburg
Region	Southeastern Transvaal	Natal Midlands	Natal Midlands
Latitude	26°50'S	29°28'S	29°32'S
Longitude	30°50'E	30°36'E	30°28'E
Altitude	1300 m	900 m	870 m
Date planted	Jan 1960	Mar 1964	Apr 1962
Date 1st felling	Jan 1970	Apr 1970	Aug 1972
Date 2nd felling	Jan 1981	Jan 1980	Sep 1986
Age 1st felling (years-months)	10-0	6-1	10-4
Age 2nd felling (years-months)	11-0	9-9	14-1
MAP (long term)	834 mm	937 mm	902 mm
MAP - plant crop	778 mm	885 mm	828 mm
MAP - coppice crop	857 mm	930 mm	853 mm
MAT	16,2°C	17,4°C	17,4°C

The designs of Experiments SE.4 and SE.5 were identical, i.e. a 3^3 factorial confounded in blocks of nine units with two replications. Each plot consisted of four rows of five trees with single row surrounds. SE.4 was planted at 2,7 m x 2,7 m (1 329 trees/ha) and SE.5 at 2,1 m x 2,1 m (2 197 trees/ha). The following treatments were applied in these two experiments:

A. <u>Time of initial reduction of coppice</u>

1. When dominant height was 1,2 m
2. When dominant height was 4,3 m
3. When dominant height was 7,3 m

B. <u>Number of coppice reductions</u>

1. One
2. Two, when dominant heights were 1,2 and 4,3m respectively (A1); or 4,3 and 7,3 m respectively (A2); or 7,3 and 9,1 m respectively (A3)
3. Three, when dominant heights were 1,2, 4,3 and 7,3 m respectively (A1); or 4,3, 7,3 and 9,1 m respectively (A2); or 7,3, 9,1 and 11,0 m respectively (A3)

C. <u>Final number of coppice shoots</u>

1. One per stool
2. Original stocking of plant crop (i.e. twenty per plot)
3. Two per stool

The plant crop of Experiment SE.7 was a planting espacement and pruning experiment (Schönau, 1974). The following planting espacements were arranged in a latin square:

S. Stocking

1. 3,0 m x 2,7 m (1 196 trees/ha)
2. 2,7 m x 2,7 m (1 329 trees/ha)
3. 2,4 m x 2,7 m (1 495 trees/ha)
4. 2,1 m x 2,7 m (1 709 trees/ha)

The coppice was reduced in two operations, the first being to four shoots per stool at 16 months of age. When the dominant height was 8 m a final reduction was done as described under C above, each allocated at random to a third of the planting espacement plots. Each subplot consisted of two rows of six trees with single row surrounds.

All experiments were regularly measured for breast height diameter (DBH), while heights were measured only in the initial stages, as a treatment control, and when the trees were felled. In addition, the tree form in each experiment was scored on a relative scale between the ages of three and six years. All trees of the plant and coppice crops in the experimental plots were measured at felling according to a sample tree procedure designed for computer processing (Schönau, 1982). Furthermore, the timber density at a height of 5 m was determined from a stratified sample in each plot when clearfelling the plant crop. The coppice of the same trees were sampled in a similar way when they were felled in Experiments SE.4 and SE.5. The results of these measurements and determinations are discussed below.

DIAMETER

In Experiments SE.4 and SE.5 the mean DBH was initially highly significantly ($P < 0,01$) reduced by delaying the first coppice reduction (see Table 2). These relatively small differences virtually disappeared or became no longer significant at a later stage, after five years in SE.5 and after ten years in SE.4, due to the decrease in absolute and relative differences with time. There was also a significant ($P < 0,05$) reduction in DBH with increasing number of coppice reductions after the final reductions were completed between three and four years of age. Table 2 shows that these differences were very small and had virtually disappeared one year later.

TABLE 2 : Effect of time of initial coppice reduction and number of coppice reductions on DBH (cm) in Experiments SE.4 and SE.5

Experiment and age (years - months)	Mean DBH in cm: Dominant height at 1st coppice reduction		Mean DBH in cm: Number of coppice reductions		Least significant difference (P = 0,05)	CV (%)
	1,2m (A1)	7,3m (A3)	One (B_1)	Three (B_3)		
SE.4						
3 - 10	11,05	10,17	10,74	10,37	0,365	5,0
11 - 0	16,83	16,44	16,53	16,33	0,538	4,7
SE.5						
3 - 8	8,88	8,34	8,79	8,42	0,221	4,4
9 - 9	12,73	12,54	12,64	12,62	0,383	3,7

In both experiments on the other hand, the number of remaining coppice shoots affected the DBH highly significantly right up to the stage when the trees were felled. The differences in DBH when one shoot was left per stool or when the number of shoots equalled the original stocking was small (17,7 vs 17,0 cm for SE.4 and 13,5 vs 13,0 for SE.5 at felling respectively). However, when two shoots were left per stool the reduction in DBH was considerable (2,9 and 2,0 cm for SE.4 and SE.5 respectively). Since the risk of wind damage is great when coppice is reduced in one operation at an early stage, the effect of the final number of coppice shoots on DBH over time is illustrated for these experiments in Figures 1 and 2 when the reductions are carried out in two operations starting when the shoot height is about 4 m. It is apparent that the original stocking of Experiment SE.5 is closer to the optimum carrying capacity of that site than that of SE.4, because the reduction in DBH is much greater for SE.5 when the number of shoots is increased from one per stool to that of the original stocking.

In Experiment SE.7 the mean DBH was decreased significantly by an increase in stocking (17.7 vs 15.6 cm, respectively for the extreme stockings, LSD (P=0,05) = 1,63) as well as highly significantly by an increasing number of coppice shoots (17,4 vs 15,6 cm for 1 and 2 shoots per stool respectively, LSD (P=0,01) = 0,88). There was, however, a significant interaction, showing that this decrease in DBH by increasing stocking did not occur when the number of coppice shoots was reduced to that of the original stocking (17,1 cm in all cases). In addition, the DBH of the single shoots was only the best at the widest spacing (19,1 cm), otherwise the original stocking gave the best DBH (17,1 vs 16,9 cm for one shoot per stool). The development of DBH for the extreme treatments and that of the best with the original stocking in this experiment is illustrated in Figure 3. The reduction in growth rate after the seventh year is caused by a series of extremely dry years with a MAP of less than 700 mm.

BASAL AREA

The total basal area at breast height per ha (BA) was significantly reduced by a delay of the first coppice reduction in Experiment SE.4 (29,8 and 27,0 m^2/ha for the extremes respectively). This was not the case in SE.5, probably due to wind damage after the first reduction. In Experiment SE.4 the BA was significantly increased by reducing the coppice in two operations (30,0 m^2/ha) instead of carrying these reductions out in one or three operations (27,0 and 28,2 m^2/ha respectively). This was also the case in SE.5 but the differences were not significant. As was to be expected, there was a highly significant increase of BA with increasing number of coppice shoots per stool (27,2 vs 31,0 m^2/ha at SE.4 and 21,9 vs 27,4 m^2/ha at SE.5 for one and two shoots per stool at harvesting respectively). This was the case in both experiments from after the last coppice reduction right until harvesting. Further scrutiny of the data revealed that overall the greatest BA was achieved when the coppice reduction started when the shoots were 4,3 m tall and when it was carried out in two operations (31,7 m^2/ha in SE.4 and 28,7 m^2/ha in SE.5).

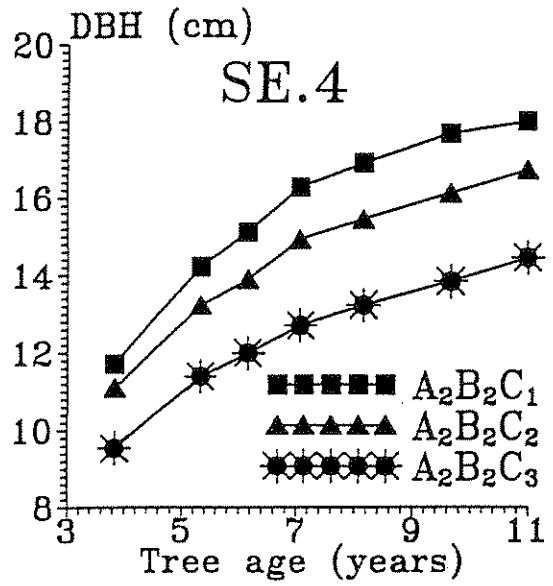

Figure 1 : Development of DBH for various coppice treatments in Experiment SE.4

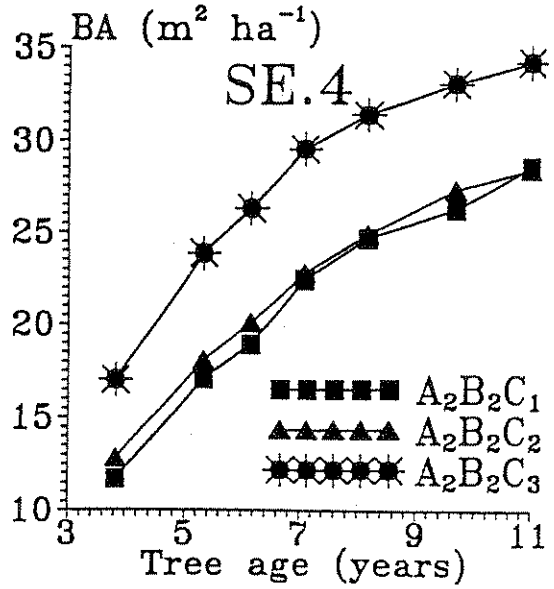

Figure 4 : Development of basal area for various coppice treatments in Experiment SE.4

NB : $A_2B_2C_1$ = Reductions at 4,3 and 7,3 m to one shoot per stool
$A_2B_2C_2$ = Reductions at 4,3 and 7,3 m to original stocking
$A_2B_2C_3$ = Reductions at 4,3 and 7,3 m to two shoots per stool

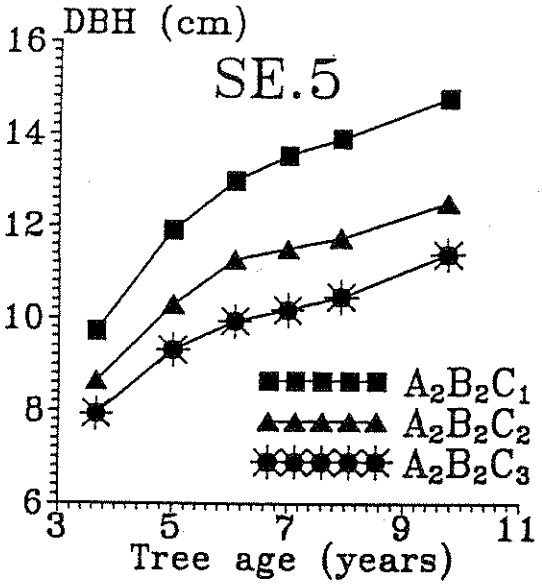

Figure 2 : Development of DBH for various coppice treatments in Experiment SE.5

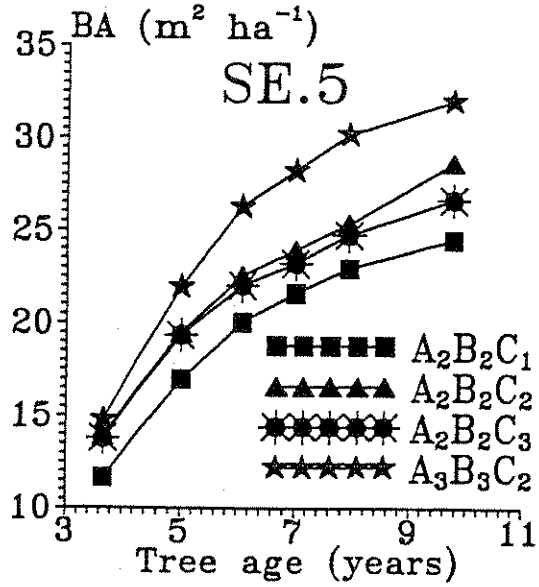

Figure 5 : Development of basal area for various coppice treatments in Experiment SE.5

NB : $A_2B_2C_1$ = Reductions at 4,3 and 7,3 m to one shoot per stool
$A_2B_2C_2$ = Reductions at 4,3 and 7,3 m to original stocking
$A_2B_2C_3$ = Reductions at 4,3 and 7,3 m to two shoots per stool
$A_3B_3C_2$ = Reductions at 7,3, 9,1 and 11,0 m to original stocking

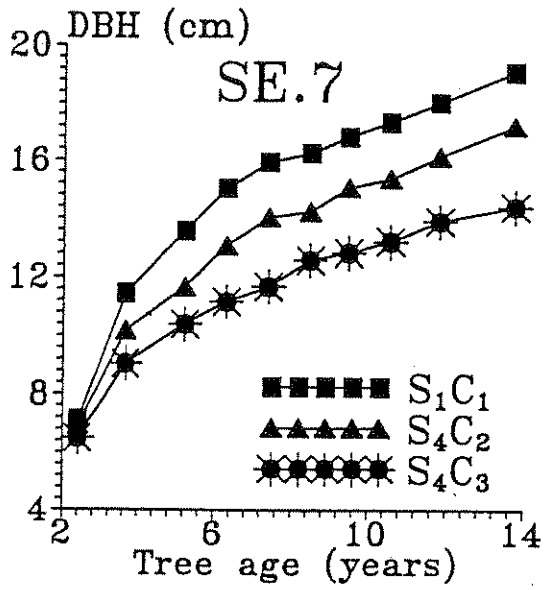

Figure 3 : Development of DBH for various treatments in Experiment SE.7

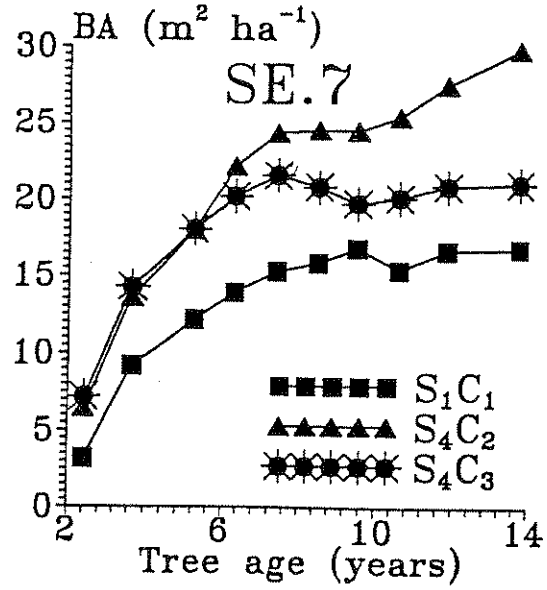

Figure 6 : Development of basal area for various treatments in Experiment SE.7

NB : S_1C_1 = 3 m x 2,7 m reduced to one shoot per stool
S_4C_2 = 2,1 m x 2,7 m reduced to original stocking
S_4C_3 = 2,1 m x 2,7 m reduced to two shoots per stool

The development of BA for the relevant treatments is illustrated in Figures 4 and 5 for SE.4 and SE.5 respectively. The influence of the higher stocking of the plant crop at SE.5, close to the carrying capacity of this site, is clearly apparent, since there is little difference in BA when two shoots per stool remained or the number of shoots equalled the original stocking. There is obviously no purpose in leaving two shoots per stool at this site. It is of interest that the best BA at SE.5 was obtained when the coppice reduction started only when the shoots were 7,3 m tall and when the reduction was carried out in three operations (see Figure 5). This was probably caused by the severe early wind damage in the earlier reduced plots and the more careful selection possible at a later stage when the potential of the remaining shoots was better expressed.

Initially at about two and a half years after felling, increased stocking increased the BA highly significantly in Experiment SE.7 (4,3 vs 5,5 m^2/ha for the extremes, LSD (P=0,01) = 0,96, CV = 7,5%). However, this increase was very small and became non significant with increasing age due to the increased variance (19,7 vs 22,7 m^2/ha for the extremes at about 14 years, LSD (P=0,05) = 4,58, CV = 11,6%). On the other hand, the usual direct relationship between BA and final number of coppice shoots per stool remained highly significant up to harvesting. Closer examination of the data revealed that one shoot per stool gave overall the lowest BA (17,2 m^2/ha) and two shoots per stool the highest BA for the two lowest stand densities (22,8 and 27,5 m^2/ha respectively). For the two highest stockings, however, maximum BA was achieved when the number of shoots equalled the original stocking (24,8 and 29,9 m^2/ha respectively), indicating again the limitations of this site.

The development of BA for the same treatments as those illustrated for DBH is shown in Figure 6. The influence of the serious drought is clearly apparent. The effect on BA, however, is more severe than on mean DBH. Apparently the smaller suppressed trees die first, enabling the bigger trees to grow again and the mean DBH to increase when the rainfall improves. On the other hand, so many trees die on the denser stocked plots and a proportionally greater number on those with the lowest stocking that not enough trees are left to compensate for this loss with increased diameter growth after improved rainfall. As a consequence, the BA remained constant after eight years of age. Only where the number of shoots were more or less equal to the original stocking did BA growth recover after better rainfall.

HEIGHT

Mean height was little affected by treatments in these experiments. After final coppice reduction in Experiment SE.4 at three years and ten months the mean height was significantly suppressed on those plots where the number of shoots was reduced to the original stocking (11,8 vs 12,1 m, LSD (P=0,05) = 0,27). However, a significant interaction showed that this was only the case when the coppice reduction was carried out in one operation (mean height = 11,6 m). This can presumably be ascribed to the wind damage after the first reduction. In SE.5 after final reduction at three years and eight months the mean height on plots where the shoots had been reduced to the original stocking were significantly superior than those where two shoots were left per stool. This was only significant however, when the reductions were carried out in two operations (11,8 vs 11,4 m, LSD (P=0,05) = 0,25). There were no significant differences in respect of height in Experiment SE.7.

Similarly the differences in height at harvesting were small. In SE.4 there was only a highly significant inverse relationship between height and stocking, with a mean height of 22,3, 21,5 and 20,3 m (LSD (P=0,01) = 1,10) for one shoot per stool, original stocking and two shoots per stool respectively. This was not the case in SE.5 where only the plots with two shoots per stool were highly significantly suppressed when compared with the lower stockings (17,7 vs 19,2 m, LSD (P=0,01) = 0,71).

BARK THICKNESS

Since it is usually closely related to DBH, bark thickness at breast height (BT) measured at harvesting was similarly affected by treatments in these experiments. It was, however, very distinct that the suppression of BT by e.g. higher stocking, later or slower reduction was more severe than that for DBH. This may influence the relationship between over-bark diameters and total timber volumes. It makes the final assessment of experiments by DBH and height measurements alone questionable and a sampling of under-bark timber volumes necessary for accurate results.

STOCKING

The planting espacements and stand densities for the three experiments are indicated in Table 3 together with the survival after the first and second rotations. The 1980 data for SE.5 were adjusted by covariance, using the 1970 stocking as the covariate which gave a 43% improvement in efficiency. The covariance analyses for SE.4 and SE.7 were not significant. This table shows that the mortality after two rotations is more severe for

higher stockings, especially in SE.7 where this was not the case after the first rotation (Schönau, 1974). In this experiment only between 44 and 54% of the original stools remained which was due to the earlier mentioned serious drought during the early 80's. In Experiment SE.5 the mortality was also heavy, i.e. 30% in two rotations, indicating again that the original stocking density was too high for this site. Number of trees per unit area is one of the main contributing factors to yield and, in order to maintain productivity, the practice of retaining as many shoots as the original stocking is recommended. Table 3 shows that this policy is not entirely successful, since even with careful selection under research conditions at least 10% of the selected shoots do not contribute to final yield or die. These results also indicate that in SE.7 the even-spaced initial espacement of 2,7 m x 2,7 m was able to carry the highest number of additional shoots.

TABLE 3 : Stand density and survival in experiments

Experiment No.	SE.4	SE.5
Planting :		
Espacement (m)	2,7 x 2,7	2,1 x 2,1
N/ha	1329 (100%)	2197 (100%)
1970 N/ha	1196 (90%)	1727 (79%)
1980/81 N/ha :		
One per stool	1104 (83%)	1537 (70%)
Original stocking	1192 (90%)	1957 (89%)
Two per stool	1794 (135%)	2700 (123%)
LSD (P = 0,05)	98	215

Experiment No.	SE.7			
Planting :				
Espacement (m)	3,0 x 2,7	2,7 x 2,7	2,4 x 2,7	2,1 x 2,7
N/ha	1196 (100%)	1329 (100%)	1495 (100%)	1707 (100%)
1970 N/ha	905 (76%)	969 (73%)	1049 (70%)	1388 (81%)
1980/81 N/ha :				
One per stool	548 (46%)	720 (54%)	747 (50%)	747 (44%)
Original stocking	847 (71%)	914 (69%)	1028 (69%)	1246 (73%)
Two per stool	947 (79%)	1301 (98%)	1277 (85%)	1210 (71%)
LSD (P = 0,05)	381	381	381	381

TIMBER VOLUME

As described above, the volumes at harvesting of the plant and coppice crops of these experiments were obtained from under-bark measurements of all individual trees. These measurements made it possible to calculate utilizable volumes up to tip diameters of 7,5, 10, 12,5, 15, 17,5 and 20 cm in addition to the total volumes up to 5 cm tip. Most of these volumes of the coppice crop in SE.4 and SE.5 were adjusted by covariance with the yields of the plant crop as covariates. Only in a few cases was the covariance analysis non significant for these experiments and were unadjusted values used. This was also the case for SE.7. The improvement by covariance varied between 22 and 66%. There were highly significant differences in total volume as a result of the final number of coppice shoots. These results for total volume and the utilizable volumes up to the various tip diameter classes are given in Table 4 for Experiments SE.4 and SE.5.

TABLE 4 : Effect of different number of coppice shoots left after final reductions in Experiments SE.4 and SE.5 for different tip diameter classes

Final numbers of coppice shoots	Volume in m^3/ha						
	5 cm	7,5 cm	10 cm	12,5 cm	15 cm	17,5 cm	20 cm
SE.4							
One per stool	248	238	213	166	103	46	12
Original stocking	244	233	205	155	91	39	13
Two per stool	282	262	216	146	70	21	2
LSD (P = 0,05)	23	23	22	21	20	15	8
SE.5							
One per stool	184	165	124	64	19	-	-
Original stocking	223	199	147	76	17	-	-
Two per stool	225	187	119	50	12	-	-
LSD (P = 0,05)	28	26	22	14	8	-	-

This table shows that total volumes are increased by between 14 and 22% if two shoots are left per stool instead of one shoot. However, these differences are no longer significant for utilizable volumes over 10 cm. It may therefore be concluded that the additional volume obtained by leaving two shoots instead of one is entirely confined to the smaller diameter classes between 5 and 10 cm. These results also confirm those for BA and stocking, i.e. that no additional yields are obtained by leaving more coppice shoots than the original stocking at SE.5 with the closer espacement. It may also be inferred that for these two experiments the optimum stocking for maximum total volume production is about 1 800 trees per ha, but that this will be less than 1 500 per ha for larger sized timber over 10 cm in diameter. Furthermore, there was also a strong trend that coppice reduction in two operations at a mean height of about 4 and 8 m increased timber volume at harvesting. This was especially the case in SE.5 where severe wind damage occurred after the earliest reduction.

The main results for SE.7 are given in Table 5, showing the much greater variance due to the severe drought. This drought affected the yields of the denser stockings to such an extent that the optimum initial stocking for this site under droughty conditions will be about 1 300 trees per ha. Only for a stand density lower than this did the total volume yield decline appreciably. However, there was no decrease for larger sized timber over 12,5 cm in diameter. Similarly to SE.5, the optimum yield in Experiment SE.7 was achieved when the coppice shoots were reduced to the original stocking, but this was applicable to all size classes except the largest one. The superiority of coppice reduction to the original stocking was also confirmed by this experiment. This was especially evident for the highest stocking where careful selection could compensate for the high mortality and resulted in the highest overall yield.

The usefulness of BA at harvesting or at an earlier age in predicting volume yield was briefly investigated. It was found that the volume/BA ratio at harvesting was in the order of a factor between 8 and 9, and that the extreme differences in this factor for the various treatments differed not more than between 1 and 2%. This factor increased to between 12 and 14 at 5 years of age with the difference between the extremes amounting to 13, 5 and 8% for SE.4, SE.5 and SE.7 respectively.

TABLE 5 : Effect of planting espacement and different number of coppice shoots left after final reduction in Experiment SE.7 for different tip diameter classes

Treatment	Volume in m^3/ha						
	5 cm	7,5 cm	10 cm	12,5 cm	15 cm	17,5 cm	20 cm
Planting espacement							
3,0 m x 2,7 m	174	166	150	120	76	35	14
2,7 m x 2,7 m	195	185	162	119	63	26	8
2,4 m x 2,7 m	200	190	166	125	74	36	13
2,1 m x 2,7 m	200	188	162	119	71	31	9
LSD (P = 0,05)	48	48	47	45	41	30	14
Final number of coppice shoots							
One per stool	149	143	129	102	64	33	13
Original stocking	220	210	187	146	89	39	11
Two per stool	207	194	165	115	59	24	8
LSD (P = 0,05)	47	45	41	35	25	16	9

TREE FORM

The total pole length and volume acceptable in respect of sweep and crookedness for preservative treatment was determined for each tree when individual trees were measured at harvesting. It has been shown that these pole volumes expressed as a percentage of the total volume in a respective tip diameter class are independent of DBH (Schönau, 1972). However, these percentages are dependent on tree form, method of pole making and can be strongly influenced by wind or other damage. They amounted to 73, 80 and 74% for the plant crops and 68, 73 and 66% for the coppice crops of SE.4, SE.5 and SE.7 respectively, indicating the expected reduced form of the coppice stands.

Further analyses showed that in SE.4 early reduction of coppice increased the pole volume highly significantly by about 20%. Only the pole volume over 12,5 cm tip was detrimentally affected by retaining two coppice shoots per stool, confirming the results from the relative form score at about half-rotation age (Stubbings and Schönau, 1980). Similar results for total acceptable pole length were significant, showing increased length for earlier coppice reductions (15 400 vs 12 800 m/ha for the extremes, LSD (P=0,05) = 1 325). In addition, two operations produced significantly longer lengths (15 100 m/ha) than one or three coppice reductions (13 400 and 13 800 m/ha respectively), and two shoots per stool longer lengths (5 900 m/ha) for the smaller diameter classes up to 10 cm tip than one shoot per stool (3 900 m/ha) or where the number of shoots were equal to the original stocking (4 100 m/ha).

The results for SE.5 were different, probably caused by the severe wind damage after the first coppice reduction in this experiment. These results showed that the total acceptable pole volume and length were significantly the greatest when the number of coppice shoots equalled the original stocking (18 500 m/ha). This was only slightly exceeded by the pole length for two shoots per stool in respect of the smallest tip diameter class of 5 cm.

In Experiment SE.7 the acceptable pole volume of the coppice crop was not affected by initial spacing but the total pole length for the widest spacing (12 000 m/ha) was significantly less than that for the other spacings (15 000 m/ha). If only one shoot was left per stool the acceptable pole volume (84 m^3/ha) and length (10 000 m/ha) were highly significantly reduced when compared with those cases where more shoots were left per stool (90 m^3/ha and 16 000 m/ha respectively). This is, however, almost entirely due to the larger volumes for the latter treatments and not to an increase in proportions. Again, the greatest volumes and lengths were obtained on the closest espacements and when the number of shoots were reduced to the original stocking, i.e. 112 m^3/ha and 20 000 m/ha respectively.

STUMP AND KERF WASTAGE

When measuring the individual sample trees the stump and kerf wastage of the individual trees was also determined since it was expected that this would increase for coppice crops when compared with plant crops. This wastage amounted only to 4,2 , 2,9 and 1,0 m^3 per ha for the plant crops of SE.4, SE.5 and SE.7 respectively. For the coppice crops these losses were nearly doubled to 7,6 , 4,3 and 4,3 m^3 per ha when one shoot was retained per stool, indicating the effect of the increased stool height. These volumes were highly significantly increased to 9,9 , 6,1 and 8,4 m^3 per ha respectively when two shoots were left per stool. This is between 3 to 4% of the total volume to 5 cm tip diameter and representative for a stump height and kerf wastage of about 20 cm under average conditions (Schönau, 1982). None of the other treatments had a significant effect.

CHANGES IN PRODUCTIVITY

These three experiments may give some answers to the controversial question whether the yield of coppice crops decline or increase when compared with the planted crop. Evans (1982) stated that coppice does not raise site growth potential, apart from any effects of increased stocking, but only accelerates growth. It is therefore difficult to compare these experiments, since their stocking densities are different, and only comparisons on individual sites are meaningful.

In Experiment SE.4 the mean annual increment of timber (MAI) was 19,5 m^3 per ha during the first rotation. Despite an eight percent reduction in stocking, the MAI in the second rotation amounted to 22,6 m^3 per ha when only one shoot per stool was retained. This amounts to a 16% increase when the MAP increased by 10%. Herbert (1984) suggested that a closer relationship exists with rainfall during the growing season. The mean rainfall from November to March was 537 mm during the plant crop and 633mm during the coppice crop, an 18% increase and close to the average increase in productivity. The yield can, however, be improved by changes in coppice management, e.g. when two reductions are carried out, starting when the trees are about 4 m high. In that case the MAI became 23,7 m^3 per ha when one shoot was left per stool or 30,1 m^3 per ha for two shoots, or increases of 22% and 54% respectively over the productivity during the plant crop. This shows that productivity is dependent on rainfall during the growing season but that it can be manipulated by changes in management techniques.

In SE.5 the MAI of the plant crop was 26,9 m^3 per ha and of the coppice crop 18,9 m^3 per ha when only one shoot was retained per stool. This 30% reduction in MAI occurred despite a 5% increase in MAP or a 14% increase in rainfall during the growing season. This comparison is not entirely equitable as the rotation of the plant crop was only just over 6 years and that of the coppice crop nearly 10 years. It may also indicate an early culmination of MAI when the stocking is too high for the capacity of a site. The MAI of the coppice crop could be improved slightly to 21,7 m^3 per ha, by carrying out the coppice reductions in two operations and starting at a height of about 4 m. Only by starting the reductions when the trees were 1,2 m tall and spreading them over three operations and retaining two shoots per stool could the MAI be raised to 26,8 m^3 per ha, similar to that of the plant crop. These results may also have been caused by the severe wind damage after the first reduction. Spreading the coppice selection over a further two reductions may have counteracted the early damage.

For SE.7 the MAI of the plant crop was 13,0 m^3 per ha, varying between 11,6 and 15,9m^3 for the different stocking densities. At 13 years and 9 months the coppice crop had a MAI of 14,0 m^3 per ha (12,6 - 14,6) or a 7% increase. This compares with a 3% increase in MAP or a 9% increase in rainfall during the growing season for the coppice crop, again a close relationship between the latter and MAI. By reducing the number of coppice shoots to the original stocking at the highest density the MAI could be increased to 20,5 m^3 per ha. However, the MAI of the coppice crop was appreciably affected by the severe drought of the early 80's. As was shown for BA, the volume increment in SE.7 from an age of 8 years onwards must have virtually been negated by the increased mortality. Therefore the MAI could have been as high as 24 m^3 per ha if the trees would have been felled when they were eight years old.

TIMBER DENSITY

The relationship between timber density of coppice and that of its planted parent crop is another matter of contention. For further reference stratified disc samples at a height of about 5 m were taken for basic specific gravity (SGP) determinations at clearfelling of the parent crop in all three experiments. These determinations were repeated on samples of the coppice from the same trees in SE.4 and SE.5. The relationship between the timber density of the coppice (SGC) and that of the parent crop was preliminarily investigated for SE.5 directly after harvesting this experiment (Schönau, 1980). This relationship was further investigated for SE.4 and SE.5 taking into account all quantifiable factors such as DBH, height, disc diameter, stocking and number of shoots per stool (NSPS) which could influence this relationship.

In Experiment SE.4 the two timber densities were highly significantly related ($r = 0,515$ with 149 df). None of the other variable factors was significantly related on its own to either of the two timber densities. Multiple regression showed that only the number of coppice shoots per stool contributed significantly to the relationship between the timber densities of the plant and coppice crops which can be expressed as follows:

$$SGC = 0,2347 + 0,4394\ SGP - 0,00865\ NSPS \quad \ldots\ldots (1)$$

The coefficient of determination was only 0,277 indicating that a large part of the variation in timber density of coppice should probably be ascribed to such factors as growth rate, treatment, site and weather, which are difficult to measure. That growth rate affects timber density inversely was shown by the author (Schönau, 1974) and is confirmed by the higher average density (0,433) for the slower growing plant crop and lower average density (0,416) for the coppice crop with a faster growth rate as shown in the previous sections.

In SE.5 the timber densities of the plant and coppice crops were also highly significantly related (r = 0,581 with 141 df). DBH and number of coppice shoots per stool significantly contributed to this relationship which can be expressed as follows:

$$SGC = 0{,}2037 + 0{,}5027\ SGP + 0{,}003462\ DBH + 0{,}01192\ NSPS \quad ..... (2)$$

The coefficient of determination was 0,390, slightly better than that for SE.4. However, there was a small increase in timber density with increasing DBH. On the other hand, the inverse effect of growth rate was shown again by the lower average density (0,409) for the faster growing plant crop, and higher density (0,469) for the slower growing coppice crop. Possible causes of these conflicting results have been ascribed to sampling technique and inherent genetic and site differences (Schönau, 1980).

Further confirmation of the inverse relationship between timber density and growth rate was obtained when the treatment effect on average timber density per plot was studied. In SE.4 the average density was significantly increased by increasing number of coppice reductions when they started early (0,405 vs 0,419). In SE.5 the average density was significantly increased by leaving two shoots per stool, particularly when more than two reductions were carried out (0,460 vs 0,483). In addition, there was a strong trend in both experiments that the timber density increased with increasing number of shoots per stool when the coppice reductions were carried out in two operations and started when the trees were about 4 m tall. All these results show a general tendency of increasing timber density with decreasing growth rate.

CONCLUSIONS AND RECOMMENDATIONS

In addition to the system of reducing the large number of coppice shoots, the performance of a coppice crop is influenced by many other factors. They include weather and site conditions, stocking and uniformity of the initial stand, correct techniques used in establishing the plant crop, suitable stand density for the particular site capacity, felling technique, method of timber extraction, and management of trash after harvesting. These have been described in detail by Stubbings and Schönau (1980), especially the detrimental effect of poor and uneven initial stocking, substandard establishment techniques, inadequate weeding, felling the trees too high and during dry seasons resulting in high stools and increased stool mortality, damage to stools by careless timber extraction and burning, and incorrect spacing of remaining coppice shoots.

These experiments showed that DBH will be reduced by late and delayed coppice reduction but that the main reduction is due to an increased number of shoots per stool and increased stocking. Reduction to the original stocking gives generally the best results, especially when the stand density and mortality are high.

Basal area is decreased by late coppice reduction but if these reductions are carried out in two operations, starting at a shoot height of 4,3 m, the largest BA overall is obtained. On the other hand, BA is increased by higher stocking and greater number of coppice shoots per stool. However, there are limitations to these increases and reduction of shoots to the original stocking gives overall the best results, especially for the higher stand densities.

Height is little affected by coppice management.At an early age the greatest influence is due to wind damage when drastic coppice reductions are carried out in one operation. Reduction to the original stocking in two operations results generally in the best height growth. Retaining more than one shoot per stool reduces mean height at harvesting, but this reduction is generally less than 10%.

The effect of coppice reduction on bark thickness is closely related to that on DBH. However, the influence is more severe indicating that a complete measurement of under-bark volumes is required to obtain reliable results.

Stool mortality is affected by site capacity as well as increased by higher stand densities and unusual dry conditions. There are indications that evenly spaced trees can carry a greater number of shoots, and that later reductions of higher stockings give the opportunity for better selection.

Timber volume can be increased by up to 25% if more than one shoot is left per stool, but the additional yields are almost entirely confined to the smaller sized timber with a diameter less than 10 cm. The optimum stand density for maximum timber production on average sites seems to be 1 800 trees per ha but not more than 1 300 per ha for droughty conditions. If only large-sized timber in excess of 10 cm diameter is the object of management, the stand density should not exceed 1 500 trees per ha. Basal area is a reliable predictor of future yields and can be used as early as 5 years of age with acceptable errors.

Under average conditions the tree form of coppice stands is poorer than that of their parent crops. Early reduction gives better form, but later reductions provide the opportunity for improved selection after early wind or other damage. More shoots per stool affect total acceptable pole yields detrimentally but this is only slight when the number of shoots equals the original stocking.

Stump and kerf wastage of a coppice crop is about double that of its parent crop and it is increased by retaining more than one shoot per stool. Under average conditions and careful felling this wastage amounts to at least 4%.

There is no decline in productivity as expressed by MAI when that of a coppice crop is compared with that of its parent crop. The MAI seems to be closely related to the average rainfall during the growing season. On the other hand, MAI can be improved by manipulating the coppice management regime.

Timber density of a coppice crop is closely related to that of its plant crop. This relationship is affected by the unaccounted variations in site and genetic characteristics as well as differences in coppice management and growth rate. The latter shows the greatest influence and its relation to timber density is usually inverse, i.e. fast-growing timber is lighter and slow-growing timber is heavier.

From these results and general observations the following basic considerations, practices, principles and recommendations for the successful coppice management of *E.grandis* and other similar eucalypts are formulated:

A. Basic Considerations :

1. A uniform diameter of the parent crop is of primary importance for an even coppice crop.

2. More coppice rotations are possible if the parent crop is fully stocked.

3. Optimum stocking depends on site quality and object of management.

4. More than 2 shoots per stool will not increase total yield per ha.

5. More than 2 shoots per stool results in poorer form and more butt sweep.

6. More than 2 shoots per stool reduces utilizable volume.

7. Stool mortality can be compensated for by retaining more than one shoot per stool.

8. Adequate compensation for stool mortality is no longer possible after the number of viable stools has dropped below 1 000 per ha.

9. Severe suppression of diameter occurs after 2 to 3 years of age when more than 2 shoots are carried per stool.

10. During the first 2 years, coppice growth is faster than the growth of its plant crop, thereafter growth depends more on management and climatic conditions.

11. Timber densities of coppice and plant crop are closely related, but the inverse relationship between growth rate and timber density is overriding.

B. Felling Practice :

1. Stool mortality is higher from felling during dry periods or winter.

2. Fewer shoots develop per stool from felling during winter.

3. Frosty areas should be felled early in the growing season or during winter.

4. Fell areas liable to strong winds as long as possible before main winds occur or in spring.

5. Use a saw in felling and not an axe which produces high stools and poor bark attachment.

6. Cut bark at convenient height (1 m) and remove bark above cut when tree is standing.

7. Fell tree as low as possible with a saw in order to obtain firm bark attachment. Dead and loose bark increase windthrow of shoots due to strangulation of shoot collars by dead bark.

8. Retain bark collar on butt log if it will be used for treated poles. This will reduce splitting of the timber.

9. Slash and trash can be used to protect stools during in-field harvesting.

10. After extraction slash and trash must be removed from stools in order to prevent poor stem form.

C. Reduction Principles :

1. Remaining shoots should be dominant.

2. Remaining shoots should be firmly attached.

3. Remaining shoots should be lowest on stool.

4. Remaining shoots should be well spaced on stool.

5. Remaining shoots should preferably be on windward side of stool if only one shoot remains.

6. Remaining shoots should be well matched and the difference in their diameters less than 1 cm.

7. Early reduction makes trees straighter.

8. Early reduction produces heavier and more branches, and late reduction lighter and less branches.

9. Early reduction produces trees with greater taper, and late reduction trees with less taper.

10. Late reduction leads to trees with poorer form.

D. Recommendations for Coppice Management :

1. It is advisable to carry out the coppice reductions in two operations. These should be gradual in order to reduce so-called feathering.

2. The first reduction should be carried out when the dominant shoot height is about 4 m and 2 to 3 shoots should be retained per stool, depending on the object of management or stool size.

3. The second reduction should be carried out when the dominant shoot height is about 8 m and 1 to 2 shoots should be retained, depending on object of management.

4. To compensate for mortality of stools, additional best-matched shoots should be left on larger-diameter stools, especially near gaps or on edges of the plantation or adjacent to stools that have not coppiced.

5. The number of shoots per ha should not exceed the original stocking or the optimum stand density for the particular site.

6. When the production of poles for preservative treatment is the objective, retain only one shoot per stool and advance the second reduction.

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INTEGRATED SYSTEM FOR PRESENT AND FUTURE GROWTH AND YIELD PREDICTION AND ANALYSIS OF *Pinus caribaea hondurensis* IN AGUDOS, SAO PAULO, BRAZIL

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SUMMARY

The objective of this study has been to develop and apply some fundamental components of an integrated system of growth and yield prediction.

Data were collected in 120 permanent sample plots with an area of 500 m2, installed in stands of *Pinus caribaea hondurensis* plantations located in the county of Agudos, state of São Paulo, Brazil. The rotation age for these stands is 25 years, with selective thinnings of the smallest and worse trees at the ages of 8, 10, 12, 15, 19 and 22 years, reducing the number of trees per hectare to 1500, 1000, 700, 500, 300 and 200, respectively.

Firstly, a growth and yield model at stand level was generated. The Weibull distribution with two parameters was used to generate the model per diameter class. The "a" parameter was estimated as a function of the minimum diameter. The representation of selective thinnings of the worst trees, with priority to remove the smallest ones, was independently obtained from the model per diametric class. The adoption of an expression which represents the thinnings allowed for the consistent simulation of various thinnings, so that the biological standard verified was satisfactory.

KEYWORDS: *Pinus caribaea*, growth, yield, prediction.

INTRODUCTION

Thinnings have been, particularly in the case of Pinus sp, one of the most important silvicultural alternatives. They influence growth and yield greatly by distributing trees according to classes of vigor, size, and quality, by regulating stand spacing and density. The development of growth and yield models which include this variable becomes, therefore, very important.

The generalization of growth and yield models to predict parameters, both in thinned and in unthinned ones, has become a pressing necessity.

Amongst the methods for future growth and yield prediction at stand level in Brazil there are those by Campos (1980), Campos and Ribeiro (1983), Trevisol Jr. (1985), besides the model per diameter class developed by Glade (1986). None of these contemplates, however, the effects of thinnings or the compatibility between the model at stand level and by diameter class.

Basically, there are three ways of representing thinnings in a growth and yield system.

The first method, adopted by Bailey and Ware (1983), Bailey et al. (1985), Piennar and Shiver (1986), Murphy and Farrar (1988), consists in including, within the expression of basal area yield, an independent variable that reflects the type and method of thinning, and provides global stand information.

The second model used to represent estimates for stands subjected to thinnings is that used by Cao et al. (1982), and Matney and Sullivan (1982), who adopted the Weibull distribution in its truncated form. Likewise, Hafley and Buford (1985), and Smith and Hafley (1984, 1986), truncate the SB bivariate distribution.

The third possibility is found in Knoebell et al. (1986), and Baldwin and Feduccia (1987). The expression which quantifies the amount to be removed from the stand is independent of the distribution function.

EXPERIMENTAL METHODS

Data for this research were collected from thinned Pinus caribaea hondurensis plantations located in the southwest of São Paulo state, Brazil. According to Golfari (1978), the climate of this region is submontane or subtropical and humid. The altitude is between 600 and 1100 m above sea level.

The data used for the development of the assortment equation came from 1300 trees that were cubed according to the Smalian method. A second data set came from 900 circular permanent sample

plots with an area of 500 m^2, providing dendrometric information before and after each thinning, as well as on the remaining trees. These stands underwent six thinnings, at the ages of 8, 10, 12, 15 , 19, and 22 years, the number of trees being reduced to 1500, 1000, 700, 500, 300, and 200 per hectare, respectively.

The growth and yield model was designed in two stages. The first one consisted in the identification of stand attributes, and the other one involved the estimation of the probability of Weibull density function parameters.

The compatibility between the model at stand level and by diametric class was in terms of basal area, since a thinning simulator based upon the reduction of tree number or upon basal area reduction will be considered.

After the development of a prediction model for volume and basal area, the fitting for each time of thinning was done as follows: before the first thinning; after the first thinning; after the second thinning; after the third thinning; after the fourth thinning; after the fifth thinning; after the sixth thinning; after the first and second conjugated thinning; after the second and third conjugated thinning.

Finally, the analysis of variance in random blocks was performed. A sub-sample from the 900 plots previously specified was used. This sub-sample was defined on the basis of plots which were remeasured the six times mentioned above. This procedure allowed for the verification of whether data coming from the various thinnings could be grouped, so that greater flexibility was achieved for simulations of different management regimes.

The Weibull function with two parameters was used to generate the distribution of diameters in classes.

$$fx\ (x;\ b,\ c) = \begin{cases} \left(\frac{c}{b}\right)\left(\frac{x}{b}\right)^{c-1} \exp\left[-\left(\frac{x}{b}\right)^{c}\right] & b,\ c > 0 \\ 0, \text{ in other circumstances} & x > 0 \end{cases}$$

where: b = parameter of scale; c = parameter of form; x = accidental variable diameter.

The location parameter "a" is estimated independently of the Weibull function, being based upon the evolution of the minimum stand diameter.

The estimate of yield per unit of area obtained from prediction of diameter frequencies was based upon class intervals defined arbitrarily. The method developed by Hyink (1980) was adopted for estimation of the Weibull distribution parameters. The generation of stand average atributes followed the Burk and Newberry (1984) theory that the arithmetic average diameter must always be smaller than or equal to the square root of the square quadratic diameter. Thus, the estimates of the two moments, arithmetic average diameter ($\bar{D}$), and quadratic average diameter (Dg) were not, and should not be, done independently. The concept

used to estimate these moments is based upon the following expression:

$$D_g = \sqrt{\left(\frac{n-1}{n}\right) S_{di}^2 + \bar{D}^2}$$

where

S_{di} = standard deviation of diameters.

Once the parameters a, b, and c of the Weibull distribution are known, the diameter distribution is generated, so as to obtain the basal area, according to the expression

$$G = 0{,}00007853981N \sum_{i=1}^{\infty} x_i^2 f_i \qquad (1)$$

where:

N = number of trees per unit of area

x_i = central value of diameter class i

$f_i = F(x_i + 1{,}5) - F(x_i - 1{,}5)$ is the proportion of trees in the diameter class i

$F(x) = 1 - \left\{ \exp - \left[(x - a)/b \right]^c \right\}$ is the cumulative Weibull distribution.

When the basal area obtained from expression (1) is not equal to that estimated for the stand, the parameters are recalculated until the basal area obtained from the diameter distribution equals that obtained for the stand (Table 1.)

RESULTS AND DISCUSSION

The analysis of variance and specific tests indicated that four groups could be formed for volume and basal area prediction, namely:

Group 1 - no thinnings were done in the stand

Group 2 - including the first thinning, the second thinning, and conjugated 1-2

Group 3 - including the third thinning, the fourth thinning, and conjugated 2-3

Group 4 - including the fifth and additional thinnings.

TABLE 1: Management regimes adopted to verify the effect of the number, intensity, and age of thinnings

THINNING AGE	NUMBER OF REMAINING TREES AFTER EACH THINNING — MANAGEMENT REGIME 1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7								1400		
8	1500	1700	1500	1500	1600	1500	1400		1400	
9										1400
10	1000							800		
11									800	
12	700	900	700	700	1000	900	800			800
13										
14								500		
15	500	500	500		600	600	450		500	
16										500
17										
18								300		
19	300	300	300	300	300	251	200		300	
20										300
21										
22	200	200	200							
23					CF	CF	CF	CF	CF	
24										CF
25	CF	CF	CF	CF						

CF = clearcut

The model selected was Clutter's (1963), modified by Beck and Della Bianca (1972.)

$$\ln V_2 = b_0 + b_1 I_2^{-1} + b_2 S^{-1} + b_3 (I_1/I_2) \ln G_1 + b_4 (1-(I_1/I_2)) + b_5 (1-(I_1/I_2)) \cdot S \qquad (2)$$

The equation coefficients fitted for each thinning group are presented in Table 2.

TABLE 2: Equation coefficients fitted for each thinning group and used for projection of volume and basal area.

GROUP	COEFFICIENTS					
	b_0	b_1	b_2	b_3	b_4	b_5
1	0,687924	11,231980	- 7,622297	1,455610	2,539051	0,119259
2	2,845296	-11,742683	- 9,602816	1,119425	3,735376	0,032722
3	3,390047	-14,122695	-12,857076	1,041737	2,952862	0,053735
4	-	35,799274	- 1,186727	1,363275	-4,720794	0,399150

Figure 1 illustrates the behavior of volume development for management regime 1, showing that the total volume is always greater on more productive sites, even after the thinnings. A greater differentiation in increase of volume on these sites as opposed to the less productive ones is more notable in the two last thinnings, possibly because the uniformly-done thinnings on all sites may bring density close to an ideal on the most productive sites. These same kinds of thinnings are, perhaps, too heavy in the case of less productive sites, where the reaction of the remaining trees to growth in volume may, in effect, become difficult. The same behavior was observed regarding basal area growth.

The following model was developed to estimate the present number of trees (N) and basal area (G) per hectare:

$$Ln\ (N/G) = b_1 I_1^{-1} + b_2\ Ln\ (S) + b_3\ Ln\ (S\ .\ I_1)$$

The number of trees on basal area ratio was the solution used in order to estimate the number of trees and the present basal area efficiently. Independent fittings of these variables would be unacceptable for this situation, since the thinnings were executed to reduce the population. This fact would allow for the explanation of models that express the present number of trees only through the regression constant (b_0). Such behavior hampers the simulations of different thinning options.

In order to estimate the present number of trees and basal area, five distinct groups were obtained from the results found in random-block designs according to the Tuckey test for comparison of means.

The first three groups have the same features as those presented above, and the remaining two groups are:

Group 4 - including the fifth thinning

Group 5 - including the sixth thinning

The coefficients of the equations fitted for each thinning group are presented in Table 3.

FIGURE 1: Estimated volume tendency (m3/ha) for Pinus caribaea hondurensis at site indexes 25, 29, 33 planted with 2300 trees and thinned down to 1500, 1000, 700, 500, 300 and 200 trees, respectively, at the ages of 8, 10, 12, 15, 18 and 21 years.

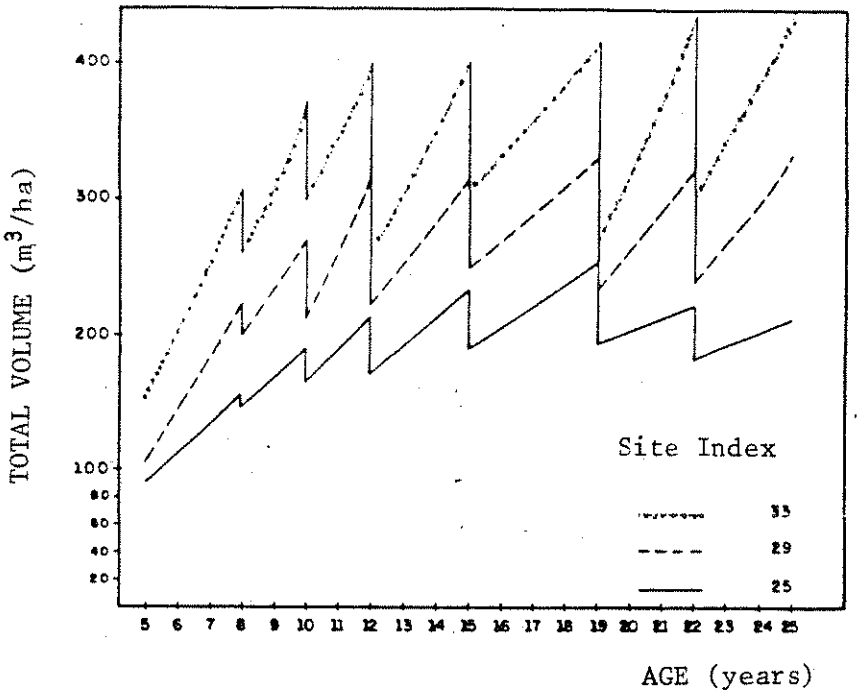

TABLE 3: Coefficients of the equations fitted for thinning groups in order to estimate the present number of trees and basal area

GROUPS	COEFFICIENTS		
	b_1	b_2	b_3
1	17,765265	- 2,875201	2,085648
2	21,632627	- 1,676004	1,220774
3	29,893565	- 1,480502	0,972282
4	36,607968	- 1,420935	0,854460
5	27,689941	- 0,938081	0,650178

It was observed, on the basis of the R^2 values, of the standard error of estimate (SE), and of the residuals analysis, that the fitted models presented entirely satisfactory estimates, both for the number of trees and basal area.

In the case of the equations that express diameter variation estimates, there was no clear evidence to allow grouping for different thinnings.

Table 4 presents the additional equations which make up the integrated system for prediction of growth and yield on the basis of diametrical class.

TABLE 4: Equations for prediction of stand variables and which make up the growth and yield system.

ATTRIBUTE	EQUATION	R^2(%)	S.E.
Site Index	$Hd = S \left[\frac{1 - EXP(-3,454065 \cdot I^{0,162451})}{1 - EXP(-3,454065 \cdot I_{REF}^{0,162451})} \right]^{125,683073}$	99,62	1,446000 m
Minimum diameter	$Dmin = 10,891531 + 18,975244 \cdot (1/I_2^2) + 2,05388 \cdot (Hd/I_2) - 0,005381 \cdot N + 0,02357 \cdot (D_g^2 \cdot I_2)^{1/2}$	92,53	1,967500 cm
Survival before 1st thinning	$N_2 = N(I_2/I_1)^{0,00403912} \cdot exp\left[-0,00379080 - (0,00018571 \cdot S) \cdot (I_2 - I_1)\right]$	94,09	41,710000 árvores
Average height per diameter class	$Ln(\bar{H}) = 0,344917 + 0,929915\ Ln(Hd) - 0,126947 \cdot Ln(D_g/D_i) + 4,292072/(I_2 \cdot D_i) - 4,654672\ D_i$	96,74	0,072186 Lncm
Commercial volume	$V_c = V(1-0,28502414 \cdot D^{4,29190978} \cdot D_c^{-3,96881555})$	99,67	0,026450 m^3
Variance of DHB:			
Before 1st thinning	$Ln(D_g^2-\bar{D}^2)=1,231830 \cdot LnHd-0,104429 \cdot Ln(N/G)-0,323432 \cdot (Hd/I)$	97,95	0,299296 $Lncm^2$
After 1st thinning	$Ln(D_g^2-\bar{D}^2)=1,467161 \cdot LnHd-0,368307 \cdot Ln(N/G)-0,330136 \cdot (Hd/I)$	97,71	0,338426 $Lncm^2$
After 2nd thinning	$Ln(D_g^2-\bar{D}^2)=1,728632 \cdot LnHd-0,787452 \cdot Ln(N/G)-0,154431 \cdot (Hd/I)$	97,69	0,355009 $Lncm^2$
After 3rd thinning	$Ln(D_g^2-\bar{D}^2)=1,449643 \cdot LnHD-0,804471 \cdot Ln(N/G)+0,062603 \cdot (Hd/I)$	96,92	0,285000 $Lncm^2$
After 4th thinning	$Ln(D_g^2-\bar{D}^2)=1,692346 \cdot LnHd-0,637314 \cdot Ln(N/G)-0,926253 \cdot (Hd/I)$	95,22	0,206900 $Lncm^2$
After 5th thinning	$Ln(D_g^2-D^2)=2,205404 \cdot LnHd-0,975195 \cdot Ln(N/G)-2,063499 \cdot (Hd/I)$	93,69	0,550098 $Lncm^2$
After 6th thinning	$Ln(D_g^2-D^2)=2,999533 \cdot LnHd-2,993005 \cdot Ln(N/G)-1,083318 \cdot (Hd/I)$	88,69	0,635706 $Lncm^2$
1st and 2nd thinning	$Ln(D_g^2-D^2)=1,475295 \cdot LnHd-0,291031 \cdot Ln(N/G)-0,725722 \cdot (Hd/I)$	98,58	0,236942 $Lncm^2$
2nd and 3rd thinning	$Ln(D_g^2-D^2)=1,921475 \cdot LnHd-0,888962 \cdot Ln(N/G)-0,516087 \cdot (Hd/I)$	97,99	0,335452 $Lncm^2$

Where:

I = age in years; I_{REF} = Index age (21 years); I_1 = present age; I_2 = Projection age; N_1 = Number of trees per hectaer at age I_1; N_2 = survival trees at I_2; V = total volume i.b.; H_d = Dominant height in m; G = basal area per hectare in m^2; S = Site index; D = DBH o.b. in cm; D_c = minimum commercial diameter o.b. in cm.

The thinning expression was the same one used by Knoebel et al. (1986).

$$P_i = \exp\left[b_1 \left(d_i^2 / d_g^2 \right)^{b2} \right]$$

Where,

P_i = proportion of basal area or number of removed trees in the diameter class "i"

d_i = central value of the diameter class "i"

d_g = average quadratic diameter before the thinning.

Tables 5 and 6 present the coefficients and respective statistics for the expressions of tree and basal area removal.

TABLE 5: Statistics and coefficients of the fitted equations for each thinning, in order to remove the number of trees

THINNINGS	COEFFICIENTS b_1	COEFFICIENTS b_2	R^2 (%)	SE	AVERAGE RESIDUAL
1	-1,72832742	2,1891446	90,92	0,1212023	0,00924067
2	-1,66772739	2,6804186	86,69	0,1360140	0,00878878
3	-2,28936124	4,6951476	88,94	0,1345734	0,01051560
4	-3,25217112	7,0762733	85,38	0,1536220	0,01043680
5	-2,15622880	10,2957106	88,57	0,1450860	0,00783275
6	-4,89466220	15,6883291	91,28	0,1233280	0,00499895

TABLE 6: Statistics and coefficients of the fitted equations to remove basal area for each thinning

THINNINGS	COEFFICIENTS b_1	COEFFICIENTS b_2	R^2 (%)	SE	AVERAGE RESIDUAL
1	-1,76079163	2,1454380	90,49	0,12377	0,00921651
2	-1,70147236	2,6631080	86,77	0,13630	0,00907598
3	-2,35649674	4,6493322	88,74	0,13531	0,01108120
4	-3,31629804	7,0131174	85,33	0,15329	0,01033320
5	-2,23620450	10,2827893	88,42	0,14587	0,00816480
6	-4,93628710	15,4500959	91,28	0,12284	0,00500462

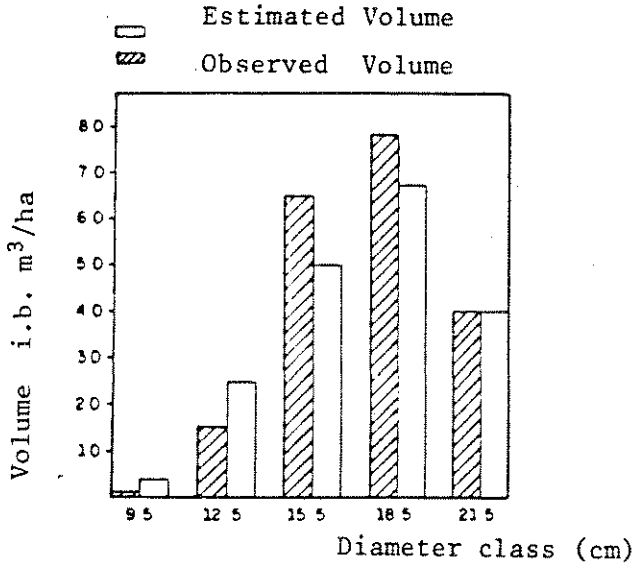

FIGURE 2: volume obtained through thinning simulator and the real one at 7, 9 years.

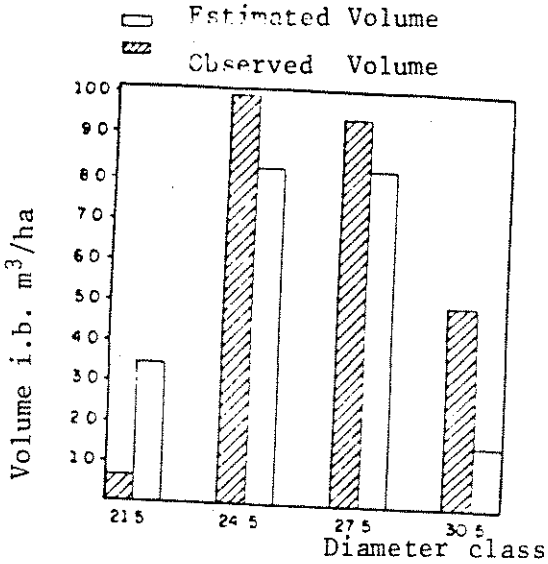

FIGURE 3: volume obtained through thinning simulator and the real one at 10 years.

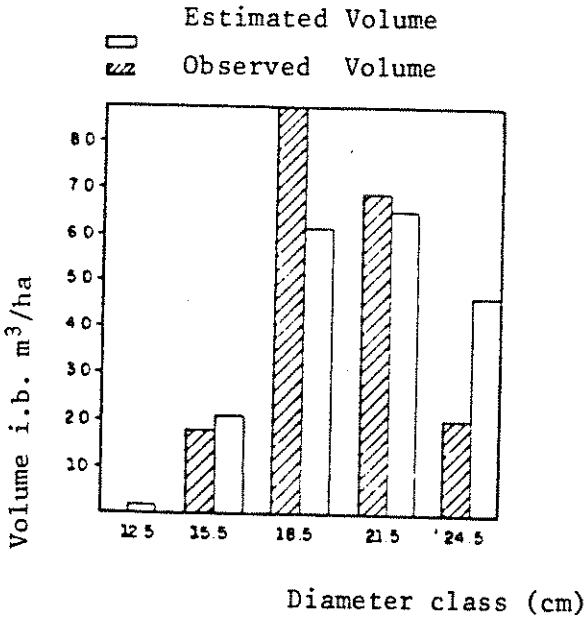

Figure 4: volume obtained through thinning simulator and the real one at 15 years.

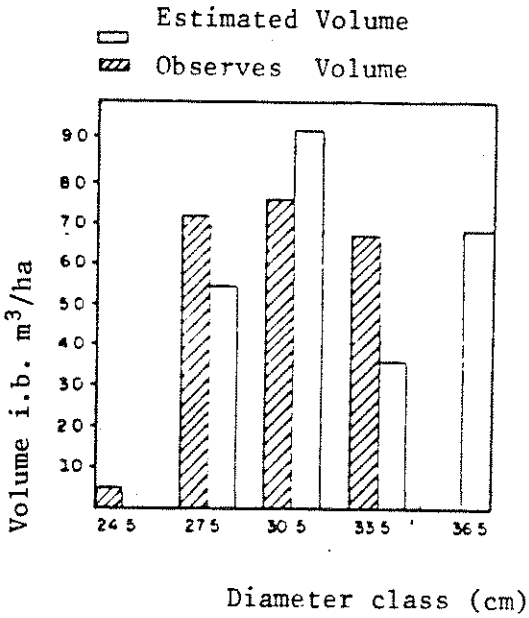

Figure 5: volume obtained through thinning simulator and the real one at 19 years.

Figures 2, 3, 4, and 5 show the observed and the predicted volume obtained through the simulation of thinnings upon management regime 1, at the ages of 7, 9, 10, 15, and 19 years.

It can be observed in the situations presented that the diameter class inferior and superior limits, both for the actual situation and for the one generated by the Weibull distribution, are perfectly consistent. One also observes that a greater accuracy in future volume projection implies a greater efficiency of the expression to quantify the amount removed in each thinning. At the age of 19 years, for example, the projected volume is 323m3/ha, while the volume observed is 347m3/ha. This difference of 24m3 is not taken into account when the thinning expression is used.

Table 7 presents the summary of estimates based upon the diameter classes for management regime 1.

Out of the ten management regimes presented in Table 1, the first four ones consider different thinning numbers; the following three regimes express different thinning weights; the last three ones express different thinning times. Given space constraints, this paper just presents the results for management regime 1.

A greater yield was observed for regime 1 when its volumes for veneer and sawmill were compared with those of additional regimes. This is in conformity with the expected standard.

Management regime 4 (results not included herein), which considers three thinnings, presented a greater total volumetric yield than the other ones, as expected.

For all the simulated management regimes, consistent results were observed in relation to those expected, and for all situations the expression which represents the remotion of number of trees presented a very satisfactory performance.

CONCLUSIONS

The developed system is an objective tool for the management of planted forests, allowing for precise and reliable estimates of the population parameters.

The Weibull function was efficient and flexible, providing, in most cases, precise diameter distribution estimates. The equations which represent the remotion standard of the number of trees and of basal area appeared to be very efficient to express the selective thinning of the worse trees, although the procedure can involve a certain degree of inflexibility. The dendrometrical relationships adopted demonstrate the subjective character of the selective thinning of the worse trees very well.

TABLE 7 - Summary of estimates based upon the diametrical classes for the number of trees (N), basal area (G), total volume (V), volume for veneer (VL), volume for sawmill (VS), and volume for particle-board (VA) taking into account the situation immediately before thinning, the trees removed in thinnings and the ones remaining for site indexes 25, 29 and 33, starting with 2250 trees and at the age of 5 years, for management regime I.

SITE INDEX	AGE	BEFORE THINNING						REMOVED TREES						AFTER THINNING						TOTAL YIELD			
		N	G	V	VL	VS	VA	N	G	V	VL	VS	VA	N	G	V	VL	VS	VA	V	VL	VS	VA
25	8	2198	40,27	179,0	0,0	40,1	177,9	698	7,7	30,2	0,0	0,4	29,7	1500	32,6	148,8	0,0	39,7	148,2	179,0	0,0	40,1	177,9
	10	1500	36,60	199,4	0,0	75,2	198,7	500	8,4	42,0	0,0	3,4	41,7	1000	28,2	157,4	0,0	71,8	157,0	229,6	0,0	75,6	228,4
	12	1000	31,84	202,6	0,0	113,0	202,3	300	6,8	40,4	0,0	9,2	40,3	700	25,0	162,3	0,0	103,8	162,0	274,8	0,0	116,8	273,7
	15	700	30,00	223,8	0,0	167,0	223,6	200	6,6	47,6	0,0	26,1	47,5	500	23,3	176,2	0,0	138,8	176,1	336,4	0,0	180,0	335,3
	19	500	28,60	247,6	33,6	211,9	247,5	200	9,0	75,5	0,0	58,8	75,5	300	19,6	172,1	33,6	153,1	172,0	407,8	33,6	253,0	406,7
	22	300	21,60	199,8	55,5	181,4	199,7	100	6,0	55,0	0,7	48,0	54,9	200	15,6	144,9	54,8	133,8	144,8	435,5	55,5	281,3	434,4
	25	200	17,40	174,6	91,0	163,9	174,6	200	17,4	174,6	91,0	163,9	174,6	0	0,0	0,0	0,0	0,0	0,0	465,3	91,7	311,8	464,2
29	8	2193	45,54	244,1	0,0	76,3	242,9	693	8,7	41,3	0,0	1,6	40,8	1500	36,8	202,8	0,0	74,7	202,1	244,1	0,0	76,3	242,9
	10	1500	41,32	269,4	0,0	126,6	268,6	500	9,5	57,1	0,0	8,8	56,8	1000	31,8	212,2	0,0	117,8	211,8	310,7	0,0	128,2	309,4
	12	1000	35,93	271,6	0,0	172,8	271,2	300	7,6	53,9	0,0	18,3	53,7	700	28,4	217,8	0,0	154,8	217,5	370,0	0,0	183,2	368,8
	15	700	34,50	310,0	14,9	250,4	309,8	200	7,4	63,0	0,0	42,1	62,9	500	27,2	247,0	14,9	208,3	246,9	462,3	14,9	279,1	461,1
	19	500	33,70	350,6	93,0	314,1	350,4	200	10,6	106,6	0,0	89,4	106,5	300	23,1	244,0	93,0	224,7	243,9	565,9	93,0	384,9	564,6
	22	300	29,10	328,1	198,3	311,5	328,0	100	7,9	88,3	38,5	82,0	88,3	200	21,1	239,7	159,8	229,5	239,7	650,0	198,3	471,9	648,7
	25	200	26,10	315,6	246,6	306,8	315,5	200	26,1	315,6	246,6	306,8	315,5	0	0,0	0,0	0,0	0,0	0,0	725,5	285,1	543,0	724,5
33	8	2188	49,10	307,0	0,0	112,2	305,7	688	9,3	51,9	0,0	3,1	51,3	1500	39,7	255,1	0,0	109,1	254,4	307,0	0,0	112,2	305,7
	10	1500	44,20	342,7	0,0	181,5	341,9	500	10,1	71,5	0,0	13,8	71,0	1000	34,1	271,3	0,0	167,6	270,8	394,6	0,0	184,6	393,2
	12	1000	38,80	344,0	12,2	235,0	343,5	300	8,0	66,0	0,0	25,7	65,8	700	30,8	278,0	12,2	209,3	277,7	467,3	12,2	251,9	465,8
	15	700	38,50	406,6	49,7	343,7	406,3	200	8,2	81,5	0,0	59,3	81,4	500	30,3	325,0	49,7	284,4	324,8	595,9	49,7	386,3	594,4
	19	500	38,40	461,2	174,6	423,4	461,0	200	12,1	142,4	12,5	124,6	142,3	300	26,3	318,8	162,1	298,8	318,7	732,0	174,6	525,3	730,5
	22	300	37,90	506,3	389,3	491,4	506,2	100	10,4	136,9	92,0	131,2	136,9	200	27,5	369,4	297,3	360,2	369,3	919,5	401,8	717,9	918,0
	25	200	37,60	460,0	396,6	451,9	460,0	200	37,6	460,0	396,6	451,9	460,0	0	0,0	0,0	0,0	0,0	0,0	1010,0	501,1	809,6	1008,7

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Tree and Stand Volume Functions for Eucalyptus Saligna in Western Kenya

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Fuelwood, for both personal and industrial consumption, forms a primary use of forests in Kenya. Tea and tobacco are both important cash crops in the Kenya economy and their leaves are dried using wood fuel. A convenient source of wood is therefore essential, and plantations of Eucalyptus saligna have been established in the western part of the country by tobacco farmers and, on a larger scale, by tea estates. Use of these exotic plantations helps protect indigenous forests which would otherwise be exploited.

Eucalyptus plantations have been grown in western Kenya since at least the 1930's, but very little information is available concerning either individual tree volumes or per hectare yields. If reasonable plans for the land base dedicated to fuel plantations and rotation length are to be formulated, tree and stand volume equations are necessary. The objective of this study was to develop the necessary tree and stand volume equations for Eucalyptus saligna plantations in western Kenya.

METHODS

Sixty-three plots were located in Eucalyptus saligna plantations established for fuelwood on tea estates near Nandi Hills, Kenya.

Rainfall in the area is reasonably distributed throughout the year and averages about 1600 mm with moderate peaks in April-May and August. Nandi Hills lies near the equator, but temperatures are mild (10°-25° C) all year due to the 2000 m elevation.

Plots were established to contain approximately 60 original planting spots. Every tree was measured for dbh to the nearest centimeter using a diameter tape. The first and every fifth tree were measured for total height with a Blum-Leiss hypsometer to the nearest 0.5 m. Crown class was denoted.

In addition to plot information, 2 trees at each location were felled and sectioned into bolts, each one meter long. Average outside and inside bark diameter measurements to the nearest 0.1 cm were made at each cut including the stump cut. Tip length from the last cut to the tip of the tree was measured to the nearest 0.1 m. Dbh was measured before felling and total height was measured before sectioning the felled tree.

Cubic meter volumes were calculated for each section of each sample tree. The butt section volume was calculated using Newton's formula, intermediate section volumes were calculated using Smalian's formula, and the tip volume was calculated assuming it was a cone. Summation of butt, intermediate, and tip volumes resulted in total volume of each sample tree.

A volume equation was obtained by fitting the sectioned tree data to the logarithmic volume equation (Schumacher and Hall, 1933; Bredenkamp, 1982).

Total tree volumes for trees with heights measured were obtained by using dbh and total height in the logarithmic volume equation. For

those trees without measured heights, heights were predicted using a regression equation developed from trees with measured heights. This equation predicted height as a function of dbh and stand quadratic mean diameter. Predicted heights were then used in the logarithmic equation to predict total volume.

Merchantable volumes were obtained using a merchantable volume equation which allowed calculation of merchantable volume to any top diameter limit. The equation is of the form:

$$MV = TV * \left(1. - \beta_1 d^{B2} D^{B3}\right)$$

where:

MV = merchantable volume (m^3) to a top diameter limit outside bark (ob) of d cm,

TV = total volume

d = merchantable diameter limit in cm (ob), and

D = dbh (cm).

This equation form has been used with success by several previous investigators (Queen and Pienaar 1977; Brister et al. 1980). Individual tree volumes were summed to obtain plot volume which was then expanded to a per hectare basis. A dataset was compiled which consisted of volume per hectare, age, height of dominant and codominant trees, basal area per hectare, stems per hectare and quadratic mean diameter for each plot. The dataset contained separate observations for total volume per hectare and merchantable volume per hectare to 2, 4, 6, and 8 cm top diameters. Merchantable volumes per hectare were obtained by summing merchantable individual tree volumes and expanding to a per hectare basis as with total volume.

An equation of the following form was fit to the dataset:

$$V_d = \beta_4 BA^{\beta_5} * H_D^{\beta_6} * A^{\beta_7} * \exp((\beta_8 * (d/D_q)^{\beta_9}))$$

where:

V_d = volume of stand (m^3/ha) to a top diameter limit outside bark (ob) of d cm,

BA = basal area per hectare (m^2/ha),

H_D = height of dominants and codominants (m),

A = age (years),

d = top diameter limit in cm (ob),

D_q = quadratic mean diameter, cm, and

β_4-β_9 = parameters estimated from the data.

The equation consists of a total volume function which is modified by:

$\exp(\beta_8 * d/D_q{}^{\beta_9})$.

This modifier was proposed by Amateis et al. (1986), to convert any total volume yield equation into a variable top, merchantable volume yield equation. Rather than fit the total volume equation and the merchantable modifier separately, one equation was formed for both total and merchantable volume and fit. A desirable property of the function is that when d = o, V_d = total volume.

Since the real utility of any yield system is for planning, it was desirable to obtain estimates of how the components of the model could be expected to change over time. Without remeasurement data this is difficult, but equations were fit to predict height of dominants and codominants, stems per hectare, and basal area per hectare for varying ages. The model forms were:

$$H_D - \beta_{10} * (1 - \exp(-\beta_{11} * A))^{\beta_{12}}$$

$$SPH = \beta_{13} A^{\beta_{14}}$$

$$BA = \beta_{15} SPH^{\beta_{16}} * H_D^{\beta_{17}}$$

where:

SPH = stems per hectare.

RESULTS

The individual tree volume equations obtained were:

$$V = .000030306 D^{2.03712722} H^{0.994221}$$

$$MV = V(1 - .985647257 d^{3.53961524} D^{-3.52981363})$$

where:

V = total volume in m^3(ob),

D = dbh in cm,

H = total height in m,

MV = merchantable volume to top diameter limit of d cm (ob), and

d = merchantable top diameter limit in cm (ob).

The stand volume equation was:

$$V_D = 0.546039060 * BA^{0.975960516} * H_D^{0.898622151} * A^{0.026674208} * \exp\left(-2.343135547 * (d/D_q)^{5.140285645}\right)$$

The height, stems per hectare, and basal area per hectare equations fitted were:

$$H_D = 26.88788020 * (1. - \exp(-0.44048674 * A))^{1.67721622}$$

$$SPH = 2209.248892 * A^{-0.337856270}$$

$$BA = 0.001954458 * SPH^{0.627336642} * H_D^{1.57822693}$$

DISCUSSION

Fuelwood plantations are single product forests. Unlike situations where pulpwood, sawlogs, and veneer logs are obtained from the same stand, only an estimate of the total or merchantable volume is required. For these types of populations, a complicated diameter distribution yield system is not necessary. A single equation giving volume per hectare yield as a function of the usual yield system inputs (age, site index or height of dominants and codominants, and some measure of stand density) is all that is required. The ability to produce one equation for both total and merchantable volume makes the system even simpler to use.

The stand volume equation derived for Eucalyptus saligna plantations explained more than 95% of the total variation in volume and produced uniform residual plots when plotted over all independent variables.

The height-age curve is a guide curve which predicts the average height of dominants and codominants that can be expected at any given age. Use of the guide curve requires acceptance of the assumption that older plots, on average, represent the future state of the younger plots, on average. If, for any reason, the older plots are on better or worse sites than the younger plots, the height growth pattern will be different than the one derived. This assumption is always necessary

when attempting to model changes over time from temporary plot data. An anamorphic set of site index curves (Figure 1) can be derived from the guide curve as:

$$S = H_D((1.-\exp(-0.44048674 * A_o))/(1.-\exp(-0.44048074 * A)))^{1.67721622}$$

where:

S = site index, and

A_o = base age.

A reasonable base age for these plantations might be 10 years since that is the average current rotation age. To use the site index function, measure several dominant and codominant trees in a stand of a given age to obtain H_D. Then calculate S from the site index equation. The expected average height of dominants at different ages can then be determined from inverting the site index equation:

$$H_D = S/((1.-\exp(-0.44048674 * A_o))/1.-\exp(-0.44048674 * A)))^{1.67721622})$$

The advantage of using the site index system is that a more precise estimate of H_D can be obtained by incorporating site index than by simply using the guide curve for the expected value given a stand age.

Site index was not important for predicting stems per hectare. This is probably because at these ages very little, if any, mortality had resulted from competition. It is also interesting that the number of trees planted was not important in predicting stems per hectare. This is probably due to the fact that almost all plantations were planted at about the same density and survival has been generally excellent.

As expected, the basal area per hectare equation does include H_D and SPH. The higher the dominant height and the higher the SPH, the higher the amount of basal area per hectare produced.

Use of these equations to estimate changes in stand volume equation variables over time allows use of the stand volume equation to estimate mean annual increment (MAI) (Figure 2). MAI culminates for a fairly wide range of site indices at about age 5. This indicates that managers could cut their current rotation by half (10 to 5 years). According to the volume per hectare values in Figure 2, approximately 50 percent more wood could be produced by growing two, five year rotations in place of one ten year rotation. As a result, a smaller land base could be used to produce the same amount of wood fuel as is currently produced.

It is impossible to determine, using these data, the effect of planting more or fewer stems per hectare due to the limited original planting densities across all sample plots. The height growth curves are steep up to age 5 after which they are flat (Figure 1). This may be largely responsible for the culmination of MAI at this age.

SUMMARY

A function was derived which predicts volume per hectare for total volume and volume to any merchantable top diameter limit. This yield function is all that is necessary for planning in situations such as fuelwood plantations where a single product is desired. An attempt was made to examine trends over time even though no remeasurement data was available. Equations predicting dominant height, stems per hectare, and

basal area per hectare were derived. Volumes were predicted for varying site index values over time and the resulting mean annual increment values culminated at about age 5 for all site indices.

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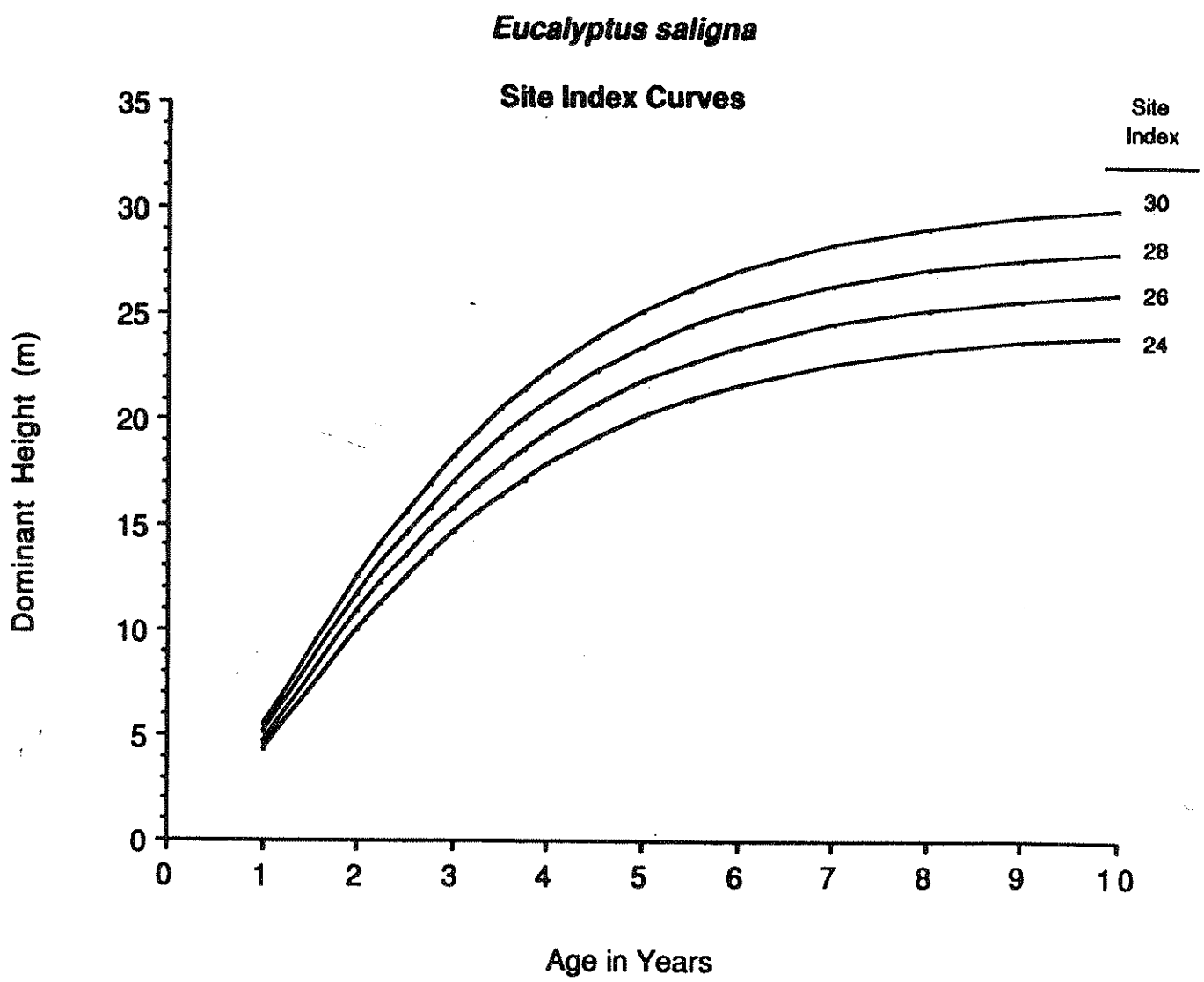

Figure 1. Site index curves for Eucalyptus saligna in western Kenya.

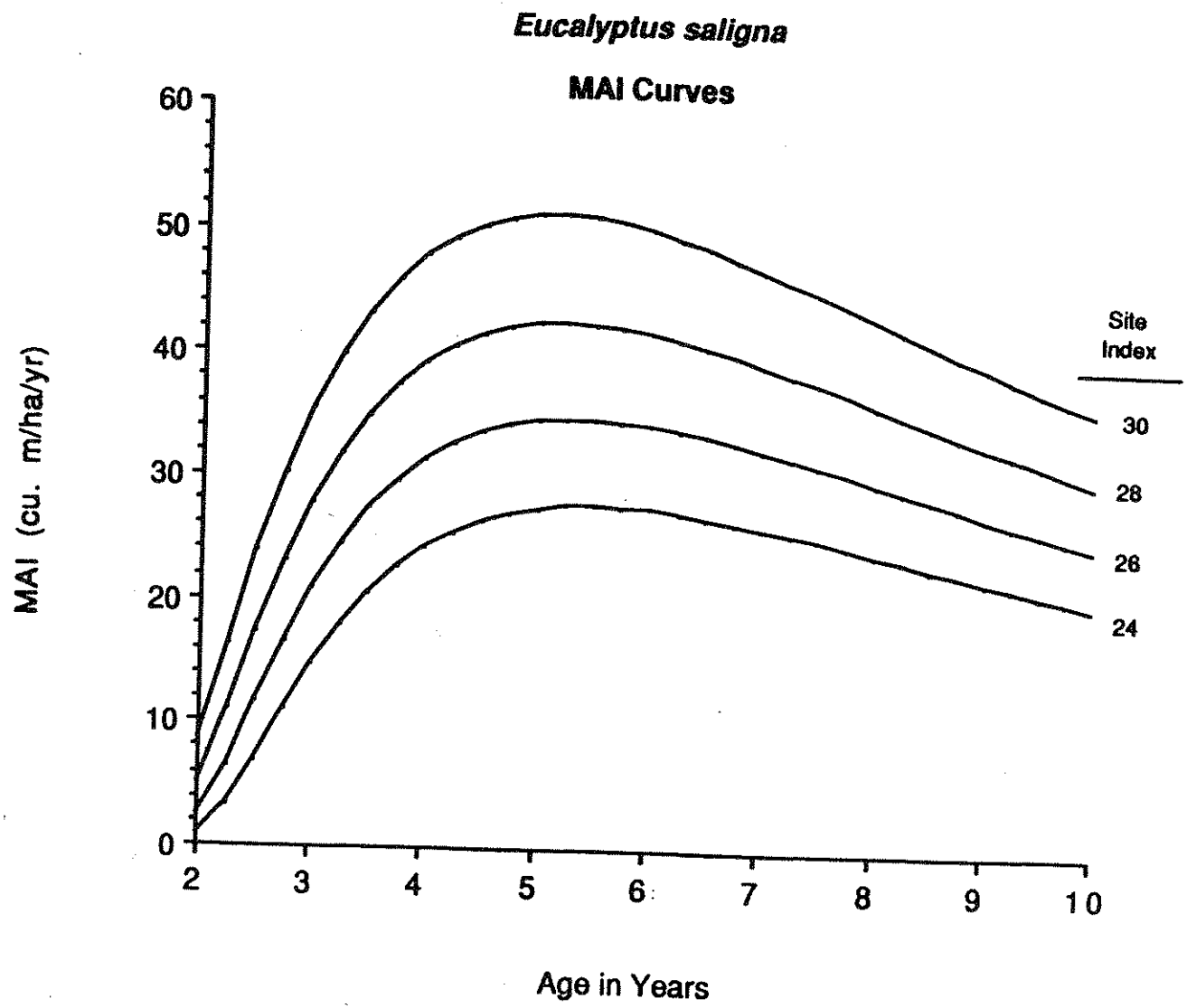

Figure 2. MAI (m^3/ha/yr) for Eucalyptus saligna plantations in western Kenya on different sites.

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EFFECTS OF PLOT-SIZE ON THE PRECISION OF GROWTH ESTIMATION ON AN EXPERIMENT IN PINE

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INTRODUCTION

Designed experiments in forestry are by nature long-term, expensive to perform, and occupy large areas of land. It is known that the larger the plot size and the greater the number of plots, the better the estimate in terms of smaller variance. If there are many silvicultural treatments to perform in a block there are often difficulties to find place for them in the same stand. Therefore it is very important to use optimum plot and block sizes to maximize the information from an experiment. In growth and yield research the aim is to be able to judge if there is a difference between treatments measured for example as growth per unit area. The needed plot area for an experiment should also be dependent on how many trees there are per unit area.

Strand (1957) found that the coefficient of variation of basal area or volume decreases approximately in proportion to the inverse of the square root of the plot size. Freese (1961) reported that the squared coefficient of variation is inversely proportional to the square root of the relative plot size. Whittle (1956) states that for small plots the variance is proportional to the square of the area, while for large regions the variance is proportional to the area.
Arvanitis & O'Regan (1967) found that the addition of a quadratic term to the variance equation resulted in a better fit.
In general most studies within the field plot-size and variance are concerned with sampling.

The aim with this study is to study the variance at different plot sizes in terms of increment per hectare and year at different times of the rotation period.

MATERIAL AND METHODS

The field-data for this pilot-study comes from a permanent sample plot established in 1903 by the Department of Forest Yield Research, Swedish University of Agricultural Sciences. The stand was born in 1866 by natural seeding and described in 1903 as moss-rich pine forest with suppressed spruce saplings. The stand was located approximately 100 kilometer Southwest of Stockholm. The ground is sea-sediment on a light slope westwards and mesic herb-type. The latitude is 58 degrees north and the height above sea-level is 50 m. The plot is rectangular with the sides 65 respective 37.66 meters, giving a plot-size of 0.2448 hectares. The plot is low-thinned in 1903, 1909 and 1912. Free thinning is made 1917. Extra heavy thinnings are made 1925, 1930 and 1935. The plot is examined thirteen times. A yield table with numbers of trees, standing and removed crop at the different examinations is given in Table 1.

The spatial position of all trees is known. At each examination the diameter at breast height was measured and registered individually on living trees. Mortality was registered at each examination for each tree and trees removed by thinning were coded. Measurement of tree and crown heights were only made on sample trees. Estimation of the standing volume at each examination and thereby also the growth, is done with help of a special computer program developed within the Department of Forest Yield Research. In order to obtain heights at every examination time the relationship between diameter and height on sample trees was used (Näslund, 1936) to allot tree-heights on all trees.

The plot will be divided in 4, 9, 16 and 25 parts. Each part will be regarded as a new plot and the growth per unit area calculated at each examination. In this study the growth measure is volume cubic meters per year and hectare. The distribution of trees within the plot and how many trees in each subdivision is shown in Figure 1 and the mean, minimum and maximum values for annual growth is given in Table 2. The relationship between the variance per plot (V) and the size (A) of the plot will be studied over time. The data will be analyzed with Fairfield Smiths law (Smith, 1938).

$$V=c*A**(-b)$$

Where;
V= Variance
A= Plot area
c=constant
b=Fairfield Smiths exponent

Table 1. Yield table for Plot 9:2 Södermanland, Jönåker.

			STAND AFTER THINNING			REMOVED			TOTAL YIELD
	age	dom height	no of stems	basal area	volume	no of stems	basal area	volume	volume
	years	dm	no/ha	m2/ha	m3sk/ha	no/ha	m2/ha	m3sk/ha	m3sk/ha
PINE	38	15.9	1585	25.1	162.0	0	.0	.0	
SPRUCE			580	2.4	16.5	4	.0	.0	
TOTAL			2165	27.5	178.5	4	.0	.0	178.5
PINE	43	17.0	1254	25.6	181.2	294	2.6	16.2	
SPRUCE			494	2.5	18.8	78	.3	3.3	
TOTAL			1748	28.1	200.0	417	3.5	19.5	221.0
PINE	46	17.5	1013	24.4	186.5	253	2.6	23.6	
SPRUCE			462	2.8	20.7	25	.1	0.9	
TOTAL			1475	27.2	207.2	278	3.4	24.5	253.0
PINE	51	18.7	670	19.5	154.5	323	7.6	59.8	
SPRUCE			437	3.4	26.2	37	.2	0.9	
TOTAL			1107	22.8	180.7	372	7.8	61.6	288.0
PINE	55	19.5	433	15.5	128.1	225	5.5	44.7	
SPRUCE			404	3.6	27.4	45	.8	7.0	
TOTAL			837	19.1	155.5	274	6.4	52.6	315.0
PINE	60	20.6	413	17.5	154.2	20	0.6	5.1	
SPRUCE			400	4.6	35.5	8	.0	.0	
TOTAL			813	22.2	189.8	28	0.6	5.1	354.0
PINE	65	21.5	323	16.9	156.2	90	3.4	29.8	
SPRUCE			372	5.0	38.1	37	.9	7.7	
TOTAL			695	21.9	194.3	127	4.3	37.4	396.
PINE	70	22.3	278	16.1	157.4	45	2.4	23.2	
SPRUCE			298	4.7	36.4	78	1.4	11.2	
TOTAL			576	20.7	193.9	123	3.8	34.5	430.
PINE	75	23.1	274	17.9	175.7	4	0.3	3.1	
SPRUCE			372	5.4	43.4	53	0.6	4.9	
TOTAL			645	23.3	219.1	57	0.9	8.0	463.
PINE	80	23.6	274	19.2	193.6	8	0.6	6.0	
SPRUCE			294	5.4	45.0	74	1.0	8.2	
TOTAL			568	24.6	238.6	82	1.6	14.2	497.
PINE	85	24.3	270	20.7	211.3	4	0.3	2.9	
SPRUCE			172	4.3	39.2	139	2.0	16.1	
TOTAL			442	25.0	250.6	143	2.3	19.0	528.
PINE	90	25.1	270	22.3	235.5	4	0.2	1.7	
SPRUCE			118	3.7	35.1	49	0.9	7.5	
TOTAL			388	26.1	270.6	53	1.1	9.2	557.
PINE	97	26.2	208	19.3	209.7	57	5.4	57.3	
SPRUCE			102	3.6	36.7	20	1.2	14.5	
TOTAL			310	22.9	246.3	77	6.6	71.7	604.

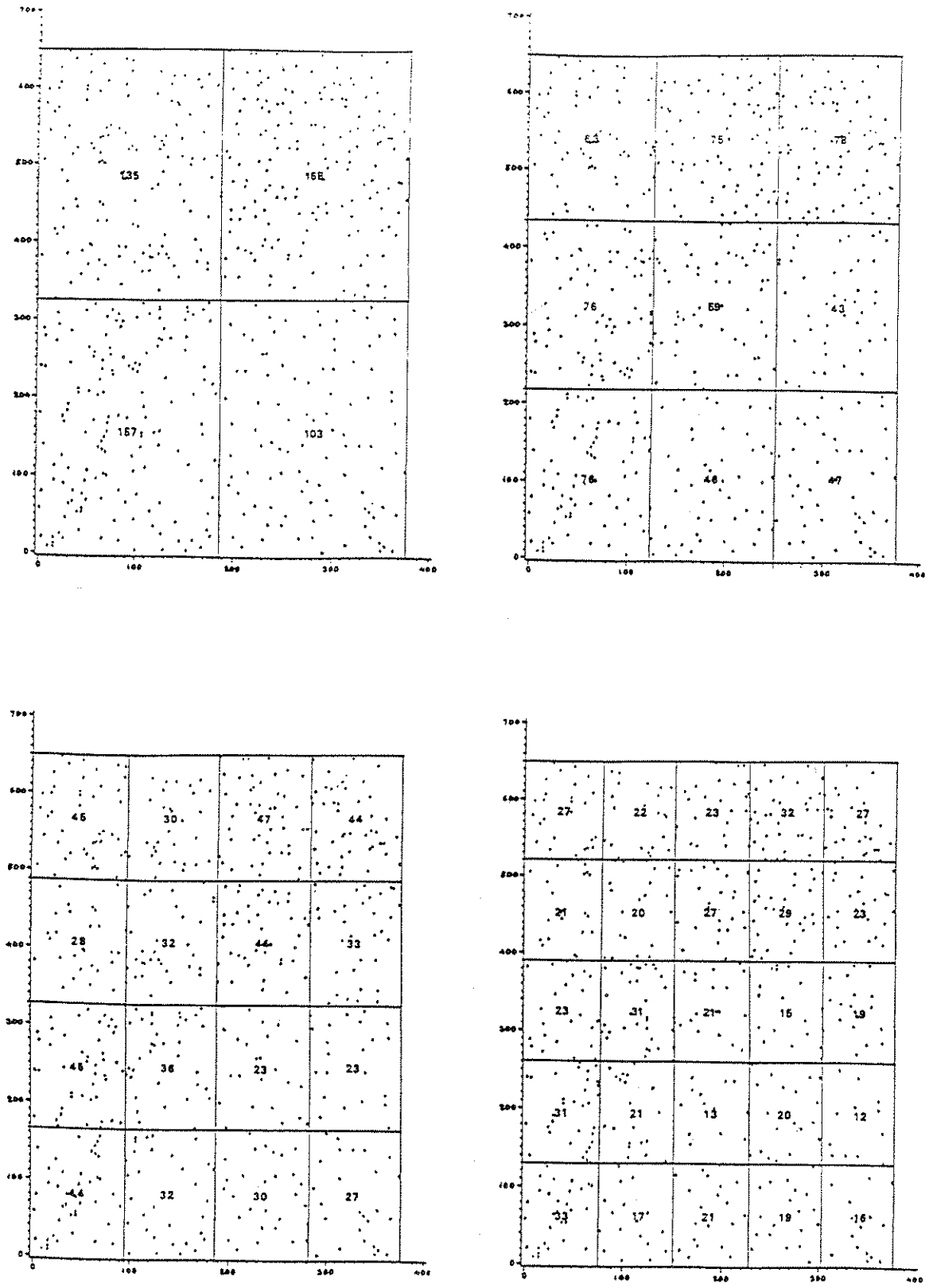

Figure 1. The plot and the distribution of trees in each subplot. The number of trees in each subplot is given.

Table 2. Mean, minimum and maximum values for annual growth in cubic meter per hectare and year at different examinations. Minimum and maximum values are given in parentheses.

Examination	Mean 1/4	Mean 1/9	Mean 1/16	Mean 1/25
2	7.1(6.0-8.4)	7.2(5.1-9.2)	7.2(5.0-9.7)	7.2(3.7-10.9)
3	10.6(9.3-12.2)	10.4(7.7-13.5)	10.4(7.6-15.7)	10.5(6.6-16.0)
4	7.0(6.4-7.5)	6.9(4.7-8.8)	6.9(3.9-8.8)	6.9(3.5-11.4)
5	9.2(7.8-11.1)	8.6(4.0-12.6)	9.2(6.1-13.9)	9.2(5.9-17.3)
6	7.9(6.6-10.0)	7.9(5.5-10.6)	7.9(4.8-11.7)	7.9(5.2-16.3)
7	8.4(7.4-10.2)	8.4(5.6-11.0)	8.4(5.1-11.2)	8.4(5.2-13.2)
8	6.8(5.5-9.1)	6.8(5.0-8.9)	6.8(3.5-10.3)	6.8(3.8-15.2)
9	6.6(4.7-8.0)	6.6(3.7-9.5)	6.6(2.3-10.2)	6.6(2.9-14.5)
10	6.7(5.5-7.9)	6.7(4.6-8.6)	6.7(3.3-8.5)	6.7(3.3-14.1)
11	6.2(4.2-8.8)	6.4(3.2-13.7)	6.2(1.6-18.6)	6.8(1.6-24.9)
12	6.0(5.3-7.0)	5.9(3.9-4.7)	5.6(2.9-8.7)	6.0(2.6-13.7)
13	6.8(5.4-7.9)	6.8(4.1-9.2)	6.6(2.5-10.0)	6.8(2.8-17.0)

RESULTS

The variance for the different plot sizes is shown in Figure 2. The variance is greatest for the smallest plot size (97.92 m2) where the original plot is divided in 25 different parts. The variance is smallest in eleven out of twelve cases for the largest plot size (612 m2) where the original plot is divided in four different parts. In examination eleven which is different from all the other examinations all ingrowth trees where recorded and numbered. The regression coefficients and the R-square(r2) are given in Table 3 together with root mean square error (RMSE). Dependent variable is the natural logarithm of the (variance+10) and independent variable natural logarithm of the area.

Table 3.

Examination	Var 1/4	Var 1/9	Var 1/16	Var 1/25	c	b	r2	RMSE
2	0.96	1.87	1.65	3.83	3.06	0.10	0.72	0.064
3	1.45	3.48	3.41	6.20	3.51	0.17	0.87	0.062
4	0.26	1.90	1.69	4.94	3.44	0.18	0.80	0.086
5	1.90	5.25	6.00	8.55	3.96	0.22	0.96	0.044
6	2.47	3.09	4.18	7.21	3.53	0.16	0.82	0.073
7	1.69	3.23	2.47	5.38	3.53	0.12	0.66	0.083
8	2.50	1.85	3.52	5.34	3.18	0.11	0.62	0.085
9	2.02	3.00	4.62	7.21	3.67	0.19	0.92	0.055
10	0.98	2.06	2.70	7.05	3.72	0.21	0.79	0.107
11	3.71	12.24	19.12	34.93	6.59	0.62	0.98	0.080
12	0.51	2.17	2.59	6.69	3.76	0.22	0.84	0.094
13	1.17	3.13	4.85	8.77	4.10	0.27	0.94	0.064

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DISTANCE DEPENDENT COMPETITION MEASURES TO MODEL GROWTH OF INDIVIDUAL TREES

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ABSTRACT

Distance-dependent competition measures of different types -area overlap indices, point density, distance-weighted size functions and area potencially available - were evaluated as well as a distance-independent index. Each one of the selected indices was used in its original version and in an unilateral version in which the trees smaller than the subject tree were not considered as competitors. For area potencially available two variants with weights 2 and 4 were tested and for area overlap and distance-weighted size function indices the variants developed by Tomé and Burkhart (1989) were also considered.

The degree of intraespecific competition in the stands under study was first evaluated and its symmetry and two-sidedress were quantified. The dynamics of competition over time was also studied. With the results of these analysis the initial data set was splitted into two subsets one designed by AC (asymmetric competition) and the other by LC (light competition). The statistical analysis of the competition measures was made for the initial data set and for each one of the subsets AC and LC. From the results of this study we may conclude that simulation of competition should be based on a previous study of the status and type of competition in order to take them into account in the choice of the more adequate competition measure.

INTRODUCTION

Effects of competition on growth of individual trees have long been studied in an attempt to predict tree growth as accurately and precisely as possible. Models for predicting individual tree growth usually include competition effects through measures of stand density and/or competition measures. A wide variety of competition measures have been developed, but the evaluation of its effectiveness in predicting growth of individual trees have relied almost exclusively on its statistical properties such as simple correlations with tree

growth or significance in multiple regressions to predict tree growth (Spurr, 1962; Moore et al., 1973; Alemdag, 1978; Noone and Bell, 1980; Daniels et al., 1986; Nance et al., 1987; Tomé and Burkhart, 1989).

The purpose of the present paper is to try to evaluate how the competition measures really account for intraespecific competition. We first evaluated the degree of intraespecific competition in the stands under study and quantified its symmetry and two-sidedness. We also looked at the dynamics of competition over time. With the results of this previous study in mind we analysed the concomitant behaviour of several competition measures as well as their statistical properties for the stands under study.

METHODS

Data

The data used in this study are from a spacing study established in Eucalyptus globulus plantations. This study, consisting of five different spacings with two replicates, was established by CELBI in March 1975, using seedlings from commercial seed. At least two buffer strips separate the plots. All plots are stem-mapped. Each plot was first measured at 71 months (5.9 years) and was later remeasured at 94, 108, 119, 130, 143, 156 and 168 months. Diameter at breast height (dbh) was measured for each tree and dead trees were recorded; heights were measured for the 10% largest (in dbh) trees in all remeasurements and in a bigger sample in the last two remeasurements. A characterization of these plots is contained in Table 1.

TABLE 1. Characterization of plots used in the study.

Plot	Area (m^2)	N. trees at planting	N. trees at 168 months	Spacing (m)	Site index (base age 10)
			replicate 1		
P1	1584	264	231	2 x 3	22.0
P3	2160	240	215	3 x 3	21.5
P5	2160	180	164	3 x 4	22.0
P7	2016	126	122	4 x 4	20.4
P9	1600	80	77	5 x 4	20.8
			replicate 2		
P2	2160	360	340	2 x 3	20.5
P4	2187	243	235	3 x 3	21.7
P6	2304	192	186	3 x 4	22.8
P8	2464	154	148	4 x 4	22.3
P10	2280	114	111	5 x 4	23.7

Competitive status of the stands

As the plantations were relatively young and planted at wide spacings it was important to be sure that intraespecific competition was already occuring. In order to detect if competition was present we examined: (a) the relationship between size of average individual and stocking density (-3/2 thinning rule); (b) the relationship

between size of the average individual and initial density at successive dates from planting (C-D effect); (c) the frequency distribution of individual plant sizes in stands of the same age but different initial densities.

The relationship between relative growth rate (RGR) - one year basal area growth relative to initial basal area - and social status of the tree - size class - on successive ages was observed for all the stands in order to have an indication of the intensity of competition and quantify its asymmetry. The relationship between quadratic mean dbh of dominants (100 largest in dbh trees per ha) and initial density at successive dates from planting helped us to assess whether two-sided competition was occurring. With the results of these analysis the initial data set was splitted into two subsets, one designed by AC (asymmetric competition) in which competition was considered to be strongly asymmetric (small trees with smaller RGRs than large ones) and the other designed by LC (light competition) with the remaining data.

Competition measures

Numerous competition measures have been used to quantify effects of competition on growth of individual trees. According to the terminology of Munro (1974) competition measures can be classified in distance-independent and distance-dependent. While the last incorporate, in some way, the number, dimensions and location of certain neighbors, the first are simply functions of stand-level variables and initial dimensions of the tree. Detailed descriptions of the existing indices can be found in several studies (e.g. Daniels 1976, Alemdag 1978, Ottorini 1978, Daniels et al., 1986).

Distance-dependent competition measures of different types - area overlap indices, point density, distance-weighted size function indices and area potencially available - were evaluated in this study as well as a distance-independent index, the tree basal area relative to mean stand basal area (Rbm). An analysis of the results in Tomé and Burkhart (1989) that, using the same data (except for the last two remeasurements), compared several variants of competition indices in each one of the above mentioned types, allowed the selection of four indices for further study - AO, DR, PD and APA (table 2). Each one of the selected indices was used in its original version and in an unilateral version in which the trees smaller than the subject tree were not considered as competitors. As the philosophy of area potencially available was not adequate to obtain an unilateral version, we evaluated two APA variants with weights 2 and 4 that discriminate in a different way between small and big trees. For area averlap and distance-weighted size function indices the variants developed by Tomé and Burkhart (1989) were also considered. These modified indices are sums of positive and negative values, according to the dominance-supression relationships between the subject tree and each neighbor; dominant neighbors give a positive contribution to the index while the influence of supressed neighbors subtract from the index. They can be considered a measure of local dominance of the subject tree.

Table 2. Distance-dependent competition measures evaluated

1. Distance-independent indices

$$Rbm = b/bm$$

2. Area averlap indices

$$AO_i = \sum_{j=1}^{n} (ao_{ij}/AI_i)\ R_{ji}$$

$$UAO_i = \sum_{j=1}^{n1} (ao_{ij}/AI_i)\ R_{ji}$$

$$MAO_i = \sum_{j=1}^{n1} (ao_{ij}/AI_i)\ R_{ji} - \sum_{j=1}^{n2} (ao_{ij}/AI_j)\ R_{ij} - \sum_{j=1}^{m} (ao_{ij0}/AI_{j0})\ R_{j0}$$

3. Point density

$$PD_i = 2500/n \sum_{j=1}^{n} (j+0.5)\ (d_j/D_{ij})^2$$

$$UPD_i = 2500/n1 \sum_{j=1}^{n1} (j-0.5)(d_j/D_{ij})^2$$

4. Distance-weighted size function indices

$$DR_i = \sum_{j=1}^{n} R_{ji}(1/D_{ij})$$

$$UDR_i = \sum_{j=1}^{n1} R_{ji}(1/D_{ij})$$

$$MDR_1 = \sum_{j=1}^{n1} R_{ji}(1/D_{ij}) - \sum_{j=1}^{n2} R_{ij}(1/D_{ij}) - \sum_{j=1}^{m} R_{ij0}(1/D_{ij})$$

$$MDD = \sum_{j=1}^{n} (d_j - d_i)(1/D_{ij}) + \sum_{j=1}^{m} (d_{j0} - d_i)(1/D_{ij})$$

5. Area potentially available

$$w_2 = d_i^2/(d_i^2 + d_j^2) \longrightarrow APA_2$$

$$w_4 = d_i^4/(d_i^4 + d_j^4) \longrightarrow APA_4$$

where b is tree basal area; bm is mean basal area; d_i is tree dbh; ao_{ij} is the overlap area between subject tree i and competitor j; AI_i = 0.25 dbh is the area of influence of tree i; $R_{ij} = d_i/d_j$; D_{ij} is distance between subject tree i and competitor j; d_{j0} = min ($d_0/(2ny)$, dmin) where d_0 is the tree dbh before dying, ny is the number of years since it died, and dmin is the dbh of the smallest tree in the stand; n is the number of competitors; n1, n2 and m are the number of dominant, supressed and dead neighbors; w_i are the weights used in the determination of bissection points to intertree lines. In the designation of the indices U stands for the unilateral version and M for the Tomé and Burkhart (1989) modified version.

Statistical analysis

Several procedures were used to evaluate the effectiveness of the competition measures under study to predict growth of individual trees:

1. Analysis on a spacing x age basis. It is convenient to study the effectiveness of distance-dependent competition measures (influence of neighbors) while holding all other influences, such as tree initial characteristics and general environment of competition, as constant as possible. With this in mind Tomé and Burkhart (1989) computed simple correlations between tree growth and each competition index on a spacing x age basis. In this way they tried to control most of the factors that influence growth of trees, but initial characteristics of each tree were left free. In the present study partial-F tests were used to evaluate the contribution of each index to multiple regressions, fitted on a spacing x age basis, in which initial tree basal area was already present.

2. Analysis of the whole data set. The most important predictor of how a tree grows is its own size (Perry, 1985), because past competitive interactions are integrated in current tree size and also because variability is introduced as a consequence of genotypic differences in response to competition and of environmental heterogeneity (Cannell et al. 1977). A second evaluation of the competition measures was then based on the significance of its contribution to a multiple regression in which initial tree size was present through tree basal area. Since most tree growth models include also some stand density measure, the competition indices should also be compared in the presence of tree size and stand density measures. Based on the results of Tomé and Burkhart (1989) the inverse of stand basal area was chosen as a density measure.

3. All possible regressions. All the competition measures initial tree basal area and the inverse of stand basal area were used in an all possible regressions algorithm in order to select the best linear models with 4, 3, 2 and 1 regressors to predict tree growth. The models were evaluated using several measures of multiple linear regression quality: R^2, residual mean square (RMS), adjusted-R^2 and PRESS-statistic. The inexistence of strong colinearity was also considered as a desirable characteristic of the models.

In all these procedures tree basal area growth was used as the dependent variable because from a biological standpoint it has undoubtly a better relationship with tree growth than dbh growth.

The last two analysis were carried out for the whole data set and for each one of the subsets AC and LC. In all the three subsets all trees considered as border trees for at least one index were excluded, as well as two points that showed a significant value of R-student (at the 0.001 level) in the multiple regression with initial tree basal area and the inverse of stand basal area.

RESULTS AND DISCUSSION

Competitive status of the stands

The relationship between the mean tree basal area and the surviving density in log scales is given in figure 1 for both replicates. As can be seen none of the stands has already began to suffer density-dependent mortality. Comparision of stands of different initial densities show that, at the same age, they differ substantially in mean tree basal area. These fact, consequence of competition, suggests that these stands will arrive at the thinning line at different times. We may conclude that, although the stands do

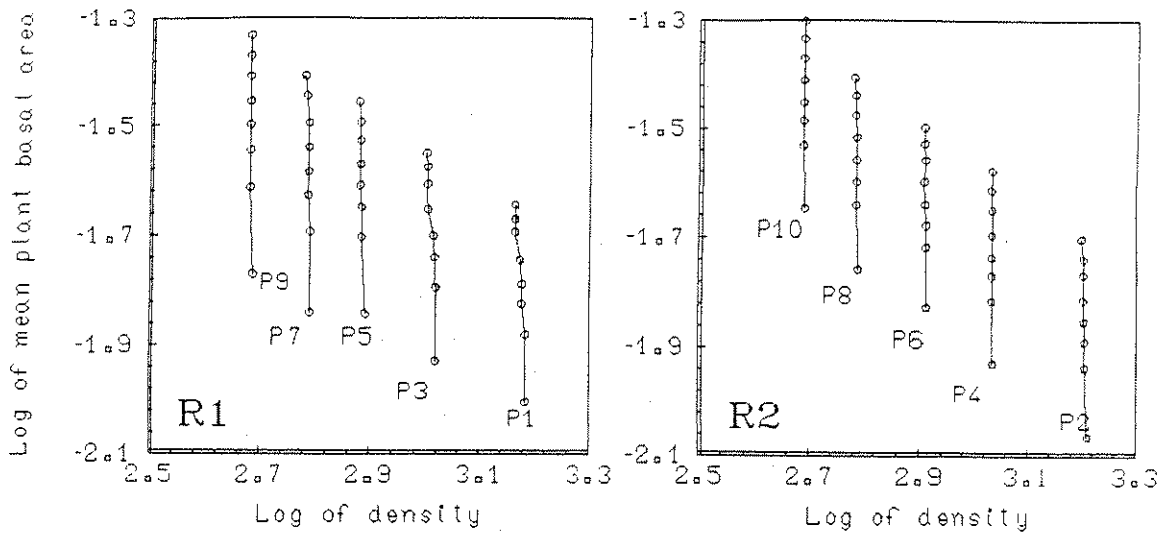

Figure 1. Relationship between size of average individual and stocking density on log scales (R1 - replicate 1; R2 - replicate 2)

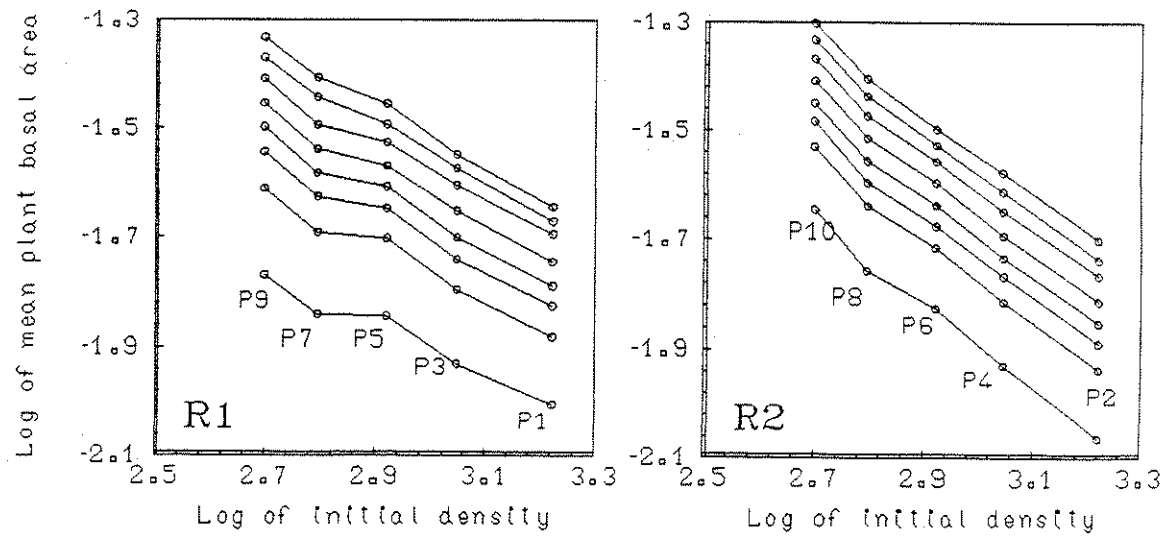

Figure 2. Relationship between size of average individual and initial density log scales at sucessive dates from planting (R1 - replicate 1; R2 - replicate 2)

not suffer mortality from crowding, competition is present. The same conclusion may be drawn from the analysis of figure 2. This figure shows how the stands go through a series of competition-density effects as they grow. If competition was not present at all, each one of the lines in figure 2 would have a zero slope; if density--dependent mortality was present at a certain point in time the line for that time point would have a -1 slope. Slopes for the last remeasurements are -0.59 and -0.74 for replicates 1 and 2 respectively.

Frequency distributions of individual plant diameter are all negatively skewed usually with a minimum in some of the younger ages and approaching to zero as time goes on. Similar results were found by Gates et al. (1983) in Pinus radiata plantations that changed from zero (normal) to negative and back to positive within 25 years and for Pseudotsuga menziesii stands in the Oregon Cascades by Perry (1985). Observation of the diameter distributions in stands of the same age but different initial densities showed that skewness is usually higher in the wider spacings. It can also be seen that the wider spacings present leptokurtic distributions (k>3) and that all

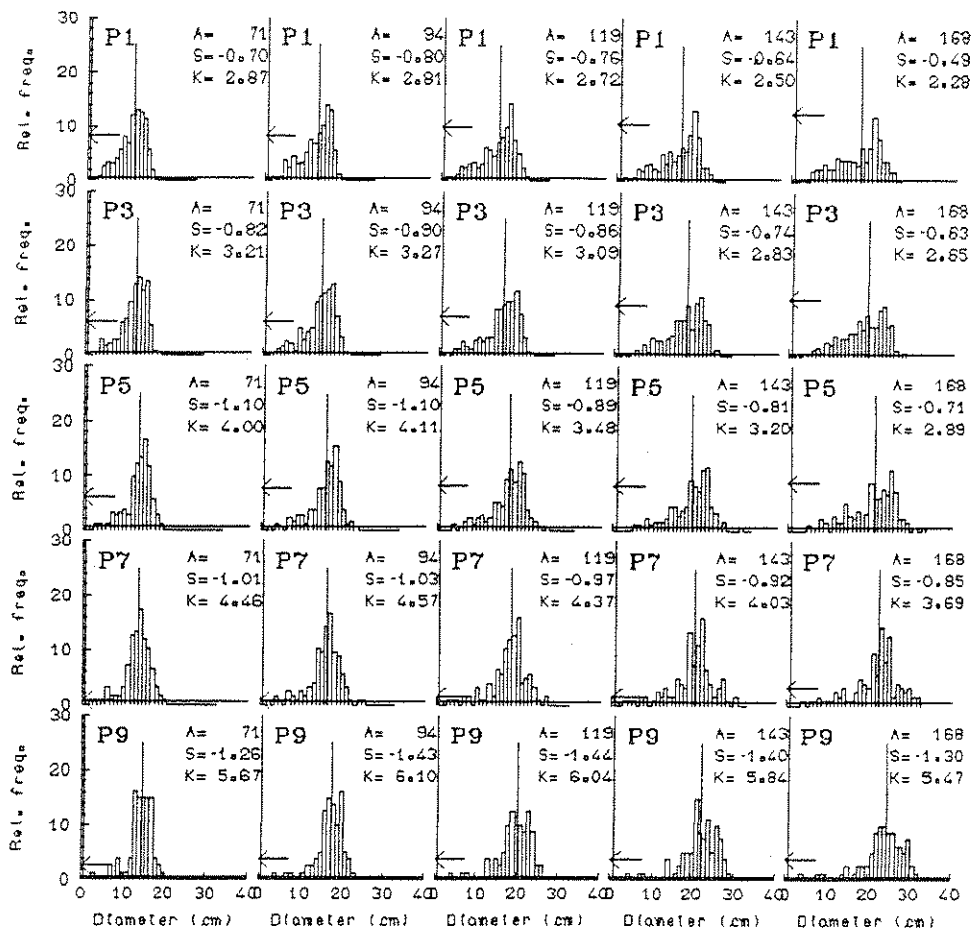

Figure 3. Frequency distributions individual plant sizes in stands of the same age but different densities for replicate 1 (A - age; S - Skewness; K - Kurtosis)

the distributions are more flat-topped as time goes on. Distributions at older ages are more broad that at younger ages as a consequence of supression of smaller plants. Figure 3 exemplifies these findings for replicate 1. Results in replicate 2 are similar although less evident probably because it is less homogeneous in what concerns site index.

Figure 4 shows the relationships between mean RGRs (2 cm diameter classes) and social status of the tree at successive ages for the stands of replicate 1. Results for replicate 2 are similar. In the younger ages smaller trees have larger RGRs than large ones which

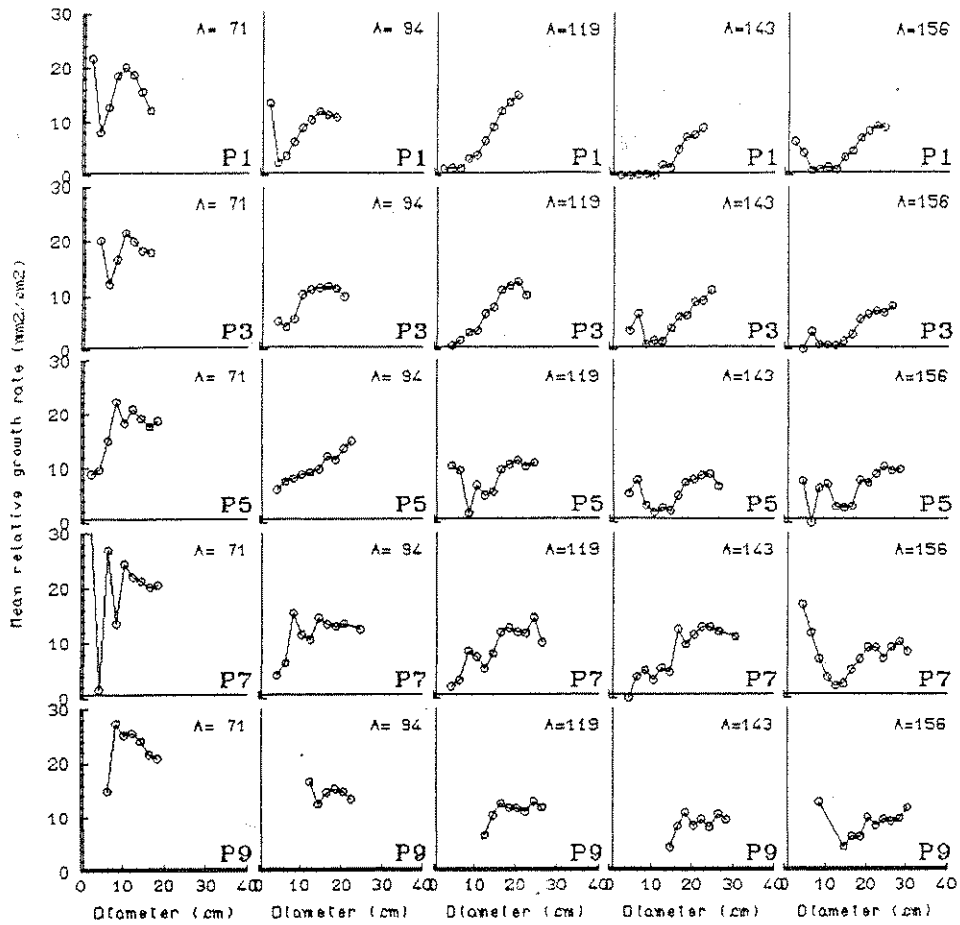

Figure 4. Relationship between RGRs (mean RGR for 2cm diameter classes) and diameter at sucessive ages for the stands of replicate 1 (A - age).

means that smaller trees are not beeing supressed; competition, if present, is surely in its early stages. As time goes on trees in the lower diameter classes begin to be clearly supressed after a period of time in which RGRs differ little among diameter classes. Supression of small trees is noted earlier in the higher density stands. In order to split the initial data set into the subsets AC and LC linear regressions were fitted between the RGRs and diameter on a plot X age basis. A plot X age set was classified as AC if it

presented a positive slope significantly different from zero ($p>0.001$). From the 3050 data points in the initial set 2271 were included in the AC subset and the remaining 779 in the LC subset. The significance level was selected according to a subjective classification based on the analysis of the plots of mean RGRs on diameter. The relationship between quadratic mean dbh of dominants and initial density at successive dates from planting (figure 5) leads to the conclusion that although competition is asymmetric, it is not completely unilateral because the larger trees (the dominants) are smaller in the stands with higher densities at all points in time and differences increase with time.

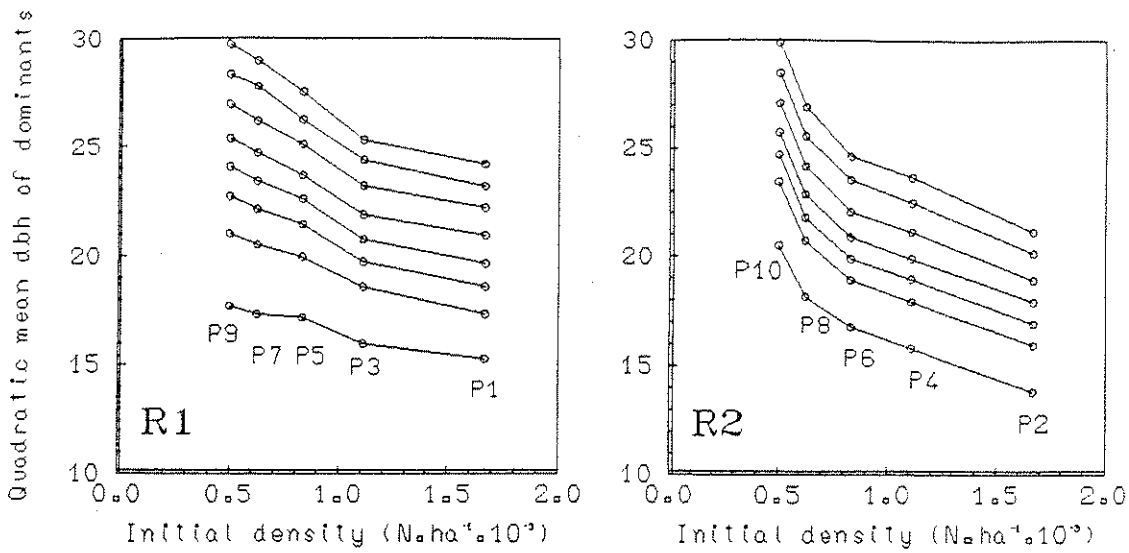

Figure 5. Quadratic mean dbh of dominants and initial density at sucessive dates from planting (R1 - replicate1; R2 - replicate 2)

Statistical analysis

Analysis on a spacing X age basis

The contributions of the several competition indices to a multiple regression, fitted on a spacing X age basis, in which initial tree basal area was already present were disappointingly low, most of the times not significant at the 0.05 level. Only at the first age most of the indices gave a significant contribution to the linear model. The unilateral version of point density (UPD) and the APA_4 showed the best results beeing significant respectively in 50% and 43.7% of the cases. An explanation for these results may be that past competitive interactions are integrated in current tree size (Perry, 1985); in a stand occupying a uniform site and before heavy mortality occurs tree size may be highly correlated with competition measures which will have in that case non-significant contributions to a linear model if tree size is already present.

Analysis on the whole data set

As can be seen in table 3 the contribution of a competition measure to the linear models under consideration was always significant both in the initial data set and in the two subsets AC and LC. Table 4 gives the ranking of the competition measures in each of the three data sets according to their contribution to tree basal area growth models.

Table 3. Contribution of the competition measures to tree basal area growth models (partial-F's for the competition term)

COMPETITION MEASURE	Independent variables in the model					
	b			b, 1/Bha		
	data set			data set		
	initial	AC	LC	initial	AC	LC
RBM	603.22	378.70	153.43	438.68	311.95	162.80
DR	520.46	117.45	129.87	22.77	6.55*	36.27
UDR	671.24	269.33	151.41	186.89	127.13	178.59
MDR	387.82	282.01	77.41	456.38	307.20	154.09
MDD	468.87	326.39	125.87	481.02	334.12	169.02
PD	802.27	248.97	215.66	28.21	41.91	29.52
UPD	1150.51	576.26	191.11	389.93	350.74	89.21
AO	685.58	229.27	179.60	96.12	68.64	61.77
UAO	80.96	158.32	10.76	325.47	260.42	49.83
MAO	383.97	293.13	64.38	420.10	309.24	103.90
APA2	727.35	220.13	256.73	66.56	56.34	74.64
APA4	958.00	501.93	210.90	275.55	296.41	95.93

Where b is tree basal area; 1/Bha is the inverse of stand basal area; and the competition measure symbols are as in table 2.
* - significant at 0.05 level; other values are significant at 0.01 level.

The rankings of the several competition measures in each data set depend heavily on the tree and stand variables that are already in the model. For instance in the initial data set point density is 3rd if just tree basal area is present and it is almost the worst if the inverse of stand basal area is in the model. This is a consequence of its high correlation with stand basal area. In fact the multicolinearities between the competition measures and other variables present in the model make the analysis completely dependent on the linear models under consideration. However some interesting conclusions can be drawn from the analysis of the rankings in the subsets AC and LC. In the first subset (stands with asymmetric competition) the unilateral and modified version of each competition measure behaved better than its original versions in both linear models. In the subset LC initial versions of the indices are best when tree basal area is the only variable in the linear model but the results are difficult to interpret when both the tree basal area and the inverse of stand basal area are present.

It is interesting to note that the distance-independent competition measure Rbm is ranked most of the times between the three best competition measures.

Table 4. Ranking of the competition measures according to their contribution to tree basal area growth models

Independent variables in the model					
b			b, 1/Bha		
data set			data set		
initial	AC	LC	initial	AC	LC
UPD	UPD	APA2	APA2	UPD	MDD
APA4	APA4	PD	PD	MDD	Rbm
PD	Rbm	APA4	APA4	Rbm	MDR
APA2	MDD	UPD	UPD	MAO	MAO
AO	MAO	AO	AO	MDR	APA4
UDR	MDR	Rbm	Rbm	APA4	UPD
Rbm	UDR	UDR	UDR	UAO	UDR
DR	PD	DR	DR	UDR	APA2
MDD	AO	MDD	MDD	AO	AO
MDR	APA2	MDR	MDR	APA2	UAO
MAO	UAO	MAO	MAO	PD	DR
UAO	DR	UAO	UAO	DR	PD

Where b is tree basal area; 1/Bha is the inverse of stand basal area; and the competition measure symbols are as in table 2.

All possible regressions

The "best" models selected for the initial, AC and LC data sets with the all possible regressions algorithm are shown in table 5. The results are clearly different for the two subsets. The best model with 4 variables for the AC subset presents the tree and stand density measures and two competition measures APA4 and MDD. Although two competition measures are present colinearity is acceptable which means that none of the indices by itself explains competition but that they are complementary. Based on its form of expression and method of calculation, APA may be regarded as a "growing space" (Moore et al., 1973) while MDD may be interpreted as measuring the dominance of the tree. Both indices seem adequated to express asymmetric competition. In the same subset the best model with 3 variables discards the stand density measure and again MDD and APA4 appear to express competition. In the model with 2 variables the unilateral version of point density appears as the best competition measure.

Tree initial dimension is never present in all the models selected for the LC data set. This result is in conformity with the analysis on a spacing x age basis: when competition is at its early stages the dimension of the tree does not seem to be a good predictor of its growth as it becomes later on, being its dominance (measured in the models by Rbm and MDR) more important in determining its ability to grow. Note that in that case area potentially available still is present in all the models but in the version APA2 that makes less difference between small and big trees which agrees with the characteristics of the subset.

The results for the initial data set are similar but the competition measures that are included do not show any consistency as for the two subsets; for instance the model with 2 variables includs

APA2 while the others include APA4.

Finally, the best independent variable to explain the basal area growth is area potentially available in the version APA4 if competition is asymmetric and in the version APA2 for the early stages of competition.

Table 5. Tree basal area growth models selected with an all possible regressions algorithm

Data set	variables in the model	R^2	adj-R^2	RMS 10^6	PRESS 10^6	APRESS 10^3	VIF
models with 4 variables							
INITIAL	b, 1/Bha, MDD, APA4	0.685	0.684	0.619	0.621	0.588	4.84
AC	b, 1/Bha, MDD, APA4	0.705	0.704	0.618	0.620	0.585	5.65
LC	1/Bha, Rbm, MDR, APA2	0.639	0.636	0.546	0.551	0.557	6.04
models with 3 variables							
INITIAL	b, 1/Bha, MDD	0.672	0.672	0.644	0.645	0.602	1.92
AC	b, MDD, APA4	0.699	0.699	0.630	0.632	0.591	1.96
LC	1/Bha, Rbm, APA2	0.634	0.632	0.552	0.556	0.561	1.60
models with 2 variables							
INITIAL	MDD, APA2	0.651	0.650	0.686	0.687	0.619	1.54
AC	b, UPD	0.688	0.687	0.653	0.654	0.603	1.69
LC	Rbm, APA2	0.592	0.590	0.615	0.617	0.598	1.70
models with 1 variable							
INITIAL	APA4	0.603	0.603	0.779	0.780	0.676	1.00
AC	APA4	0.647	0.647	0.738	0.739	0.655	1.00
LC	APA2	0.547	0.546	0.681	0.684	0.643	1.00

Where PRESS and APRESS values are divided by the number of data points in each data set in order to make results comparable; VIF is the largest variance inflation factor; and other variables are as before

C O N C L U S I O N

As an overall conclusion of this study we may say that there is still a lot of work to be done to improve the simulation of intraspecific competition.

The data that we used are not for sure the more adequated to study competition. The stands were in an early stage of competition although a severe supression of small trees was evident for the last remeasurements. Anyway they allowed us to conclude that simulation of competition should be based on a previous study of the status and

type of competition present in order to take them into account in the choice of the more adequate competition measure and in the selection of the tree growth model.

From all the competition measures evaluated area potentially available seems to express better the complex competitive process and be able by modifying the weight to simulate competition with different levels of asymmetry. Measures of the dominance are also important to complement the information on "growing space".

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FRACTAL ANALYSIS OF CROWN STRUCTURE

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Summary

Tree crown is difficult to describe in terms of classical geometry because it is a neither three-dimensional volume, nor a two-dimensional surface. Rather, the crown is a fractal, a hybrid of surface and volume formed by a convoluted and disjunctive photosynthetic layer. Each crown has its own fractal dimension. This dimension describes the overall distribution of foliage within a crown and can be broken down into structural and functional components. These components refer to two reasons for the presence of foliage inside the crown: (1) its ability to function in low light, or self-tolerance of foliage (functional depth); (2) the abundance of light due to widely separated branches, or self-avoidance of foliage (structural depth). The concept of the field of crown depth is introduced to help visualize the measurement of functional and structural components of fractal dimension.

Keywords: fractal dimension, functional crown depth, structural crown depth, self-tolerance, tolerance.

Fractal dimensions of crowns

Tree productivity is closely related to the distribution of light and the pattern of carbon utilization within the crown. These processes are determined by the structure of tree crowns. For this reason, crown structure is included as a major component in many models of tree and forest growth.

The evaluation of tree growth and productivity requires repeated measurements or coring. In contrast, crown structure can

be grasped instantly by direct observation. This makes crown structure an attractive indicator of tree productivity. This instant grasp, however, is qualitative and cannot be utilized in modelling. To use crown structure for predicting tree growth, one has to describe this structure quantitatively. It is not an easy task. Overall crown shape can be approximated by geometrical solids such as cones, ellipsoids, spheres, and their combinations. Because the actual surface of a tree crown is not the surface of the convex hull that envelopes it, these approximations are of limited use. The actual surface is composed of discrete units (leaves) situated at varying distances from the hull's surface. Each leaf or needle, no matter how deeply located in the midst of the crown, is a part of the crown surface. Otherwise, the leaf would not exist. In principle, it is possible to measure the area, position, and angle of each leaf. Such a quantification would produce an unmanageable amount of data, hardly suitable for modelling. We need much more concise measures of crown structure.

Fractal dimension

Fractal geometry (Mandelbrot 1983) provides the concepts and tools needed to condense information on crown structure into a few meaningful numbers. One of the key concepts is fractal dimension. It is a generalization of spatial dimension known from classical geometry. Length, area, and volume of smooth lines, rectangles, and cylinders, or their approximations which we see in our buildings, lumber, and most man-made things, do not change with the unit of measurement. Their spatial dimensions are integers: one for lines, two for areas, and three for volumes. It seems obvious that any line has one dimension, length. It is equally evident that two dimensions, length and width, are required to define area. It is also clear that volumes have all three dimensions, length, width, and height. It looks improbable that these basic ideas need any revision. As far as the dimension of a line is concerned, what else besides length could be detected?

However obvious, these concepts of classical geometry do not work for many natural objects such as tree crowns, coastlines, or boundaries between forest types. A common feature of these objects is self-similarity. Each bay of a coastline has smaller bays, which in their turn contain yet smaller bays. The idea that the structure of plants is also based on the repetition of a basic unit was expressed by Darwin (1859, p. 437): "... in flowering plants, we see a series of successive spiral whorls of leaves. An indefinite repetition of the same part or organ is the common characteristic (as Owen has observed) of all low or little-modified forms."

There are several ways to ascertain self-similarity of the tree crown. The density of foliage decreases toward the crown's center. The ratio of the densities in adjacent layers of the crown probably remains constant. Thus, in some ways, trees are self-similar in the manner of Russian dolls which reside one

inside another. Self-similarity of crowns can also be derived from the modular construction of trees (Halle et al. 1978). Trees, like most plants, are composed of a hierarchy of modules. For a tree a module is a single shoot, or unramified branch. Each branch resembles branches in the higher and lower orders, as well as the entire tree centered around the stem. In any case of self-similarity, each piece of an object is similar to the whole.

Self-similarity implies that natural lines do not have unique invariable lengths. Smaller units of measurement reveal finer details of a curve and, within a certain range of units, produce a greater length for the same curve. The relationship between the length and measurement unit is determined by an analog of spatial dimension, called fractal dimension, which is specific for each line. Fractal dimensions of natural lines are greater than one and the excess indicates the degree of the lines' convolution. Similarly, fractal dimensions of natural surfaces are greater than two (the spatial dimension of areas in classical geometry) but less than three. Mathematically, self-similarity leads to a power relationship between size and measurement unit:

$$S = ar^{E-D} \tag{1}$$

where S - size of a fractal object (length, area, or volume),
r - unit of measurement,
E - Euclidean dimension (an integer, 1, 2, or 3),
D - fractal dimension,
a - constant (prefactor).

Methods for determining fractal dimensions

The standard method for determining fractal dimensions, the box-counting method, requires slicing a given object into many layers without distortion of its structure, subdividing them into cubic boxes, and counting the number of nonempty boxes. This procedure is repeated many times using various box sizes. The fractal dimension of the studied object is one of the parameters of the relationship between the number of boxes and their size. Despite its many theoretical advantages, it is all but impossible to apply this method to trees.

"The two-surface method" (Zeide 1990) for determining fractal dimensions was designed specifically for tree crowns. The method is based on the assumption that the relationship between leaf area of a tree, A, and surface area of a convex hull that envelopes its crown, C, contains a parameter that can be identified as a fractal dimension of crown surface, D. This relationship is

$$A = aC^{D/2} \tag{2}$$

where a is a constant. Because leaves have a fixed thickness, their area is proportional to the foliage mass, which can be measured more easily and reliably than area. Procedures for

estimating foliage mass and surface area, C, are well-established in biomass studies. Therefore, the proposed method presents no technical problems.

The results produced by the two-surface method showed that tree crowns are indeed self-similar. Fractal dimensions for dominant trees of ten conifer species and intermediate trees of four species varied between two and three (more precisely, from 2.21 for intolerant ponderosa pine (Pinus ponderosa)to 2.81 for very tolerant hemlock (Tsuga heterophylla)), that is, within the theoretical limits of two and three. A value of three indicates that needles are uniformly distributed throughout the crown volume and their mass is proportional to the volume. This would be possible only if needles were extremely tolerant to low light. A dimension of two could be realized in the opposite situation when needles would not tolerate any shading and would be located exclusively on the crown periphery (so that their mass and surface are proportional to the surface of the hull).

These reasons suggest some correlation between fractal dimensions and species tolerance which was empirically detected for the ten conifer species.

Primary unit of tree crowns

The photosynthetic surface of a tree is organized into discrete units such as chlorophyll molecules, chloroplasts, cells, leaves (or needles), fascicles of needles, and branches of various order. In ecological literature, a module, represented by a shoot on a tree, seems to gain preference as the basic unit of structure (Halle et al. 1978, Gill 1986, Waller 1986). Harper (1984) refers to all plants and some animals (the hydroids, bryozoans, corals, etc.) as 'modular organisms'. Modules usually consist of smaller structural units, metamers (White 1984). In plants, each metamer is made up of a leaf, axillary buds, and the associated internode.

In the past, the selection of a unit was often determined by the specialty of a researcher and was somewhat arbitrary. With the help of fractals, the size of the unit can now be calculated. The two-surface method allows one to determine fractal dimensions without using the box-counting method and equation (1). When the dimensions and foliage mass are known, this equation can be used to calculate the unit size, r. Preliminary calculations for ten investigated conifers showed that the average mass of the unit was about 0.02 g, which is close to the mass of a single needle. This result suggests that the unit might vary among species and even within a crown since the size of leaves changes predictably within the crown.

Perhaps the mass of a leaf does not represent a unique "natural crown unit." If, instead of leaf area (represented by foliage mass), the unit is calculated on the basis of the surface area of chloroplasts, the result would be different. Still, because foliage mass is a basic and unique crown characteristic, the corresponding unit is of special significance.

Functional and structural crown depths

Considering fractal dimensions of crowns, two related questions arise: (1) Is species tolerance sufficient to explain its fractal dimension? (2) Can one number (fractal dimension) characterize distribution of foliage within a tree crown?

Looking at crowns, it appears that foliage distribution results from two kinds of adaptation, functional (or physiological) and structural. The same high fractal dimension of surface can be achieved by the ability of foliage to function in a wide range of light intensities (as evidenced by tolerant species) and by structural adaptation (such as a permeable crown). Deep penetrating cavities in the crown let the sunlight in and result in a substantial density of foliage inside the crown of even intolerant species.

Accordingly, we can introduce the concepts of functional depth and structural depth of the crown. Foliage is present inside functionally deep crowns because of its ability to function in low light. In dominant trees, functional depth is an indication of tolerance of foliage. Foliage within structurally deep crowns exists because of the abundance of light entering between widely separated branches. Both kinds of depth distinguish crown surface from the flat, two-dimensional surface of Euclidean geometry.

These arguments suggest that the questions posed at the beginning of this section should be answered in negative. Because fractal dimension reflects the combined action of two kinds of adaptation, tolerance alone is not a sufficient explanation of a particular fractal dimension. At least two numbers are required to characterize distribution of foliage within tree crowns, functional and structural depths of crowns.

Measuring crown depth

The concept of functional and structural depths implies that a crown's fractal dimension, or rather its excess over Euclidean dimension, D - E, has two components, functional and structural. This poses the problem of breaking down the fractal dimension and measuring each component separately. As with determining fractal dimension, two groups of methods could be used to measure functional and structural depths of crowns: general methods suitable for any object and methods specifically designed for trees.

Lacunarity as a measure of structural depth

General methods developed in fractal geometry for measuring lacunarity might be suitable for estimating structural depth. Mandelbrot (1983, p. 310) coined the term "lacunarity" (from Latin for gap) and proposed several measures of it in order to describe an object's texture, that "elusive notion which mathematicians and scientists tend to avoid because they cannot grasp it. Engineers and artists [and, we can add, foresters] cannot avoid it, but mostly fail to handle it to their satisfaction." Lacunarity of an object is high if its gaps tend to be large. This parameter is distinct from fractal dimension.

For a broad class of fractals lacunarity can be measured by the prefactor a of equation (1). The outer cutoff can also serve as a measure of lacunarity. The outer cutoff and inner cutoff refer to two critical points on the line representing the relationship of the measurement unit and the resulting size of a fractal object. These points indicate the ends of the straight segment (on the log-log scale) of the line, that is, the range where the object is self-similar.

Analysis of the distribution of gaps, as they appear on a two-dimensional projection (such as a photograph) of an object, is another way to estimate lacunarity. The rolling disk method, described by Peter Pfeifer (1988), can be used to obtain the distribution of gaps. A disk is placed on a photograph or on a digitized image of a crown so that it touches the crown border at one point. The disk is then "rolled" until it touches two points simultaneously. Here we stop rolling the disk and calculate the area of the "bay" bounded by the disk and foliage. Then we resume rolling the disk until it closes the next bay, calculate its area and add it to the previous bay area, and so on until we return to the original point. Repeating this procedure for disks of various radii, we obtain the distribution of the bay areas which can be used to estimate lacunarity and fractal dimension.

Normalized foliage mass and functional depth

While the structural depth can be estimated by geometrical methods, the functional depth deals with the ecophysiology of trees and its estimation requires some ecological reasoning. Because, by definition, a functionally deep crown has a smaller volume of cavities, one can assume that, other things being equal, such a crown should have a larger foliage mass than a structurally deep crown. To make comparison possible, crowns with fixed volume and stem diameter at the crown base are considered. The logarithm of foliage mass in such a crown will be referred to as the normalized mass. Normalized mass is a direct measure of functional depth for trees with identical fractal dimensions. In general, functional depth depends on both normalized mass and fractal dimension.

The assumption that the normalized foliage mass might be an ecological indicator of functional depth and, therefore, of species tolerance is supported by research. Parker and Long

(1989) tested eight commonly cited indicators of relative tolerance such as stand density, bole taper, and foliage retention time. These indicators were evaluated by their ability to discriminate between populations of various species differing in tolerance, and between populations of a given species at different stages of succession, as well as by the collective ability to make these inter- and intraspecific distinctions. Empirical findings failed most of the common indicators so that "much of the conventional wisdom concerning relative tolerance may not be generally applicable" (Parker and Long 1989, p. 187).

This study produced positive results as well: "Of the stand and tree attributes studied, only leaf-area:sapwood-area ratio is both independent of stand history and structure. It appeared to be useful in making inter-specific distinctions on the basis of relative tolerance" (Parker and Long 1989, p. 187). Because leaf area is proportional to foliage mass and sapwood area is proportional to the stem's cross-section at crown base, the ratio singled out by Parker and Long is very similar to the normalized foliage mass.

The field of crown depth

To visualize the calculation of both kinds of crown depth, the concept of the field of crown depth is introduced. Construction of this field is based on the logarithmic form of equation (1):

$$M = k + \ln(r)(2 - D) \tag{3}$$

where M is the normalized foliage mass, D is the fractal dimension of a given species, ln(r) is the natural logarithm of the linear crown unit, and k is constant.

The field is located on a plane with fractal dimension plotted on the horizontal axis and normalized foliage mass on the vertical axis. The field is bounded laterally by lines corresponding to the extreme values of fractal dimension (two and three). From above and below the field is bounded by the upper and lower limits of the normalized foliage mass.

The standard deviation of fractal dimension and normalized mass were used to calculate these lower and upper limits for the ten investigated species mentioned earlier. The distance between the actual smallest fractal dimension (for example, D = 2.21) and the theoretical minimum of this variable (D = 2) was expressed in terms of the standard deviation of fractal dimension.

The same portion of the standard deviation of mass was subtracted from the actual smallest mass. It was assumed that this value lies on the line representing the lower limit of normalized mass. According to equation (3), the slope of this line is equal to the logarithm of crown unit, ln(r), which was found earlier. This slope and one point on the line made it possible to determine the lower limiting line of normalized mass. Analogous calculations produced the upper limiting line. The

total area of the field, A, is equal

$$A = (M_{max} - M_{min})(3 - 2) = M_{max} - M_{min} \quad (4)$$

where $(M_{max} - M_{min})$ is the height of the field and (3 - 2) is its width (Figure 1). Thus, although the field depicts fractals, it is constructed using Euclidean geometry.

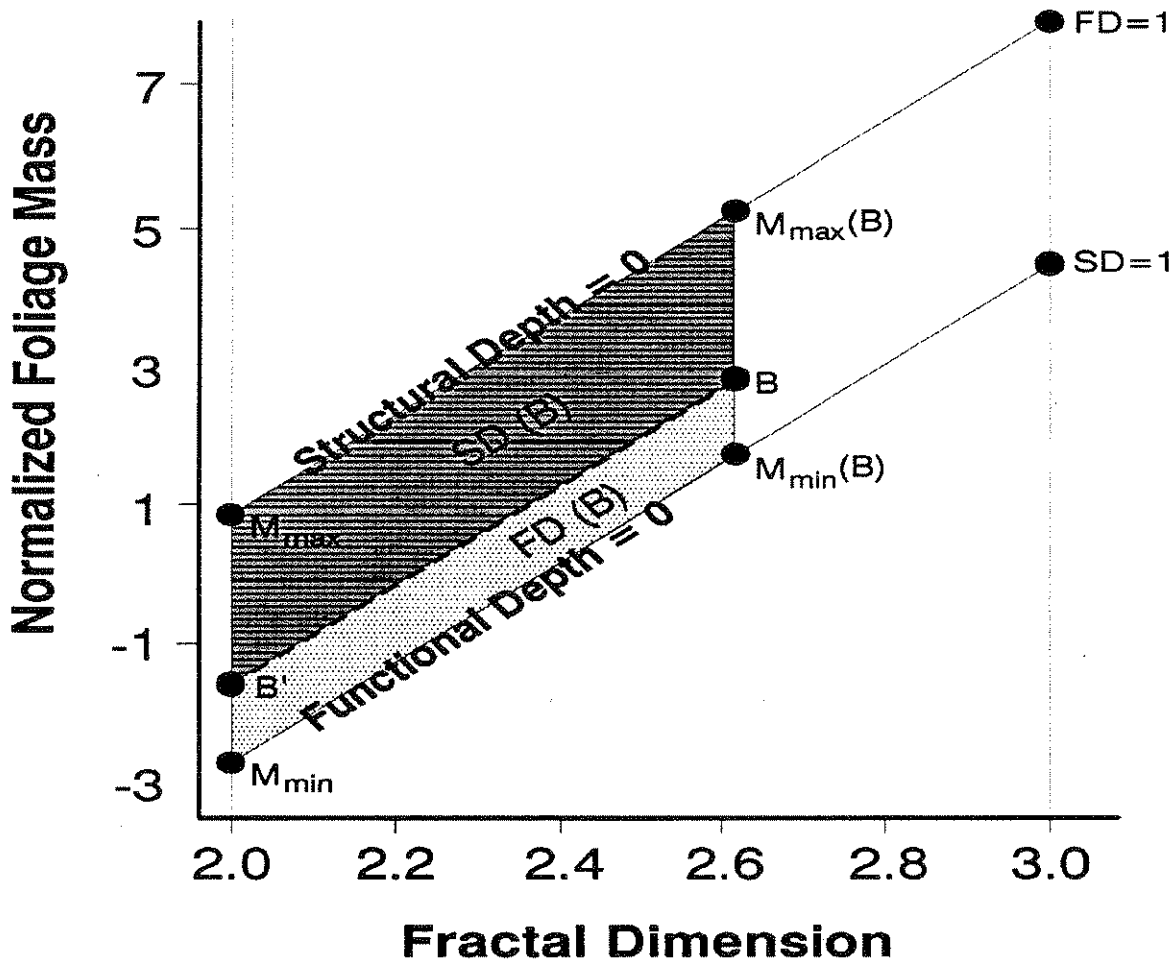

Figure 1. Construction and use of the field of crown depth.
M_{min} and M_{max} = minimum and maximum values of normalized foliage mass for the fractal dimension of two;
$M_{min}(B)$ and $M_{max}(B)$ = minimum and maximum values of normalized foliage mass for the fractal dimension of species B;
FD(B) = area representing functional depth for species B;
SD(B) = area representing structural depth for species B;
FD=1 and SD=1 are the points with the greatest depth.

It is reasonable and convenient to require that the measures of functional depth, FD, and structural depth, SD, should satisfy the following conditions:

1. At their maximum, FD = 1 and SD = 1.
2. At their minimum, FD = 0 and SD = 0.

3. When FD = 1 or SD = 1, D = 3.
4. When D = 2, FD = 0 and SD = 0.
5. Because functional and structural depths are components of fractal dimension, FD + SD = D - 2.

For a certain species B, the ratio of the area of parallelogram B-B'-M_{min}-M_{min}(B) to the total area, A, satisfies these conditions as a measure of functional depth (Figure 1). Similarly, the ratio of the area of parallelogram B-B'-M_{max}-M_{max}(B) to the total area, A, can be accepted as a measure of structural depth. If, for example, the fractal dimension of species B is D=2.63, M(B)=2.38, M_{max}(B)=5.12 and M_{min}(B)=1.17, then its functional depth is equal to:

$$FD(B) = \frac{[M(B) - M_{min}(B)](2.63 - 2)}{M_{max}(B) - M_{min}(B)} = 0.19 \qquad (5)$$

and its structural depth is:

$$SD(B) = \frac{[M_{max}(B) - M(B)](2.63 - 2)}{M_{max}(B) - M_{min}(B)} = 0.44 \qquad (6)$$

Discussion

The concepts of fractal geometry appear to be effective in the analysis of structure of tree crowns. By removing the rigid division of areas and volumes, fractal geometry offers a totally new insight: the tree crown is neither a three-dimensional volume, nor a two-dimensional surface. Rather, it is a fractal, a surface-volume formed by a convoluted and disjunctive photosynthetic layer adapted to optimize light interception. Each crown has its own dimension determined by hereditary and environmental factors.

These concepts lead to a deeper understanding of crown structure which goes beyond geometrical considerations. Self-similarity is not limited to physical characteristics. Ecophysiological variables, such as the distribution of foliage sensitivity to light, can also be self-similar. Fractal dimension describes the overall self-similarity of a crown and can be broken down into structural and functional components.

Thus, fractals are essential to understanding the functioning of trees as well as their geometrical forms. For this reason the term "fractal geometry" is too restrictive. The qualifier "fractal" as in fractal ecology or fractal plant physiology would also be superfluous. Nonfractal ecology or physiology would make as much sense as nonmathematical physics. Fractal dimension exceeds the corresponding Euclidean dimension not so much in quantity as in the ability to express the inner fabric and functioning of an object. Unlike classical geometry which is reserved for pure mathematicians, fractal research

requires the participation of natural scientists along with mathematicians.

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